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Advances in the Topology of Fuzzy Neutrosophic Sets and Spaces

E. Poongothai ● F. Smarandache
(editors)

NEUTROSOPHIC SCIENCE INTERNATIONAL ASSOCIATION PUBLISHING HOUSE
NSIA

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Neutrosophic Science International Association (NSIA)
Publishing House

Gallup - Guayaquil
United States of America – Ecuador
2026

Editor:

Neutrosophic Science International Association (NSIA)
Publishing House
https://fs.unm.edu/NSIA/

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ISBN 978-1-59973-848-2

TABLE OF CONTENTS

F. Smarandache	FOREWORD	5
E. Poongothai E. Padmavathi	BAIRE SPACES ON FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES	7
E. Poongothai E. Padmavathi	ON FUZZY NEUTROSOPHIC *I*-BAIRE SPACES	18
E. Poongothai E. Padmavathi	ON FUZZY NEUTROSOPHIC DIRECTED BAIRE SPACES AND CHARACTERIZATIONS OF OTHER FUZZY NEUTROSOPHIC SPACES	28
E. Poongothai E. Padmavathi	ON FUZZY NEUTROSOPHIC SEMI-OPEN(CLOSED) SETS IN FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES	45
E. Poongothai E. Padmavathi	FUZZY NEUTROSOPHIC SEMI-BAIRE SPACES	56
E. Poongothai E. Padmavathi	FUZZY NEUTROSOPHIC DENSE SETS AND BAIRE SPACES	68
E. Poongothai E. Padmavathi	*C*-ALMOST *P*-SPACES AND OTHER SPACES ON FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES	85
E. Poongothai E. Kalaivani	NEUTROSOPHIC σ-BAIRE SPACES ON FUZZY SETTING	93
E. Poongothai E. Kalaivani	A CHARACTERIZATION OF NEUTROSOPHIC σ-BAIRE SPACE ON FUZZY SETTING	105
E. Poongothai E. Kalaivani	NEUTROSOPHIC BOUNDARY AND NEUTROSOPHIC SEMI-BOUNDARY ON FUZZY SETTING	114
E. Poongothai E. Kalaivani	SOME REMARKS ON FUZZY NEUTROSOPHIC σ-BAIRE SPACES AND OTHER FUZZY NEUTROSOPHIC SPACES	132
E. Poongothai E. Kalaivani	ON FUZZY NEUTROSOPHIC PRE σ-BAIRE SPACES	145
E. Poongothai E. Kalaivani	FUZZY NEUTROSOPHIC REGULAR σ-BAIRE SPACES	155

E. Poongothai E. Kalaivani	FUZZY NEUTROSOPHIC SETTING ON P-SPACES, WEAK P-SPACES AND ALMOST P-SPACES	170
E. Poongothai E. Kalaivani	A SHORT NOTE ON FUZZY NEUTROSOPHIC μ-STRONGLY CATEGORY SETS AND FUZZY NEUTROSOPHIC μ-STRONGLY BAIRE SPACES	180
E. Poongothai D. Vinoba	FUZZY NEUTROSOPHIC SUPRA DENSE G_δ-SPACES	196
E. Poongothai D. Vinoba	ON FUZZY NEUTROSOPHIC SUPRA SEMI-BAIRE SPACES	207
E. Poongothai D. Vinoba	ON FUZZY NEUTROSOPHIC SUPRA REGULAR BAIRE SPACES	222
E. Poongothai D. Vinoba	SUPRA RESIDUAL SETS AND SUPRA BAIRE SPACE FUNCTIONS ON FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES	233

Foreword

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This collection of research explores the evolving landscape of topology by integrating **fuzzy set theory** with **neutrosophic sets**, which account for membership, non-membership, and indeterminacy degrees. The primary focus of these works is the systematic development and characterization of **Baire spaces** within various fuzzy neutrosophic environments.

The collection addresses several advanced variations and generalizations of these spaces, including:

- **Fundamental Concepts**: The introduction of fuzzy neutrosophic F_σ-sets, G_σ-sets, and the definitions of dense, nowhere dense, and residual sets.
- **Space Categorization**: Detailed studies on first and second category sets and spaces to establish the foundational properties of fuzzy neutrosophic Baire spaces.
- **Specialized Frameworks**: Exploration of complex structures such as **σ-Baire spaces**, **Directed Baire spaces**, and **Supra Baire spaces** (including supra semi-Baire and supra regular Baire varieties).
- **Ideal and P-Spaces**: Research into fuzzy neutrosophic *I*-Baire spaces using ideal space theory, as well as characterizations of P-spaces, submaximal spaces, and resolvable spaces.

By establishing interrelations between these diverse fuzzy neutrosophic topological structures, this collection enhances the ability to model complex systems under uncertainty. These theoretical advancements provide critical support for practical applications in fields such as **artificial intelligence**, **decision-making**, **pattern recognition**, and **image processing.**

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BAIRE SPACES ON FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT

In this paper a property which can be used to Baire spaces in fuzzy neutrosophic topological spaces are introduced and studied. For this purpose, introduced fuzzy neutrosophic $F_\sigma -$ set, fuzzy neutrosophic $G_\delta -$ set, fuzzy neutrosophic dense, fuzzy neutrosophic nowhere dense, fuzzy neutrosophic first category, fuzzy neutrosophic second category and fuzzy neutrosophic residual set are defined. Also, some characterizations about these concepts are obtained.

Keywords: Fuzzy neutrosophic dense set, Fuzzy neutrosophic nowhere dense set, Fuzzy neutrosophic residual set, Fuzzy neutrosophic Baire spaces, Fuzzy neutrosophic first and second category.

1. INTRODUCTION

The concept of fuzzy sets was introduced by L.A. Zadeh in 1965 [10]. Then the fuzzy set theory is extension by many researchers. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [9] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

In this paper, we define new concepts of fuzzy neutrosophic $F_\sigma -$ set, fuzzy neutrosophic $G_\delta -$ set, fuzzy neutrosophic dense, fuzzy neutrosophic nowhere dense, fuzzy neutrosophic first and second category sets, fuzzy neutrosophic residual set, fuzzy neutrosophic Baire spaces, fuzzy neutrosophic first and second category spaces in fuzzy neutrosophic topological spaces, and we also discussed some new properties and examples based of this defined concept.

2. PRELIMINARIES

Throughout the present paper, X denote the fuzzy neutrosophic topological spaces. Let A_N be a fuzzy neutrosophic set on X. The fuzzy neutrosophic interior and closure of A_N is denoted by $fn(A_N)^+$, $fn(A_N)^-$ respectively. A fuzzy neutrosophic set A_N is defined to be fuzzy neutrosophic open set $(fnOS)$ if $A_N \leq fn\,(((A_N)^-)^+)^-$. The complement of a fuzzy neutrosophic open set is called fuzzy neutrosophic closed set $(fnCS)$.

Definition 2.1 [2]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

With the condition $0 \leq T_{A^*}(x) + \mathrm{I}_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2 [2]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$.

Definition 2.3 [2]:

Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$

$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$

Definition 2.4 [2]:

The difference between two fuzzy neutrosophic sets A and B is defined as $A \backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.5 [2]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1\ for\ all\ x \in X.$ It is denoted by 0_N.

Definition 2.6 [2]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0\ for\ all\ x \in X.$ It is denoted by 1_N.

Definition 2.7 [2]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as

$A^C = \langle x, T_{A^C}(x), \mathrm{I}_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8 [1]:

A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X

$$(i)\ \ 0_N, 1_N \in \tau$$

$$(ii)\ \ A_1 \cap A_2 \in \tau\ \ for\ any\ A_1, A_2 \in \tau$$

$$(iii)\ \ \cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i : i \in J\}\ \in \tau$$

Satisfying the following axioms.

In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any Fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [1]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10 [1]:

Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$int(A) = \cup \{G : G\ is\ a\ fuzzy\ neutrosophic\ open\ set\ in\ X\ and\ G \subseteq A\}$$

$$cl(A) = \cap \{G : G\ is\ a\ fuzzy\ neutrosophic\ closed\ set\ in\ X\ and\ A \subseteq G\}$$

3. ON FUZZY NEUTROSOPHIC NOWHERE DENSE SETS

Definition 3.1:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $F_\sigma -$ set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$.

Definition 3.2:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $G_\delta -$ set in (X, τ_N) if $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$.

Definition 3.3:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic semi-open if $A_N \leq fn(\ ((A_N)^+)^-)$. The complement of A_N in (X, τ_N) is called a fuzzy neutrosophic semi-closed set in (X, τ_N).

Definition 3.4:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic dense if there exist no fuzzy neutrosophic closed set B_N in (X, τ_N) such that $A_N \subset B_N \subset 1_X$. That is, $fn\,(A_N)^- = 1_N$.

Definition 3.5:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic nowhere dense set if there exist no non-zero fuzzy neutrosophic open set B_N in (X, τ_N) such that $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$.

Example 3.1:

Let $X = \{a, b, c\}$ and consider the family $\tau_N = \{0_N, 1_N, A_N, B_N, C_N\}$ where

$A_N = \{\langle a, 0.3,0.3,0.5\rangle, \langle b, 0.6,0.6,0.5\rangle, \langle c, 0.6,0.6,0.5\rangle\}$

$B_N = \{\langle a, 0.6,0.6,0.6\rangle, \langle b, 0.3,0.3,0.3\rangle, \langle c, 0.6,0.6,0.6\rangle\}$

$C_N = \{\langle a, 0.3,0.3,0.4\rangle, \langle b, 0.7,0.7,0.4\rangle, \langle c, 0.3,0.3,0.4\rangle\}$

Thus (X, τ_N) is a fuzzy neutrosophic topological spaces.

Now $fn(((\overline{A_N})^-)^+) = 0_N, fn(((\overline{B_N})^-)^+) = 0_N$ and $fn(((\overline{C_N})^-)^+) = 0_N$. This implies that $\overline{A_N}$, $\overline{B_N}$ and $\overline{C_N}$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N).

Definition 3.6:

Let (X, τ_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set A_N in (X, τ_N) is called fuzzy neutrosophic first category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where ${A_N}_i{}'$s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Any other fuzzy neutrosophic set in (X, τ_N) is said to be of fuzzy neutrosophic second category.

Definition 3.7:

A fuzzy neutrosophic topological space (X, τ_N) is called fuzzy neutrosophic first category space if the fuzzy neutrosophic set 1_X is a fuzzy neutrosophic first category set in (X, τ_N). That is $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$, where ${A_N}_i{}'$s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Otherwise (X, τ_N) will be called a fuzzy neutrosophic second category space.

Definition 3.8:

Let A_N be a fuzzy neutrosophic first category set in (X, τ_N). Then $\overline{A_N}$ is called fuzzy neutrosophic residual set in (X, τ_N).

Proposition 3.1:

If A_N is a fuzzy neutrosophic closed set in (X, τ_N) with $fn(A_N)^+ = 0_N$ then A_N is a Fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proof:

Let A_N is a fuzzy neutrosophic closed set (X, τ_N). Then $fn(A_N)^- = A_N$. Now $fn(((A_N)^-)^+) = fn(A_N)^+ = 0_N$. and hence A_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proposition 3.2:

If A_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N) then $fn(A_N)^+ = 0_N$.

Proof:

Let A_N be a fuzzy neutrosophic nowhere dense set in (X, τ_N). Now $A_N \leq fn(A_N)^-$ implies that $fn(A_N)^+ \leq fn(((A_N)^-)^+) = 0_N$. Hence, we have $fn(A_N)^+ = 0_N$.

Remark 3.1:

The complement of a fuzzy neutrosophic nowhere dense set need not be a fuzzy neutrosophic nowhere dense set. For, consider the following example.

Example 3.2:

Let $X = \{a, b, c\}$ and consider the family $\tau_N = \{0_N, 1_N, A_N, B_N, C_N\}$ where

$A_N = \{\langle a, 0.5, 0.5, 0.4\rangle, \langle b, 0.5, 0.3, 0.5\rangle, \langle c, 0.5, 0.5, 0.4\rangle\}$

$B_N = \{\langle a, 0.5, 0.5, 0.3\rangle, \langle b, 0.4, 0.2, 0.5\rangle, \langle c, 0.5, 0.4, 0.3\rangle\}$

$C_N = \{\langle a, 0.5, 0.4, 0.5\rangle, \langle b, 0.5, 0.3, 0.2\rangle, \langle c, 0.5, 0.6, 0.3\rangle\}$

Now $fn(((B_N)^-)^+) = 0_N$ is a fuzzy neutrosophic nowhere dense sets in (X, τ_N).

But $fn(((\overline{B_N})^-)^+) \neq 0_N$. Therefore $\overline{B_N}$ is not a fuzzy neutrosophic nowhere dense sets in (X, τ_N).

Proposition 3.3:

If A_N is a fuzzy neutrosophic dense, fuzzy neutrosophic open set in (X, τ_N) such that $B_N \leq \overline{A_N}$, then B_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proof:

Let A_N be a fuzzy neutrosophic open set in (X, τ_N) such that $fn(A_N)^- = 1$. Now $B_N \leq \overline{A_N}$ implies that $fn(B_N)^- \leq fn(\overline{A_N})^- = (1 - A_N)$ $[(\overline{A_N})$ is a fuzzy neutrosophic closed set in

(X, τ_N)] Then we have $fn(((B_N)^-)^+) \leq fn(\overline{A_N})^+ = \overline{(fn(A_N))^-} = 1 - 1 = 0_N$. and hence $fn(((B_N)^-)^+) = 0_N$. Therefore B_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proposition 3.4:

If A_N is a non-zero fuzzy neutrosophic nowhere dense set in (X, τ_N), is a fuzzy neutrosophic nowhere dense set then A_N is fuzzy neutrosophic semi-closed set in (X, τ_N).

Proof:

Let A_N be a fuzzy neutrosophic nowhere dense set in (X, τ_N). Then $fn(((A_N)^-)^+) = 0_N$. and therefore $fn(((A_N)^-)^+) \leq A_N$. Hence, A_N is fuzzy neutrosophic semi-closed set in (X, τ_N).

Proposition 3.5:

If a fuzzy neutrosophic closed set A_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N) if and only if $fn(A_N)^+ = 0_N$.

Proof:

Let A_N be a fuzzy neutrosophic closed set in (X, τ_N) with $fn(A_N)^+ = 0_N$. Then by proposition 3.1, A_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N). Conversely, let A_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N), then $fn(((A_N)^-)^+) = 0_N$, which implies that $fn(A_N)^+ = 0_N$, [since A_N is fuzzy neutrosophic closed set in $fn(A_N)^- = A_N$].

Proposition 3.6:

If A_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N), then $\overline{A_N}$ is a fuzzy neutrosophic dense set in (X, τ_N).

Proof:

Let A_N be a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Then by proposition 3.2, we have, $fn(A_N)^+ = 0_N$. Now $fn(\overline{A_N})^- = \overline{(fn(\overline{A_N}))^+} = 1 - 0_N = 1_N$. Therefore $\overline{A_N}$ is a fuzzy neutrosophic dense set in (X, τ_N).

Proposition 3.7:

If A_N is a fuzzy neutrosophic nowhere dense set and fuzzy neutrosophic open set in (X, τ_N), then $\overline{A_N}$ is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proof:

Let A_N be a fuzzy neutrosophic open sets in (X, τ_N) such that, $fn(A_N)^- = 1$. Now $fn(((\overline{A_N})^-)^+) = \overline{(fn((A_N)^+)^-)} = \overline{fn(A_N)^-} == 1 - 1 = 0_N$. Hence $\overline{A_N}$ is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proposition 3.8:

If A_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N), then $fn(A_N)^-$ is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proof:

Let $fn(A_N)^- = B_N$, Now $fn(((B_N)^-)^+) = fn((((A_N)^-)^-)^+) = fn(((A_N)^-)^+) = 0_N$.

Hence $B_N = fn(A_N)^-$ is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proposition 3.9:

If A_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N), then $\overline{fn(A_N)^-}$ is a fuzzy neutrosophic dense set in (X, τ_N).

Proof:

By proposition 3.9, we have $fn(A_N)^-$ is a fuzzy neutrosophic nowhere dense set in (X, τ_N). By proposition 3.7, we have $\overline{fn(A_N)^-}$ is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proposition 3.10:

Let A_N be a fuzzy neutrosophic dense set in (X, τ_N).

If B_N is any fuzzy neutrosophic set in (X, τ_N), then B_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N), if and only if $A_N \wedge B_N$ is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proof:

Let B_N be a Fuzzy neutrosophic nowhere dense set in (X, τ_N).

Now,

$$fn(((A_N \wedge B_N)^-)^+) = (fn(fn(A_N)^- \wedge fn\,(B_N)^-))^+ = (fn(1 \wedge fn(B_N)^-))^+$$

$$= fn(((B_N)^-)^+) = 0_N.$$

Therefore $A_N \wedge B_N$ is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Conversely let $A_N \wedge B_N$ is a fuzzy neutrosophic nowhere dense set in (X, τ_N). Then $fn(((A_N \wedge B_N)^-)^+) = 0_N$ Implies that $(fn(fn(A_N)^- \wedge fn\,(B_N)^-))^+$.

Hence $(fn(1 \wedge fn(B_N)^-))^+ = 0_N$ and therefore $fn(((B_N)^-)^+) = 0_N$ which means that B_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proposition 3.11:

If A_N is a Fuzzy neutrosophic first category set in (X, τ_N) then $\overline{A_N} = \wedge_{i=1}^{\infty} B_{N_i}$, where $fn(B_{N_i})^- = 1$.

Proof:

Let A_N be a fuzzy neutrosophic first category set in (X, τ_N). Then $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $A_i{}'$s are Fuzzy neutrosophic nowhere dense sets in (X, τ_N). Now $\overline{A_N} = \bigvee_{i=1}^{\infty} \bar{A}_{N_i} = \bigwedge_{i=1}^{\infty} \bar{A}_{N_i}$. Now $A_{N_i}$is a Fuzzy neutrosophic nowhere dense sets in (X, τ_N). Then, by proposition 3.7, we have $\bar{A}_{N_i}$ is a fuzzy neutrosophic dense set in (X, τ_N). Let us put $B_{N_i} = \bar{A}_{N_i}$. Then $\bar{A}_N = \bigwedge_{i=1}^{\infty} B_{N_i}$ where $fn(B_{N_i})^- = 1$.

Proposition 3.12:

Every fuzzy neutrosophic nowhere dense sets is a fuzzy neutrosophic closed set.

Proof:

Let A_N be any fuzzy neutrosophic nowhere dense set in a fuzzy neutrosophic topological space (X, τ_N). Therefore, we have $fn(((A_N)^-)^+) = 0_N$ and it means that there does not exist any fuzzy neutrosophic open set in between A_N and $(A_N)^-$. Also, let us suppose that $A_N \leq B_N$, where B_N is fuzzy neutrosophic open set and obviously $(A_N)^- \leq B_N$. Therefore B_N is a fuzzy neutrosophic closed set.

4. FUZZY NEUTROSOPHIC BAIRE SPACE

Definition 4.1:

A fuzzy neutrosophic topological space (X, τ_N) is called fuzzy neutrosophic Baire space if $fn(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0_N$, where $A_{N_i}{}'s$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N).

Example 4.1:

Let $X = \{a, b, c\}$ and consider the family $\tau_N = \{0_{N,}1_N, A_N, B_N, C_N, D_N\}$ where

$A_N = \{\langle a, 0.3,0.3,0.5\rangle, \langle b, 0.6,0.6,0.5\rangle, \langle c, 0.6,0.6,0.5\rangle\}$

$B_N = \{\langle a, 0.3,0.3,0.3\rangle, \langle b, 0.6,0.6,0.5\rangle, \langle c, 0.6,0.6,0.6\rangle\}$

$C_N = \{\langle a, 0.7,0.7,0.4\rangle, \langle b, 0.4,0.3,0.3\rangle, \langle c, 0.3,0.3,0.4\rangle\}$

$D_N = \{\langle a, 0.3,0.3,0.3\rangle, \langle b, 0.7,0.7,0.7\rangle, \langle c, 0.3,0.3,0.3\rangle\}$

Now $\overline{A_N}$, $\overline{B_N}$ and $\overline{C_N}$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Also $fn(\overline{A_N} \vee \overline{B_N} \vee \overline{C_N})^+ = 0_N$. Hence (X, τ_N) be a fuzzy neutrosophic Baire Space.

Proposition 4.1:

Let (X, τ_N) be a fuzzy neutrosophic topological space. Then the following are equivalent.

i) (X, τ_N) is a fuzzy neutrosophic baire space.

ii) $fn\,(A_N)^+ = 0_N$, for every fuzzy neutrosophic first category set A_N in (X, τ_N).

iii) $fn\,(B_N)^+ = 1_N$, for every fuzzy neutrosophic residual set B_N in (X, τ_N).

Proof:

$(i) \Rightarrow (ii)$

Let A_N be a fuzzy neutrosophic first category set in (X, τ_N). Then $A_N = (\bigvee_{i=1}^{\infty} A_{N_i})$, where ${A_{N_i}}'$ s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Now $fn\,(A_N)^+ = fn\,(\bigvee_{i=1}^{\infty} A_{N_i})^+ = 0_N$. Since (X, τ_N) is a fuzzy neutrosophic Baire space. Therefore $fn\,(A_N)^+ = 0_N$.

$(ii) \Rightarrow (iii)$

Let B_N be a fuzzy neutrosophic residual set in (X, τ_N). Then $\overline{B_N}$ is a fuzzy neutrosophic first category set in (X, τ_N). By hypothesis, $fn(\overline{B_N})^+ = 0_N$ which implies that $\overline{fn(A_N)^-} = 0_N$.

Hence $fn\,(A_N)^- = 1_N$.

$(iii) \Rightarrow (i)$

Let A_N be a fuzzy neutrosophic first category set in (X, τ_N). Then $A_N = (\bigvee_{i=1}^{\infty} A_{N_i})$, where ${A_{N_i}}'$ s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Now A_N is a fuzzy neutrosophic first category set implies that $\overline{A_N}$ is a fuzzy neutrosophic residual set in (X, τ_N). By hypothesis, we have $fn(\overline{A_N})^- = 1_N$ which implies that $\overline{fn(A_N)^+} = 1_N$. Hence $fn\,(A_N)^+ = 0_N$. That is, $fn\,(\bigvee_{i=1}^{\infty} A_{N_i})^+ = 0_N$, where ${A_{N_i}}'$ s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Hence (X, τ_N) is a fuzzy neutrosophic Baire space.

Proposition 4.2:

If A_N be a fuzzy neutrosophic first category set in (X, τ_N) then $\overline{A_N} = \bigwedge_{i=1}^{\infty} B_{N_i}$, where $fn\,(B_{N_i})^- = 1_N$.

Proof:

Let A_N be a fuzzy neutrosophic first category set in (X, τ_N).

Then $A_N = (\bigvee_{i=1}^{\infty} A_{N_i})$, where ${A_{N_i}}'$ s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Now $\overline{A_N} = \overline{\bigvee_{i=1}^{\infty} A_{N_i}} = \bigwedge_{i=1}^{\infty} \bar{A}_{N_i}$ Now A_{N_i} is a fuzzy neutrosophic nowhere dense sets in (X, τ_N). Then by proposition [3.6] we have $\overline{A_{N_i}}$. is a fuzzy neutrosophic dense sets in (X, τ_N). Let us put $B_{N_i} = \overline{A_{N_i}}$. Then $\overline{A_N} = \bigwedge_{i=1}^{\infty} B_{N_i}$, where $fn\,(B_{N_i})^- = 1_N$.

Proposition 4.3:

If $fn\,(\bigvee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$ where $fn\,(A_{N_i})^{+} = 0_N$ and $A_{N_i} \in \tau_N$, then (X, τ_N) is a fuzzy neutrosophic Baire space.

Proof:

Now $A_{N_i} \in \tau_N$ implies that A_{N_i} is a fuzzy neutrosophic open sets in (X, τ_N). Since $fn\,(A_{N_i})^{+} = 0_N$. By proposition (3.2), A_{N_i} is a fuzzy neutrosophic nowhere dense sets in (X, τ_N). Therefore $fn\,(\bigvee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$, where $A_{N_i}{}'s$ is a fuzzy neutrosophic nowhere dense sets in (X, τ_N). Hence (X, τ_N) is a fuzzy neutrosophic Baire space.

Proposition 4.4:

If $fn\,(\bigvee_{i=1}^{\infty} A_{N_i})^{+}$ where $fn\,(A_{N_i})^{+} = 0_N$ and A_{N_i}'s are fuzzy neutrosophic closed sets in fuzzy neutrosophic topological space in (X, τ_N) then (X, τ_N) is a fuzzy neutrosophic Baire space.

Proof:

Let $A_{N_i}{}'s$ be fuzzy neutrosophic closed sets in (X, τ_N). Since $fn\,(A_{N_i})^{+} = 0_N$, by proposition (3.2), $A_{N_i}{}'s$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Thus $fn\,(\bigvee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$, where $A_{N_i}{}'s$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Hence (X, τ_N) is a fuzzy neutrosophic Baire space.

Proposition 4.5

If $fn\,(\bigwedge_{i=1}^{n} A_{N_i},)^{-} = 1_N$, where $A_{N_i}{}'s$ are fuzzy neutrosophic dense and fuzzy neutrosophic open sets in fuzzy neutrosophic topological space (X, τ_N) if and only if (X, τ_N) is a fuzzy neutrosophic Baire space.

Proof:

Let $A_{N_i}{}'s$ be fuzzy neutrosophic dense sets in (X, τ_N). Then $fn\,(\bigwedge_{i=1}^{n} A_{N_i},)^{-} = 1_N$ which implies that $1 - fn\,(\bigwedge_{i=1}^{n} A_{N_i},)^{-} = 0_N$. That is $fn\,((1 - \bigwedge_{i=1}^{n} A_{N_i}))^{+} = 0_N$ implies that $fn\,(1 - \bigvee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$. Since $A_{N_i}{}'s$ be fuzzy neutrosophic dense, $fn\,(A_{N_i})^{-} = 1_N$. Hence $fn\,(1 - A_{N_i})^{+} = 1 - fn\,(A_{N_i})^{-} = 0_N$. Consequently $fn\,(1 - \bigvee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$, where $fn\,(1 - A_{N_i})^{+} = 0_N$ and $A_{N_i}{}'s$ be fuzzy neutrosophic closed sets in (X, τ_N). By proposition 4.4, (X, τ_N) is a fuzzy neutrosophic Baire space. Conversely, let $A_{N_i}{}'s$ are fuzzy neutrosophic dense and fuzzy neutrosophic open sets in (X, τ_N). By proposition (3.7), $1 - A_{N_i}{}'s$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Then $A_N = \bigvee_{i=1}^{\infty} 1 - A_{N_i}$ is a fuzzy neutrosophic first category set in (X, τ_N). Now $fn\,(A_N)^{+} = fn\,(\bigvee_{i=1}^{\infty}(1 - A_{N_i}))^{+} = fn\,(1 - \bigwedge_{i=1}^{\infty} A_{N_i})^{+} = (1 - fn\,(\bigvee_{i=1}^{\infty} A_{N_i}))^{-}$. Since (X, τ_N) is a fuzzy neutrosophic Baire space, by proposition 4.1, we set $fn\,(A_N)^{+} = 0_N$. Then $(1 - fn\,(\bigwedge_{i=1}^{\infty} A_{N_i})^{-}) = 0_N$. This implies that $(fn\,(\bigwedge_{i=1}^{\infty} A_{N_i})^{-}) = 1_N$.

CONCLUSION

In this paper, the concept of a new class of sets, spaces and called them fuzzy neutrosophic dense, fuzzy neutrosophic nowhere dense, fuzzy neutrosophic residual set, fuzzy neutrosophic first category set, fuzzy neutrosophic second category sets, fuzzy neutrosophic Baire spaces, fuzzy neutrosophic first category space, fuzzy neutrosophic second category space. Some of its characterizations of fuzzy neutrosophic Baire spaces are also studied. This shall be extended in the future research studies.

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ON FUZZY NEUTROSOPHIC *I*-BAIRE SPACES

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ABSTRACT

In this paper, the concept of fuzzy neutrosophic $I-$ Baire spaces is introduced, characterizations and properties of these spaces are given. It is shown that is (X, τ_N) Baire if and only if (X, τ_N, I) is fuzzy neutrosophic $I-$Baire for any ideal space I on X.

Keywords: Fuzzy neutrosophic $*-$dense set, Fuzzy neutrosophic nowhere $*-$dense set, Fuzzy neutrosophic $*-$ residual set, Fuzzy neutrosophic $I-$Baire spaces, Fuzzy neutrosophic $*-$ one and two category.

1. INTRODUCTION

Fuzzy sets are mathematical framework that extends classical set theory to handle uncertainty and vagueness. Introduced by Lofti A. Zadeh in 1965 [11], fuzzy set provide a way to represent and manipulate imprecise or ambiguous information. In classical set theory, an element can either belong or not belong to a set, with no degrees of membership. However, in many real word situations, the membership of an element to a set is not always clear-cut. This is where fuzzy sets come into play. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [10] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

Fuzzy sets have found applications in various fields, including control systems, artificial intelligence, decision-making, pattern recognition, and data analysis. They provide a powerful tool for modelling and reasoning with imprecise and uncertain information, allowing for more flexible and nuanced representations of real world.

The aim of this paper is to introduce and study fuzzy neutrosophic $I-$ Baire spaces. Some characterizations and properties of fuzzy neutrosophic I- Baire spaces are investigated. If τ_N is a fuzzy neutrosophic topology on X and I is an ideal topology on X, then (X, τ_N, I) is called an fuzzy neutrosophic ideal topological space or simply an fuzzy neutrosophic ideal space.

2. PRELIMINARIES

Throughout the present paper, (X, τ_N, I) denote the fuzzy neutrosophic ideal topological spaces. Let A_N be a fuzzy neutrosophic set on X. The fuzzy neutrosophic ideal interior and ideal closure of A_N is denoted by $*-(A_N)^+$, $*-(A_N)^-$ respectively. A fuzzy neutrosophic set A_N is defined to be fuzzy neutrosophic $*-$ open set $(fnOS)$ if $A_N \leq *-fn\ (((A_N)^-)^+)^-$. The complement of a fuzzy neutrosophic $*-$ open set is called fuzzy neutrosophic $*-$ closed set $(fnCS)$.

Definition 2.1 [2]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $x \in X$ where $T, I, F: X \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

With the condition $0 \leq T_{A^*}(x) + \mathrm{I}_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2 [2]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$.

Definition 2.3 [2]:

Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$

$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$

Definition 2.4 [2]:

The difference between two fuzzy neutrosophic sets A and B is defined as $A \backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.5 [2]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1\ for\ all\ x \in X$. It is denoted by 0_N.

Definition 2.6 [2]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0\ for\ all\ x \in X$. It is denoted by 1_N.

Definition 2.7 [2]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as

$A^C = \langle x, T_{A^C}(x), \mathrm{I}_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8 [1]:

A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X

$$(i)\ \ 0_N, 1_N \in \tau$$

$$(ii)\ \ A_1 \cap A_2 \in \tau\ \ for\ any\ A_1, A_2 \in \tau$$

$$(iii)\ \ \cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i : i \in J\}\ \in \tau$$

Satisfying the following axioms. In this case the pair $(X,\ \tau)$ is called fuzzy neutrosophic topological space and any Fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [1]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10 [1]:

Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$int(A) = \cup \{G : G\ is\ a\ fuzzy\ neutrosophic\ open\ set\ in\ X\ and\ G \subseteq A\}$$

$$cl(A) = \cap \{G : G\ is\ a\ fuzzy\ neutrosophic\ closed\ set\ in\ X\ and\ A \subseteq G\}$$

Motivated by the classical concept of $* -$ denseness and nowhere $* -$ denseness in [12], and the fuzzy neutrosophic $* -$ dense set and fuzzy neutrosophic nowhere $* -$ dense set we shall now defined.

Let X be a non- empty set, let τ_N be a family of fuzzy neutrosophic sets of X. I is called an fuzzy neutrosophic ideal on X, if it satisfies the following condition.

(1) If $A_N \in I$ and $A_N \subset B_N$ then $B_N \in I$.
(2) If $A_N, B_N \in I$ then $A_N \cup B_N \in I$.

If τ_N is a fuzzy neutrosophic topology on X and I is an ideal topology on X, then (X, τ_N, I) is called an fuzzy neutrosophic ideal topological space.

3. ON FUZZY NEUTROSOPHIC*-DENSE SETS AND NOWHERE*- DENSE SETS

Definition 3.1:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic ideal space (X, τ_N, I) is called a fuzzy neutrosophic $* -$ dense if there exist no fuzzy neutrosophic $* -$ closed set B_N in (X, τ_N, I) such that $A_N < B_N < 1$. That is, $fn\ * -(A_N)^- = 1_N$.

Definition 3.2:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic ideal space (X, τ_N, I) is called a fuzzy neutrosophic nowhere $* -$ dense if there exist no non-zero fuzzy neutrosophic $* -$ open set B_N in (X, τ_N, I) such that $B_N < \ fn \ * -(A_N)^-$. That is, $fn * - (((A_N)^-)^+) = 0_N$.

Definition 3.3:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic ideal topological space (X, τ_N, I) is called a fuzzy neutrosophic $* -$ semi-open if $A_N \leq \ fn * - (((A_N)^+)^-)$. The complement of A_N in (X, τ_N, I) is called a fuzzy neutrosophic $* -$ semi-closed set in (X, τ_N, I).

Example 3.1:

Let $X = \{a, b, c\}$ and consider the fuzzy neutrosophic sets A_N, B_N are defined as follows.

$$A_N = \{\langle a, 0.6,0.6\rangle, \langle b, 0.6,0.6\rangle, \langle c, 0.3,0.4\rangle\}$$

$$B_N = \{\langle a, 0.6,0.5\rangle, \langle b, 0.6,0.5\rangle, \langle c, 0.4,0.5\rangle\}$$

Then $\tau_N = \{0_{N,} 1_N, A_N, B_N\}$ is a fuzzy neutrosophic ideal topological space on X.

Thus (X, τ_N, I) is a fuzzy neutrosophic ideal space, $1 - A_N, 1 - B_N$ are fuzzy neutrosophic nowhere $* -$ dense sets.

Definition 3.4:

Let (X, τ_N, I) be a fuzzy neutrosophic ideal topological space. A fuzzy neutrosophic set A_N in (X, τ_N, I) is called fuzzy neutrosophic $* -$ one category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere $* -$ dense sets in (X, τ_N, I). A fuzzy neutrosophic which is not fuzzy neutrosophic $* -$ one category set is called a fuzzy neutrosophic $* -$ two category set in (X, τ_N, I).

Definition 3.5:

Let A_N be a fuzzy neutrosophic $* -$ one category set in (X, τ_N, I). Then $1 - \ A_N$ is called fuzzy neutrosophic $* -$ residual set in (X, τ_N, I).

Proposition 3.1:

Let (X, τ_N, I) be a fuzzy neutrosophic ideal space. If A_N is a fuzzy neutrosophic nowhere $* -$ dense set in (X, τ_N, I) then

(1) $fn \ * - (A_N)^+ = 0$.
(2) $(1 - A_N)$ is fuzzy neutrosophic $* -$ dense set in (X, τ_N, I).
(3) $fn \ * -(A_N)^-$ is fuzzy neutrosophic nowhere $* -$ dense set in (X, τ_N, I).
(4) $1 - \ fn * -(A_N)^-$ is fuzzy neutrosophic $* -$ dense set in (X, τ_N, I).
(5) A_N is a fuzzy neutrosophic $* -$ semi closed set in (X, τ_N, I).

Proof:

(1) Let A_N be a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I). Then $fn*-(A_N)^+ = A_N$. Now $fn*-(((A_N)^-)^+) = fn*-(A_N)^+ = 0_N$. and hence we have, $fn*-(A_N)^+ = 0_N$.

(2) Let A_N be a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I). Then by proposition (1) we have $fn*-(A_N)^+ = 0_N$. Now $1-(fn*-(1-A_N)^+) = 1-0 = 1_N$. Therefore $(1-A_N)$ is fuzzy neutrosophic $*-$ dense set in (X, τ_N, I).

(3) Let A_N be a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I). Now $fn*-(A_N)^- \leq A_N$ gives that $fn*-(A_N)^- \leq fn*-(((A_N)^-)^+) = 0_N$. Hence by the hypothesis, $fn*-(A_N)^-$ is fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

(4) By proposition (3), $fn*-(A_N)^-$ is fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

by proposition (2), we have $1-(fn*-(1-A_N)^-)$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

(5) Let A_N be a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I). Then $fn*-(((A_N)^-)^+) = 0_N$. And therefore $fn*-(((A_N)^-)^+) \leq A_N$. Hence, A_N a fuzzy neutrosophic $*-$ semi closed set in (X, τ_N, I).

Proposition 3.2:

If A_N is a fuzzy neutrosophic $*-$ dense, fuzzy neutrosophic $*-$ open set in (X, τ_N) such that $B_N \leq 1-A_N$, then B_N is a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

Proof:

Let A_N be a fuzzy neutrosophic $*-$ open set in (X, τ_N, I) such that $fn*-(A_N)^- = 1_N$. Now $B_N \leq 1-A_N$ implies that $fn*-(B_N)^- \leq fn*-(1-A_N)^- = (1-A_N)$ [$1-A_N$ is a fuzzy neutrosophic $*-$ closed set in (X, τ_N, I)] Then we have $fn*-(((B_N)^-)^+) \leq fn*-(1-A_N)^+ = 1-(fn*-(A_N)^-) = 1-1 = 0_N$. and hence $fn*-(((B_N)^-)^+) = 0_N$. Therefore B_N is a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

Proposition 3.3:

If A_N is a fuzzy neutrosophic nowhere $*-$dense set and fuzzy neutrosophic $*-$ open set in (X, τ_N, I), then $1-A_N$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

Proof:

Let A_N be a fuzzy neutrosophic $*-$ open set in (X, τ_N, I) such that, $fn*-(A_N)^- = 1$. Now $fn*-(((1-A_N)^-)^+) = 1-(fn*-(((1-A_N)^+)^-)) = 1-fn*-(A_N)^- = 1-1 = 0_N$. Hence $1-A_N$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

Proposition 3.4:

Let A_N be a fuzzy neutrosophic $*-$ dense set in (X, τ_N, I). If B_N is any fuzzy neutrosophic set in (X, τ_N, I), then B_N is a fuzzy neutrosophic nowhere $*-$dense set in (X, τ_N, I), if and only if $A_N \wedge B_N$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

Proof:

Let B_N be a Fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

Now, $fn*-(((A_N \wedge B_N)^-)^+) = fn*-(fn*-(A_N)^- \wedge fn*-(B_N)^-)^+$

$= (fn*-(1 \wedge fn*-(B_N)^-))^+ = fn*-(((B_N)^-)^+) = 0_N.$

Therefore $A_N \wedge B_N$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

Conversely let $A_N \wedge B_N$ is a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I). Then $fn*-(((A_N \wedge B_N)^-)^+) = 0_N$ Implies that $fn*-((fn*-(A_N)^- \wedge fn*-(B_N)^-))^+$.

Hence $fn*-((1 \wedge fn*-(B_N)^-))^+ = 0_N$ and therefore $fn*-(((B_N)^-)^+) = 0_N$ which means that B_N is a fuzzy neutrosophic nowhere $*-$ dense set in (X, τ_N, I).

Proposition 3.5:

If A_N is a fuzzy neutrosophic $*-$ one category set in (X, τ_N, I) then $1 - A_N = \wedge_{i=1}^{\infty} B_{N_i}$, where $fn*-(B_{N_i})^- = 1$.

Proof:

Let A_N be a fuzzy neutrosophic $*-$ first category set in (X, τ_N, I). Then $A_N = \vee_{i=1}^{\infty} A_{N_i}$, where A_i's are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Now $(1 - A_N) = 1 - \vee_{i=1}^{\infty} A_{N_i} = \wedge_{i=1}^{\infty} 1 - A_{N_i}$. Now $A_{N_i}$is a fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Then, by proposition 3.1 (4), we have $1 - A_{N_i}$ is a fuzzy neutrosophic $*-$ dense set in (X, τ_N, I). Let us put $B_{N_i} = 1 - A_{N_i}$. Then $1 - A_N = \wedge_{i=1}^{\infty} B_{N_i}$ where $fn*-(B_{N_i})^- = 1$.

Proposition 3.6:

Every fuzzy neutrosophic nowhere $*-$ dense sets is a fuzzy neutrosophic $*-$ closed set.

Proof:

Let A_N be any fuzzy neutrosophic nowhere $*-$ dense set in a fuzzy neutrosophic ideal topological space (X, τ_N, I). Therefore, we have $fn*-(((A_N)^-)^+) = 0_N$ and it means that there does not exist any fuzzy neutrosophic $*-$ open set in between A_N and $fn*-(A_N)^-$. Also, let us suppose that $A_N \leq B_N$, where B_N is fuzzy neutrosophic $*-$ open set and obviously $fn*-(A_N)^- \leq B_N$. Therefore B_N is a fuzzy neutrosophic $*-$ closed set.

3. FUZZY NEUTROSOPHIC *I*-BAIRE SPACE

Definition 4.1:

A fuzzy neutrosophic ideal topological space (X, τ_N, I) is called fuzzy neutrosophic $I-$ Baire space if $fn*-(\vee_{i=1}^{\infty}(A_{N_i}))^{+}=0_N$, where ${A_{N_i}}'s$ are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I).

Definition 4.2:

A fuzzy neutrosophic topological space (X, τ_N, I) is called fuzzy neutrosophic $*-$ one category space if the fuzzy neutrosophic set 1_X is a fuzzy neutrosophic $*-$ one category set in (X, τ_N, I). That is $1_X = \vee_{i=1}^{\infty} A_{N_i}$, where ${A_{N_i}}'$s are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Otherwise (X, τ_N, I) will be called a fuzzy neutrosophic $*-$ two category space.

Example 4.1:

Let $X=\{a,b,c\}$ and consider the fuzzy neutrosophic sets A_N, B_N are defined as follows.

$$A_N = \{\langle a, 0.6, 0.5\rangle, \langle b, 0.6, 0.5\rangle, \langle c, 0.3, 0.4\rangle\}$$

$$B_N = \{\langle a, 0.6, 0.6\rangle, \langle b, 0.6, 0.4\rangle, \langle c, 0.3, 0.5\rangle\}$$

Then $\tau_N = \{0_{N,}1_N, A_N, B_N, A_N \vee B_N, A_N \wedge B_N\}$ is a fuzzy neutrosophic ideal topological space on X. Thus (X, τ_N, I) is a fuzzy neutrosophic ideal space. $1-A_N, 1-B_N, 1-A_N \vee B_N \vee 1-A_N \wedge B_N$ are fuzzy neutrosophic nowhere $*-$ dense sets and $[1-A_N \vee 1-B_N \vee 1-A_N \vee B_N \vee 1-A_N \wedge B_N] = 1-A_N \wedge B_N$ is a fuzzy neutrosophic $*-$ one category set. This implies that

$fn*-(1-A_N \wedge B_N)^{+}=0, 1-A_N \wedge B_N$ is a fuzzy neutrosophic $I-$ Baire space.

Proposition 4.1:

Let (X, τ_N, I) be a fuzzy neutrosophic ideal topological space. Then the following are equivalent.

i) (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space.

ii) $fn*-(A_N)^{+}=0_N$, for every fuzzy neutrosophic $*-$ first category set A_N in (X, τ_N, I).

iii) $fn*-(B_N)^{+}=1_N$, for every fuzzy neutrosophic $*-$ residual set B_N in (X, τ_N, I).

Proof:

$(i) \Rightarrow (ii)$

Let A_N be a fuzzy neutrosophic $*-$ one category set in (X, τ_N, I). Then $A_N = (\vee_{i=1}^{\infty} A_{N_i})$, where ${A_{N_i}}'$ s are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Now $fn*-(A_N)^{+}=$

$fn*-(\vee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$. Since (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space. Therefore $fn*-(A_N)^{+} = 0_N$.

$(ii) \Rightarrow (iii)$

Let B_N be a fuzzy neutrosophic $*-$ residual set in (X, τ_N, I). Then $1 - B_N$ is a fuzzy neutrosophic $*-$ one category set in (X, τ_N, I). By hypothesis, $fn*-(1 - B_N)^{+} = 0_N$ which implies that $fn*-(1-(1-A_N)^{-})= 0_N$.

Hence $fn*-(A_N)^{-} = 1_N$.

$(iii) \Rightarrow (i)$

Let A_N be a fuzzy neutrosophic $*-$ one category set in (X, τ_N, I). Then $A_N = (\vee_{i=1}^{\infty} A_{N_i})$, where ${A_{N_i}}'$ s are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Now A_N is a fuzzy neutrosophic $*-$ one category set implies that $1 - A_N$ is a fuzzy neutrosophic $*-$ residual set in (X, τ_N, I). By hypothesis, we have $1 - fn*-(1 - A_N) = 1_N$ which implies that $1 - fn*-(A_N)^{+} = 1_N$. Hence $fn*-(A_N)^{+} = 0_N$. That is, $fn*-(\vee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$, where ${A_{N_i}}'$ s are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Hence (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space.

Proposition 4.2:

If A_N be a fuzzy neutrosophic one category set in (X, τ_N, I) then $1 - A_N = \wedge_{i=1}^{\infty} B_{N_i}$, where $fn*-(B_{N_i})^{-} = 1_N$.

Proof:

Let A_N be a fuzzy neutrosophic $*-$ one category set in (X, τ_N, I).

Then $A_N = (\vee_{i=1}^{\infty} A_{N_i})$, where ${A_{N_i}}'$ s are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Now $1 - A_N = 1 - (\vee_{i=1}^{\infty} A_{N_i}) = \wedge_{i=1}^{\infty}(1 - A_{N_i})$ Now A_{N_i} is a fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Then by proposition 3.1(4) we have $1 - A_{N_i}$ is a fuzzy neutrosophic $*-$ dense sets in (X, τ_N, I). Let us put $B_{N_i} = 1 - A_{N_i}$ Then $1 - A_N = \wedge_{i=1}^{\infty} B_{N_i}$, where $fn*-(B_{N_i})^{-} = 1_N$.

Proposition 4.3:

If $fn*-(\vee_{i=1}^{\infty} A_{N_i})^{+} = 0_N$ where $fn*-(A_{N_i})^{+} = 0_N$ and $A_{N_i} \in \tau_N$, then (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space.

Proof:

Now $A_{N_i} \in \tau_N$ implies that A_{N_i} is a fuzzy neutrosophic $*-$ open sets in (X, τ_N, I). Since $fn*-(A_{N_i})^{+} = 0_N$. By proposition 3.1(1), A_{N_i} is a fuzzy neutrosophic nowhere $*-$ dense sets

in (X, τ_N, I). Therefore $fn*-(\bigvee_{i=1}^{\infty} A_{N_i})^+ = 0_N$, where ${A_{N_i}}'s$ is a fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Hence (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space.

Proposition 4.4:

If $fn*-(\bigvee_{i=1}^{\infty} A_{N_i})^+$ where $fn*-(A_{N_i})^+ = 0_N$ and A_{N_i}'s are fuzzy neutrosophic $*-$ closed sets in fuzzy neutrosophic ideal topological space in (X, τ_N, I) then (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space.

Proof:

Let ${A_{N_i}}'s$ be fuzzy neutrosophic $*-$ closed sets in (X, τ_N, I). Since $fn*-(A_{N_i})^+ = 0_N$, by proposition 3.1(1) ${A_{N_i}}'s$ are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Thus $fn*-(\bigvee_{i=1}^{\infty} A_{N_i})^+ = 0_N$, where ${A_{N_i}}'s$ are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Hence (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space.

Proposition 4.5

If $fn*-(\bigwedge_{i=1}^{\infty} A_{N_i},)^- = 1_N$, where ${A_{N_i}}'s$ are fuzzy neutrosophic $*-$ dense and fuzzy neutrosophic $*-$ open sets in fuzzy neutrosophic ideal topological space (X, τ_N, I) if and only if (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space.

Proof:

Let ${A_{N_i}}'s$ be fuzzy neutrosophic $*-$ dense sets in (X, τ_N, I). Then $fn*-(\bigwedge_{i=1}^{\infty} A_{N_i},)^- = 1_N$ which implies that $1-fn*-(\bigwedge_{i=1}^{\infty} A_{N_i},)^- = 0_N$. That is $(1-fn*-\bigwedge_{i=1}^{\infty} A_{N_i})^+ = 0_N$ implies that $(1-fn*-\bigvee_{i=1}^{\infty} A_{N_i})^+ = 0_N$. Since ${A_{N_i}}'s$ be fuzzy neutrosophic $*-$ dense, $*fn-(A_{N_i})^- = 1_N$. Hence $fn*-\left(1-A_{N_i}\right)^+ = 1-fn*-\left(A_{N_i}\right)^- = 0_N$. Consequently $fn*-(1-\bigvee_{i=1}^{\infty} A_{N_i})^+ = 0_N$, where $fn*-(1-A_{N_i})^+ = 0_N$ and ${A_{N_i}}'s$ be fuzzy neutrosophic $*-$ closed sets in (X, τ_N, I). By proposition 4.4, (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space.

Conversely, let ${A_{N_i}}'s$ are fuzzy neutrosophic $*-$ dense and fuzzy neutrosophic $*-$ open sets in (X, τ_N, I). By proposition (3.7), $1-{A_{N_i}}'s$ are fuzzy neutrosophic nowhere $*-$ dense sets in (X, τ_N, I). Then $A_N = fn*-(\bigvee_{i=1}^{\infty} 1-A_{N_i})$ is a fuzzy neutrosophic $*-$ one category set in (X, τ_N, I). Now $fn*-(A_N)^+ = fn*-\left(\bigvee_{i=1}^{\infty}\left(1-A_{N_i}\right)\right)^+ = fn*-(1-\bigwedge_{i=1}^{\infty} A_{N_i})^+ = (1-fn*-\left(\bigvee_{i=1}^{\infty} A_{N_i}\right))^-$. Since (X, τ_N, I) is a fuzzy neutrosophic $I-$ Baire space, by proposition 4.1, $fn*-(A_N)^+ = 0_N$. Then $(1-fn*-\left(\bigwedge_{i=1}^{\infty} A_{N_i}\right)^-) = 0_N$. This implies that $\left(fn*-\left(\bigwedge_{i=1}^{\infty} A_{N_i}\right)^-\right) = 1_N$.

CONCLUSION

In this paper, the concept of a new class of sets, spaces are called them fuzzy neutrosophic $*-$ dense, fuzzy neutrosophic nowhere $*-$ dense, fuzzy neutrosophic $*-$ residual set, fuzzy neutrosophic $*-$ one category set, fuzzy neutrosophic $*-$ two category sets, fuzzy neutrosophic $I-$Baire spaces, fuzzy neutrosophic $*-$ one category space, fuzzy neutrosophic $*-$ two category space. Some of its characterizations of fuzzy neutrosophic $I-$ Baire spaces are also studied. This shall be extended in the future research studies.

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ON FUZZY NEUTROSOPHIC DIRECTED BAIRE SPACES AND CHARACTERIZATIONS OF OTHER FUZZY NEUTROSOPHIC SPACES

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Abstract. We study fuzzy neutrosophic directed Baire spaces and their relevant fuzzy neutrosophic topological spaces. A characterization of fuzzy neutrosophic directed Baire spaces is given using fuzzy neutrosophic $G_\delta-$ sets. Further, we prove that the weakly directed Baire spaces with a directed Baire spaces. Also we discuss several characterizations of other fuzzy neutrosophic spaces namely fuzzy neutrosophic $P-$ space, fuzzy neutrosophic submaximal space, fuzzy neutrosophic almost resolvable space and fuzzy neutrosophic almost irresolvable space.

Keywords: Fuzzy neutrosophic $G_\delta-$ set, fuzzy neutrosophic directed Baire space, fuzzy neutrosophic weakly directed Baire space, fuzzy neutrosophic $P-$ space, fuzzy neutrosophic submaximal space, fuzzy neutrosophic almost resolvable space and fuzzy neutrosophic almost irresolvable space.

1. Introduction

The concept of fuzzy sets were introduced by L.A. Zadeh in 1965 [22]. Then the fuzzy set theory is extension by many researchers. The important concept of fuzzy topological space was offered by C. L. Chang [6] and from that point forward different ideas in topology have been reached out to fuzzy topological space. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy toplogy has been developed. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [19] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

A. K. Mishra [10] introduced the concepts of $P-$ spaces as a generalization of $\omega_\alpha-$ additive spaces of Sikorski [16] and L. W. Cohen and C. Goffman [7]. The concept of $P-$ spaces in fuzzy setting was introduced by G. Balasubramanian in [4] . Almost $P-$ spaces in classical topology was introduced by A. L. Veksler [20] and was also studied further by R. Levy [9].

The concept of almost $P-$ spaces in fuzzy setting was introduced by the authors in [18]. The term "submaximal space" was first used by Bourbaki in [5]. Since its birth, the class of submaximal spaces has attracted the attention of general topologists. The first systematic study of submaximal spaces was undertaken in the paper of A. V. Arhangel'Skiand P. J. Collins [3]. The notion of submaximal spaces in fuzzy topology was introduced by G. Balasubramanian in [4] . The concepts of almost resolvable spaces was introduced by Richard Bolstein [13] as a generalization of resolvable spaces of E. Hewit [8]. In recent years, fuzzy topology has been found to be very useful in solving many practical problems.
In this paper, we introduce the concept of fuzzy neutrosophic directed Baire spaces and fuzzy neutrosophic weakly directed Baire spaces. Also, we discuss several characterizations and inter-relations between fuzzy neutrosophic $P-$ space, fuzzy neutrosophic submaximal space, fuzzy neutrosophic almost resolvable space and fuzzy neutrosophic almost irresolvable spaces. Several examples are given to illustrate the concepts introduced in this paper.

2. **Preliminaries**

Throughout this present paper, X denotes the fuzzy neutrosophic topological spaces. Let A_N be a fuzzy neutrosophic set on X. The fuzzy neutrosophic interior and closure of A_N is denoted as $fn(A_N)^+$, $fn(A_N)^-$ respectively. A fuzzy neutrosophic set A_N is defined to be fuzzy neutrosophic open set ($fnOS$) if $A_N \leq fn(((A_N)^-)^+)^-$. The complement of a fuzzy neutrosophic open set is called fuzzy neutrosophic closed set ($fnCS$).

Definition 2.1. [2] A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F : X \to [0, 1]$ and $0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3$. With the condition $0 \leq T_{A^*}(x) + I_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2. [2] A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

Definition 2.3. [2] Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then
$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$
$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$

Definition 2.4. [2] The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$.

Definition 2.5. [2] A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0, I_A(x) = 0, F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N.

Definition 2.6. [2] A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N.

Definition 2.7. [2] The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$. The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8. [2] A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X
(i) $0_N, 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$
Satisfying the following axioms.
In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9. [2] The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10. [2] Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by
$int(A) = \cup\{G : G$ is a fuzzy neutrosophic open set in X and $G \subseteq A\}$
$cl(A) = \cap\{G : G$ is a fuzzy neutrosophic closed set in X and $A \subseteq G\}$

Theorem 2.11. *[2] Let (X, τ_N) be a fuzzy neutrosophic topological space over X. Then the following properties hold.* $(i) cl(A^c) = (intA)^c$ $(ii) int(A^c) = (clA)^c$

Definition 2.12. [11] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (P, τ_N) is called a fuzzy neutrosophic $F_\sigma-$ set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$for$i \in I$.

Definition 2.13. [11] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (P, τ_N) is called a fuzzy neutrosophic $G_\delta-$ set if $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$.

Definition 2.14. [11] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (P, τ_N) is called a fuzzy neutrosophic semi $-$ open if $A_N \leq fn(((A_N)^+)^-)$. The complement of A_N in (P, τ_N) is called a fuzzy neutrosophic semi $-$ closed set in (P, τ_N).

Definition 2.15. [11] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (P, τ_N) is called a fuzzy neutrosophic dense if there exist no fuzzy neutrosophic closed set B_N in (P, τ_N) such that $A_N \subset B_N \subset 1_X$. That is, $fn(A_N)^- = 1_N$.

Definition 2.16. [11] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (P, τ_N) is called a fuzzy neutrosophic nowhere dense set if there exist no non $-$ zero fuzzy neutrosophic open set B_N in (P, τ_N) such that $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$.

Definition 2.17. [11] Let (P, τ_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set A_N in (P, τ_N) is called fuzzy neutrosophic one category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere dense sets in (P, τ_N). Any other fuzzy neutrosophic set in (P, τ_N) is said to be of fuzzy neutrosophic two category.

Definition 2.18. [11] A fuzzy neutrosophic topological space $(P, \tau_N)$is called fuzzy neutrosophic one category space if the fuzzy neutrosophic set 1_X is a fuzzy neutrosophic one category set in (P, τ_N). That is $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fuzzy neutrosophic two category space.

Definition 2.19. [11] Let A_N be a fuzzy neutrosophic one category set in (P, τ_N). Then $\overline{A_N}$ is called fuzzy neutrosophic residual set in (P, τ_N).

Definition 2.20. [12] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $P-$ space if each fuzzy neutrosophic $G_\delta-$ set in (X, τ_N) is fuzzy neutrosophic open set in (X, τ_N).

Definition 2.21. [12] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic hyperconnected space if every non – null fuzzy neutrosophic open subset of (X, τ_N) is fuzzy neutrosophic dense in (X, τ_N).

Definition 2.22. [12] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic submaximal space if for each fuzzy neutrosophic set A_N in (X, τ_N) such that $(A_N)^- = 1$ then $A_N \in \tau_N$ in (X, τ_N). That is, (X, τ_N) is a fuzzy neutrosophic submaximal space if each fuzzy neutrosophic dense set in (X, τ_N) is a fuzzy neutrosophic open set in (X, τ_N).

Definition 2.23. [12] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic resolvable space if there exists a fuzzy neutrosophic dense set A_N in (X, τ_N) such that $(1 - A_N)^- = 1$. Otherwise, (X, τ_N) is called a fuzzy neutrosophic irresolvable space.

Definition 2.24. [12] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic almost resolvable space if $\bigvee_{i=1}^{\infty} (A_{N_i}) = 1$, where (A_{N_i})'s in (X, τ_N) are such that $(A_{N_i})^+ = 0$. Otherwise, (X, τ_N) is called a fuzzy neutrosophic almost irresolvable space.

3. ON FUZZY NEUTROSOPHIC DIRECTED AND WEAKLY DIRECTED BAIRE SPACES

Definition 3.1. Let (X, τ_N) be a fuzzy neutrosophic topological space. Then (X, τ_N) is called a fuzzy neutrosophic directed Baire space if $(\bigvee_{i=1}^{\infty} (A_{N_i})^+ = 0)$, where (A_{N_i})'s are fuzzy neutrosophic $G_\delta-$ sets and fuzzy neutrosophic nowhere dense sets in (X, τ_N).

Example 3.2. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N and C_N defined on X as follows

$A_N = \{\langle a, 0.5, 0.4, 0.5\rangle, \langle b, 0.6, 0.5, 0.5\rangle, \langle c, 0.4, 0.5, 0.4\rangle\}$

$B_N = \{\langle a, 0.4, 0.5, 0.5\rangle, \langle b, 0.5, 0.5, 0.5\rangle, \langle c, 0.4, 0.6, 0.5\rangle\}$

$C_N = \{\langle a, 0.6, 0.5, 0.5\rangle, \langle b, 0.5, 0.5, 0.4\rangle, \langle c, 0.4, 0.4, 0.3\rangle\}$

Then, $\tau_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, A_N \vee C_N, B_N \vee C_N, A_N \wedge B_N, B_N \wedge C_N, A_N \wedge C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.

Then $((1 - A_N)^-)^+ = (A_N)^+ = 0$, $((1 - B_N)^-)^+ = (B_N)^+ = 0$, $((1 - C_N)^-)^+ = (C_N)^+ = 0$, $((1 - B_N \wedge C_N)^-)^+ = (B_N \wedge C_N)^+ = 0$, $((1 - A_N \wedge C_N)^-)^+ = (A_N \wedge C_N)^+ = 0$.

Therefore, fuzzy neutrosophic nowhere dense sets are $1 - A_N$, $1 - B_N$, $1 - C_N$, $1 - B_N \wedge C_N$ and $1 - A_N \wedge C_N$ in (X, τ_N).

The fuzzy neutrosophic sets are $\bigwedge_{i=1}^{\infty} (B_N \wedge (A_N \wedge B_N) \wedge (A_N \wedge C_N)) = A_N \wedge C_N$ is a fuzzy neutrosophic $G_\delta-$ sets in (X, τ_N).

This implies that $(\bigvee_{i=1}^{\infty} (A_N \wedge C_N) \vee (1 - A_N) \vee (1 - B_N))^+ = 0$.

Hence (X, τ_N) is a fuzzy neutrosophic directed Baire space.

Definition 3.3. Let (X, τ_N) be a fuzzy neutrosophic topological space. Then (X, τ_N) is called a fuzzy neutrosophic weakly directed Baire space if $(\bigvee_{i=1}^{\infty} (A_{N_i})^+ \neq 0)$, where (A_{N_i})'s are fuzzy neutrosophic $G_\delta-$ sets and fuzzy neutrosophic nowhere dense sets in (X, τ_N).

Example 3.4. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N and C_N defined on X as follows

$A_N = \{\langle a, 0.5, 0.6, 0.4\rangle, \langle b, 0.7, 0.6, 0.5\rangle, \langle c, 0.6, 0.5, 0.4\rangle\}$

$B_N = \{\langle a, 0.5, 0.5, 0.4\rangle, \langle b, 0.7, 0.5, 0.6\rangle, \langle c, 0.6, 0.5, 0.6\rangle\}$

$C_N = \{\langle a, 0.5, 0.6, 0.7\rangle, \langle b, 0.5, 0.4, 0.5\rangle, \langle c, 0.3, 0.4, 0.7\rangle\}$

Then, $\tau_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, A_N \vee C_N, B_N \vee C_N, A_N \wedge B_N, B_N \wedge C_N, A_N \wedge C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.

Then $((1 - C_N)^-)^+ = (C_N)^+ = 0$, $((1 - B_N \wedge C_N)^-)^+ = (B_N \wedge C_N)^+ = 0$, $((1 - A_N \wedge C_N)^-)^+ = (A_N \wedge C_N)^+ = 0$.

Therefore, fuzzy neutrosophic nowhere dense sets are $1 - C_N$, $1 - B_N \wedge C_N$ and $1 - A_N \wedge C_N$.

The fuzzy neutrosophic sets are $\bigwedge_{i=1}^{\infty} ((A_N \wedge B_N) \wedge (A_N \vee B_N) \wedge (B_N \vee C_N) \wedge (A_N \vee C_N) = A_N \wedge B_N$

is a fuzzy neutrosophic $G_\delta-$ sets in (X, τ_N).
This implies that $(\bigvee_{i=1}^{\infty}(A_N \wedge B_N) \vee (1 - B_N \vee C_N) \vee (1 - A_N \wedge C_N))^+ = A_N \wedge B_N \neq 0$.
Hence (X, τ_N) is a fuzzy neutrosophic weakly directed Baire space.

Proposition 3.5. *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then the following are equivalent.*
(i) (X, τ_N) is a fuzzy neutrosophic directed Baire space.
(ii) $(A_N)^+ = 0$, for every fuzzy neutrosophic one category set A_N in (X, τ_N).
(iii) $(A_N)^- = 1_N$, for each fuzzy neutrosophic residual set B_N in (X, τ_N).

Proof. $(i) \implies (ii)$
Let A_N be a fuzzy neutrosophic first category set in (X, τ_N). Then $A_N = (\bigvee_{i=1}^{\infty}(A_{N_i}))^+$, where A'_{N_i} s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Then $(A_N)^+ = (\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0$. Since (X, τ_N) is a fuzzy neutrosophic directed Baire space, $(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0$. Hence, $(A_N)^+ = 0$ for fuzzy neutrosophic one category set A_N in (X, τ_N).

$(ii) \implies (iii)$
Let B_N be a fuzzy neutrosophic residual set in (X, τ_N). Then $(1 - B_N)$ is a fuzzy neutrosophic one category set in (X, τ_N). By hypothesis, $(1-B_N)^+ = 0$ then by theorem 2.11, $1-(B_N)^- = 0$. Hence $(B_N)^- = 1$, for fuzzy neutrosophic residual set B_N in (X, τ_N).

$(iii) \implies (i)$
Let A_N be a fuzzy neutrosophic first category set in (X, τ_N). Then $A_N = (\bigvee_{i=1}^{\infty} A_{N_i})$, where A_{N_i}' s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Now A_N is a fuzzy neutrosophic one category set implies that $(1 - A_N)$ is a fuzzy neutrosophic residual set in (X, τ_N). By hypothesis, we have $(1 - A_N)^- = 1$ then by theorem 2.11, $1 - (A_N)^+ = 1$. Hence $(A_N)^+ = 0$. That is, $(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0$, where A_{N_i}' s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Hence (X, τ_N) is a fuzzy neutrosophic directed Baire space. □

Proposition 3.6. *If A_N is a fuzzy neutrosophic set in a fuzzy neutrosophic directed Baire space (X, τ_N), then (X, τ_N) is a fuzzy neutrosophic Baire space.*

Proof. Let A_N be a fuzzy neutrosophic $G_\delta-$ set in a fuzzy neutrosophic directed Baire space (X, τ_N). Also, the fuzzy neutrosophic $G_\delta-$ set is a fuzzy neutrosophic nowhere dense set in (X, τ_N). Therefore, A_N is a fuzzy neutrosophic nowhere dense set in in (X, τ_N) and hence (X, τ_N) is a fuzzy neutrosophic Baire space. □

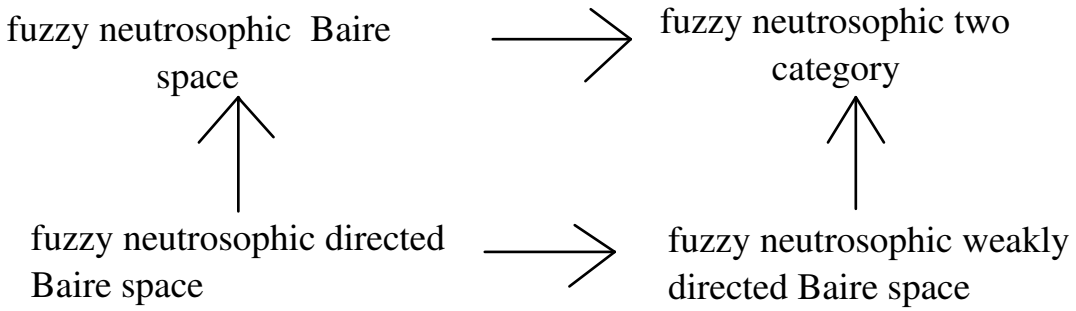
Proposition 3.7. *If the fuzzy neutrosophic topological space (X, τ_N) is a fuzzy neutrosophic directed Baire space, then (X, τ_N) is a fuzzy neutrosophic two category space.*

Proof. Let (X, τ_N) be a fuzzy neutrosophic directed Baire space. Then $(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0$, where A_{N_i}' s are fuzzy neutrosophic $G_\delta-$ sets and fuzzy neutrosophic nowhere dense sets in (X, τ_N). Then $\bigvee_{i=1}^{\infty}(A_{N_i}) \neq 1$ [otherwise $\bigvee_{i=1}^{\infty}(A_{N_i}) = 1$ implies that $(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = (1)^+ = 1$ and hence $0 = 1$, a contradiction]. Hence (X, τ_N) is a fuzzy neutrosophic second category space. □

Proposition 3.8. *If the fuzzy neutrosophic $P-$ space (X, τ_N) is a fuzzy neutrosophic hyperconnected space, then (X, τ_N) is a fuzzy neutrosophic directed Baire space.*

Proof. Let A_N be a fuzzy neutrosophic $G_\delta-$ set in a fuzzy neutrosophic $P-$ space (X, τ_N). Since (X, τ_N) is fuzzy neutrosophic $P-$ space is a fuzzy neutrosophic open set in (X, τ_N). Since the fuzzy neutrosophic $P-$ space (X, τ_N) is a fuzzy neutrosophic hyperconnected space, the fuzzy neutrosophic open set A_N in (X, τ_N) is a fuzzy neutrosophic dense set in (X, τ_N).That is $(A_N)^- = 1$. Hence A_N is a fuzzy neutrosophic $G_\delta-$ set and fuzzy neutrosophic nowhere dense set in (X, τ_N). Then by the proposition 3.5, $(1 - A_N)$ is a fuzzy neutrosophic one category set in (X, τ_N). Therefore $(1-A_N) = \bigvee_{i=1}^{\infty}(A_{N_i})$, where A_{N_i}' s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Then, $[\bigvee_{i=1}^{\infty}(A_{N_i})]^+ = (1 - A_N)^+ = 1 - (A_N)^- = 1 - 1 = 0$. Hence we have $[\bigvee_{i=1}^{\infty}(A_{N_i})]^+ = 0$, where A_{N_i}' s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Also, A_N is a fuzzy neutrosophic $G_\delta-$ sets in (X, τ_N). Therefore (X, τ_N) is a fuzzy neutrosophic directed Baire space. □

The following arrow diagram shows the relation between the four spaces namely, fuzzy neutrosophic Baire space, fuzzy neutrosophic two category, fuzzy neutrosophic directed baire space, fuzzy neutrosophic weakly directed baire space.


fuzzy neutrosophic Baire space
fuzzy neutrosophic two category
fuzzy neutrosophic directed Baire space
fuzzy neutrosophic weakly directed Baire space

Hence fuzzy neutrosophic Baire space, fuzzy neutrosophic two category, fuzzy neutrosophic directed baire space, fuzzy neutrosophic weakly directed baire space are independent concepts.

4. CHARACTERIZATIONS OF OTHER FUZZY NEUTROSOPHIC SPACES

Proposition 4.1. *If $(A_N)^+ = 0$ for a fuzzy neutrosophic set A_N defined on X in the fuzzy neutrosophic submaximal space (X, τ_N), then A_N is the fuzzy neutrosophic nowhere dense set in (X, τ_N).*

Proof. Suppose that $(A_N)^+ = 0$, for a fuzzy neutrosophic set A_N defined on X in (X, τ_N). Now $1 - (A_N)^+ = 1 - 0 = 1$ and by theorem 2.11, $(1 - A_N)^- = 1$. Since (X, τ_N) fuzzy neutrosophic submaximal space, $1 - A_N$ is a fuzzy neutrosophic open set in (X, τ_N) and then A_N is a fuzzy neutrosophic closed set in (X, τ_N), and thus $(A_N)^- = A_N$. This implies that $((A_N)^-)^+ = (A_N)^+ = 0$, in (X, τ_N) and hence A_N is the fuzzy neutrosophic nowhere dense set in (X, τ_N). □

Proposition 4.2. *If A_N is the fuzzy neutrosophic nowhere dense set in the fuzzy neutrosophic submaximal space (X, τ_N), then $1 - A_N$ is the fuzzy neutrosophic open set in (X, τ_N).*

Proof. Let A_N be the fuzzy neutrosophic nowhere dense set in (X, τ_N). Then, $((A_N)^-)^+ = 0$, in (X, τ_N). But $(A_N)^+ \leq ((A_N)^-)^+$, implies that $(A_N)^+ = 0$. Then, $1 - (A_N)^+ = 1$ and by theorem 2.11, $(1 - A_N)^- = 1$. Since (X, τ_N) is the fuzzy neutrosophic submaximal space, $1 - A_N$ is the fuzzy neutrosophic open set in (X, τ_N). □

Proposition 4.3. *If (X, τ_N) is the fuzzy neutrosophic submaximal space then $(A_N)^+ = 0$ iff $((A_N)^-)^+ = 0$, for the fuzzy neutrosophic set A_N defined on X in (X, τ_N).*

Proof. Suppose that $(A_N)^+ = 0$, for a fuzzy neutrosophic set A_N defined on X in (X, τ_N). Then by proposition 4.1, $((A_N)^-)^+ = 0$, in (X, τ_N). Conversely, suppose that $((A_N)^-)^+ = 0$, in (X, τ_N). Now $(A_N)^+ \leq ((A_N)^-)^+$, implies that $(A_N)^+ = 0$, in (X, τ_N). □

Proposition 4.4. *If (X, τ_N) is the fuzzy neutrosophic submaximal space, then (X, τ_N) is a fuzzy neutrosophic strongly irresolvable space.*

Proof. Let A_N be the fuzzy neutrosophic dense set in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic submaximal space, the fuzzy neutrosophic dense set A_N in (X, τ_N) is fuzzy neutrosophic open set and then $(A_N)^+ = A_N$, in (X, τ_N). Now $((A_N)^-)^+ = (A_N)^- = 1$, in (X, τ_N). Hence (X, τ_N) is a fuzzy neutrosophic strongly irresolvable space. □

Proposition 4.5. *If (X, τ_N) is the fuzzy neutrosophic almost resolvable space and fuzzy neutrosophic submaximal space, then (X, τ_N) is not a fuzzy neutrosophic two category space.*

Proof. Let (X, τ_N) be a fuzzy neutrosophic almost resolvable space. Then $\bigvee_{i=1}^{\infty}(A_{N_i}) = 1$, where the fuzzy neutrosophic sets (A_{N_i})'s in (X, τ_N) are such that $(A_{N_i})^+ = 0$. Since (X, τ_N) is a fuzzy neutrosophic submaximal space, by proposition 4.3, $(A_{N_i})^+ = 0$ in (X, τ_N), implies that $((A_{N_i})^-)^+ = 0$, in (X, τ_N) and hence (A_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Thus, $\bigvee_{i=1}^{\infty}(A_{N_i}) = 1$, where (A_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X, τ_N), implies that (X, τ_N) is fuzzy neutrosophic one category space and hence (X, τ_N) is not fuzzy neutrosophic two category space. □

Proposition 4.6. *If (X, τ_N) is the fuzzy neutrosophic submaximal space, then for any fuzzy neutrosophic set A_N defined on X, $(A_N)^- \wedge (1 - A_N)$ is the fuzzy neutrosophic closed set in (X, τ_N) .*

Proof. Let (X, τ_N) be a fuzzy neutrosophic submaximal space and A_N be a fuzzy neutrosophic set defined on X. It is claimed that $[(1-(A_N)^-) \vee A_N)]$ is a neutrosophic dense set in (X, τ_N). Assume the contrary. Suppose that $([1-(A_N)^-] \vee A_N)^- \neq 1$. Then $1-([1-(A_N)^-]^- \vee A_N) \neq 0$ and this implies that $1-([1-(A_N)^-]^- \vee (A_N)^-) \neq 0$ and $\{1-[1-(A_N)^-]^-\} \wedge \{1-(A_N)^-\} \neq 0$. Then, $((A_N)^-)^+ \wedge \{1 - (A_N)^-\} \neq 0$. This will imply that $((A_N)^-)^+ \nleq 1 - \{1 - (A_N)^-\}$. Then, $((A_N)^-)^+ \nleq (A_N)^-$, a contradiction. Hence it must be $([1 - (A_N)^-] \vee A_N) = 1$, in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic submaximal space, $[1-(A_N)^-] \vee A_N$ is the fuzzy neutrosophic open set in (X, τ_N). Then, $1-([1-(A_N)^-] \vee A_N)$ is the fuzzy neutrosophic closed set in (X, τ_N) and hence $(A_N)^- \wedge (1 - A_N)$ is the fuzzy neutrosophic closed set in (X, τ_N). □

Proposition 4.7. *If each fuzzy neutrosophic $G_\delta-$ set is a fuzzy neutrosophic dense set in a fuzzy neutrosophic submaximal space (X, τ_N), then (X, τ_N) is a fuzzy neutrosophic $P-$ space.*

Proof. Let A_N be a fuzzy neutrosophic $G_\delta-$ set in a fuzzy neutrosophic submaximal space (X, τ_N). Then, by the hypothesis, A_N is a fuzzy neutrosophic dense set in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic submaximal space, the fuzzy neutrosophic dense set A_N in (X, τ_N), is a fuzzy neutrosophic open set in (X, τ_N). That is, every fuzzy neutrosophic $G_\delta-$ set in (X, τ_N) is a fuzzy neutrosophic open set in (X, τ_N). Therefore (X, τ_N) is a fuzzy neutrosophic $P-$ space. □

Proposition 4.8. *If $(A_N)^+ = 0$, where A_N is a fuzzy neutrosophic $F_\sigma-$ set in a fuzzy neutrosophic submaximal space (X, τ_N), then (X, τ_N) is a fuzzy neutrosophic $P-$ space.*

Proof. Let A_N be a fuzzy neutrosophic $G_\delta-$ set in a fuzzy neutrosophic submaximal space (X, τ_N). Then, $(1 - A_N)$ is a fuzzy neutrosophic $F_\sigma-$ set in (X, τ_N). Then, by the hypothesis,

$(1 - A_N)^+ = 0$, for the fuzzy neutrosophic $F_\sigma-$ set in (X, τ_N). This implies that $(A_N)^- = 1$. Then A_N is a fuzzy neutrosophic dense set in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic submaximal space, the fuzzy neutrosophic dense set A_N in (X, τ_N), is a fuzzy neutrosophic open set in (X, τ_N). That is, every fuzzy neutrosophic $G_\delta-$ set in (X, τ_N) is a fuzzy neutrosophic open set in (X, τ_N). Therefore (X, τ_N) is a fuzzy neutrosophic $P-$ space. □

Proposition 4.9. *If each fuzzy neutrosophic $F_\sigma-$ set is a fuzzy neutrosophic nowhere dense set in a fuzzy neutrosophic submaximal space (X, τ_N), then (X, τ_N) is a fuzzy neutrosophic $P-$ space.*

Proof. Let A_N be a fuzzy neutrosophic $F_\sigma-$ set in a fuzzy neutrosophic submaximal space (X, τ_N) such that $((A_N)^-)^+ = 0$. Then $(A_N)^+ \leq ((A_N)^-)^+$, implies that $(A_N)^+ = 0$. Now $(A_N)^+ = 0$ for a fuzzy neutrosophic $F_\sigma-$ set in a fuzzy neutrosophic submaximal space (X, τ_N). Then by proposition 4.8, (X, τ_N) is a fuzzy neutrosophic $P-$ space. □

Proposition 4.10. *If $((A_N)^+)^- = 1$, for each fuzzy neutrosophic $G_\delta-$ set A_N in a fuzzy neutrosophic submaximal space (X, τ_N), then (X, τ_N) is a fuzzy neutrosophic $P-$ space.*

Proof. Let A_N be a fuzzy neutrosophic $F_\sigma-$ set in a fuzzy neutrosophic submaximal space (X, τ_N). Then $(1 - A_N)$ is a fuzzy neutrosophic $G_\delta-$ set in (X, τ_N). By hypothesis, $((1 - A_N)^+)^- = 1$. Then $1 - ((1 - A_N)^+)^- = 0$. This implies that $1 - [1 - ((A_N)^-)^+] = 0$. That is, $((A_N)^-)^+ = 0$ and hence A_N is a fuzzy neutrosophic nowhere dense set in (X, τ_N). Thus the fuzzy neutrosophic $F_\sigma-$ set is a fuzzy neutrosophic nowhere dense set in a fuzzy neutrosophic submaximal space (X, τ_N). Hence by proposition 4.9, (X, τ_N) is a fuzzy neutrosophic $P-$ space. □

Proposition 4.11. *If A_N is a fuzzy neutrosophic residual set in a fuzzy neutrosophic submaximal space (X, τ_N), then A_N is a fuzzy neutrosophic $G_\delta-$ set in (X, τ_N).*

Proof. Let A_N be a fuzzy neutrosophic residual set in a fuzzy neutrosophic submaximal space (X, τ_N). Then $1 - A_N$ is a fuzzy neutrosophic one category set in (X, τ_N) and hence $1 - A_N = \bigvee_{i=1}^{\infty} (A_{N_i})$, where (A_{N_i})'s are fuzzy neutrosophic nowhere dense set in (X, τ_N). Since (A_{N_i})'s are fuzzy neutrosophic nowhere dense set (X, τ_N), $((A_{N_i})^-)^+ = 0$. Then $(A_{N_i})^+ \leq ((A_{N_i})^-)^+$, implies that $(A_{N_i})^+ = 0$. This implies that $1 - (A_{N_i})^+ = 1$ and hence $(1 - A_{N_i})^- = 1$. Since (X, τ_N) is a fuzzy neutrosophic submaximal space, the fuzzy neutrosophic dense sets $(1 - A_{N_i})$'s are fuzzy neutrosophic open sets in (X, τ_N). Then (A_{N_i})'s are fuzzy neutrosophic closed sets in (X, τ_N). Hence $1 - A_N = \bigvee_{i=1}^{\infty} (A_{N_i})$, where (A_{N_i})'s are fuzzy neutrosophic closed

sets in (X, τ_N), implies that $1 - A_N$ is fuzzy neutrosophic $F_\sigma-$ set in (X, τ_N). Therefore A_N is a fuzzy neutrosophic $G_\delta-$ set in (X, τ_N). □

Proposition 4.12. *If A_N is a fuzzy neutrosophic residual set in a fuzzy neutrosophic submaximal space and fuzzy neutrosophic $P-$ space (X, τ_N), then A_N is a fuzzy neutrosophic open set in (X, τ_N).*

Proof. Let A_N be a fuzzy neutrosophic residual set in a fuzzy neutrosophic submaximal space and fuzzy neutrosophic $P-$ space (X, τ_N). Since A_N is a fuzzy neutrosophic residual set in a fuzzy neutrosophic submaximal space (X, τ_N), by proposition 4.11, A_N is a fuzzy neutrosophic $G_\delta-$ set in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic $P-$ space, the fuzzy neutrosophic $G_\delta-$ set in (X, τ_N) is a fuzzy neutrosophic open set in (X, τ_N). Hence fuzzy neutrosophic residual set A_N in a fuzzy neutrosophic submaximal space and fuzzy neutrosophic $P-$ space (X, τ_N) is a fuzzy neutrosophic open set in (X, τ_N). □

Proposition 4.13. *If A_N is the fuzzy neutrosophic nowhere dense set in a fuzzy neutrosophic submaximal space (X, τ_N), then A_N is a fuzzy neutrosophic closed set in (X, τ_N).*

Proof. Let A_N be a fuzzy neutrosophic nowhere dense set in a fuzzy neutrosophic submaximal space (X, τ_N). Then we have $((A_N)^-)^+ = 0$ and $(A_N)^+ \leq ((A_N)^-)^+$, implies that $(A_N)^+ = 0$. Then, $1 - (A_N)^+ = 1$ implies that $(1 - A_N)^- = 1$ and hence $1 - A_N$ is the fuzzy neutrosophic dense set in (X, τ_N). Since (X, τ_N) fuzzy neutrosophic submaximal space, $1 - A_N$ is the fuzzy neutrosophic open set in (X, τ_N). Therefore the fuzzy neutrosophic nowhere dense set A_N is the fuzzy neutrosophic closed set in (X, τ_N). □

Proposition 4.14. *If (A_{N_i})'s are fuzzy neutrosophic nowhere dense sets in a fuzzy neutrosophic submaximal space and fuzzy neutrosophic $P-$ space (X, τ_N), then $(\bigvee_{i=1}^{\infty}(A_{N_i}))^- = \bigvee_{i=1}^{\infty}(A_{N_i})$.*

Proof. Let (A_{N_i})'s be fuzzy neutrosophic nowhere dense sets in a fuzzy neutrosophic submaximal and fuzzy neutrosophic $P-$ space (X, τ_N). Since (A_{N_i})'s are fuzzy neutrosophic nowhere dense sets in a fuzzy neutrosophic submaximal space (X, τ_N), by proposition 4.13, (A_{N_i})'s are fuzzy neutrosophic closed sets in (X, τ_N) and hence $(1 - A_{N_i})$'s are fuzzy neutrosophic open sets in (X, τ_N). Now $B_N = \bigwedge_{i=1}^{\infty}(1 - A_{N_i})$ is a non – zero fuzzy neutrosophic $G_\delta-$ set in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic $P-$ space, B_N is a fuzzy neutrosophic open set in (X, τ_N) and hence we have $(B_N)^+ = B_N$. This implies that $(\bigwedge_{i=1}^{\infty}(1 - A_{N_i}))^+ = \bigwedge_{i=1}^{\infty}(1 - A_{N_i})$.

Then, $(1 - \bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 1 - \bigvee_{i=1}^{\infty}(A_{N_i})$ and hence $1 - (\bigvee_{i=1}^{\infty}(A_{N_i}))^- = 1 - \bigvee_{i=1}^{\infty}(A_{N_i})$. Therefore we have $(\bigvee_{i=1}^{\infty}(A_{N_i}))^- = \bigvee_{i=1}^{\infty}(A_{N_i})$. □

Proposition 4.15. *If a fuzzy neutrosophic $P-$ space (X, τ_N) is a fuzzy neutrosophic hyperconnected space, then (X, τ_N) is a fuzzy neutrosophic Baire space.*

Proof. Let A_N be a fuzzy neutrosophic $G_\delta-$ set in a fuzzy neutrosophic $P-$ space (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic $P-$ space, is a fuzzy neutrosophic open set in (X, τ_N). Since the fuzzy neutrosophic space (X, τ_N), is a fuzzy neutrosophic hyperconnected space, the fuzzy neutrosophic open set A_N in (X, τ_N) is a fuzzy neutrosophic dense set in (X, τ_N). That is, $(A_N)^- = 1$. Hence A_N is a fuzzy neutrosophic $G_\delta-$ set and fuzzy neutrosophic dense set in (X, τ_N). Then by proposition, $(1 - A_N)$ is the fuzzy neutrosophic one category set in (X, τ_N). Therefore $(1 - A_N) = \bigvee_{i=1}^{\infty}(A_{N_i})$. where (A_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Then $[\bigvee_{i=1}^{\infty}(A_{N_i})]^+ = [1 - A_N]^+ = 1 - (A_N)^- = 1 - 1 = 0$. Hence we have $[\bigvee_{i=1}^{\infty}(A_{N_i})] = 0$ where (A_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Therefore (X, τ_N) is a fuzzy neutrosophic Baire space. □

Proposition 4.16. *If a fuzzy neutrosophic $P-$ space (X, τ_N) is a fuzzy neutrosophic hyperconnected space, then (X, τ_N) is a fuzzy neutrosophic second category space.*

Proof. Let the fuzzy neutrosophic $P-$ space (X, τ_N) be a fuzzy neutrosophic hyperconnected space. Then by proposition 4.15, (X, τ_N) is a fuzzy neutrosophic Baire space and hence $[\bigvee_{i=1}^{\infty}(A_{N_i})]^+ = 0$, where (A_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). We claim that $[\bigvee_{i=1}^{\infty}(A_{N_i})] \neq 1$. Suppose that $[\bigvee_{i=1}^{\infty}(A_{N_i})] = 1$. This would imply that $[\bigvee_{i=1}^{\infty}(A_{N_i})]^+ = (1)^+ = 1 \neq 0$, a contradiction. Hence we must have $\bigvee_{i=1}^{\infty}(A_{N_i}) \neq 1$, where (A_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Therefore (X, τ_N) is a fuzzy neutrosophic second category space. □

Proposition 4.17. *A fuzzy neutrosophic topological space (X, τ_N) is a fuzzy neutrosophic resolvable space, if $\bigvee_{i=1}^{N}(A_{N_i}) = 1$, where the fuzzy neutrosophic sets $(A_{N_i})'s$ in (X, τ_N) are such that $(A_{N_i})^+ = 0$.*

Proof. Now $\bigvee_{i=1}^{N}(A_{N_i}) = 1$ implies that $1 - \bigvee_{i=1}^{N}(A_{N_i}) = 0$. Then $\bigwedge_{i=1}^{N}(1 - A_{N_i}) = 0$. Hence there must be atleast two non - zero disjoint fuzzy neutrosophic sets $1 - A_{N_i}$ and $1 - A_{N_j}$ in

(X, τ_N). Then $(1 - A_{N_i}) + (1 - A_{N_j}) \leq 1$, which implies that $(1 - A_{N_i}) \leq A_{N_j}$ and hence $(1 - A_{N_j})^- \leq (A_{N_j})^-$. Now $(A_{N_i})^+ = 0$ implies that $1 - (A_{N_i})^+ = 1$ and hence $(1 - A_{N_i})^- = 1$. Then $1 \leq (A_{N_j})^-$. That is, $(A_{N_j})^- = 1$. Also $(A_{N_j})^+ = 0$ implies that $(1 - A_{N_j})^- = 1$. Hence the fuzzy neutrosophic topological space (X, τ_N) has a dense fuzzy neutrosophic set A_{N_j} such that $(1 - A_{N_j})^- = 1$. Hence (X, τ_N) is a fuzzy neutrosophic resolvable space. It is clear that from the definition that every fuzzyneutrosophic resolvable space is a fuzzy neutrosophic almost resolvable space. □

Proposition 4.18. *If the fuzzy neutrosophic topological space (X, τ_N) is a fuzzy neutrosophic first category space, then (X, τ_N) fuzzy neutrosophic almost resolvable space.*

Proof. Since the fuzzy neutrosophic topological space (X, τ_N) is of fuzzy neutrosophic first category space, we have $1 = \bigvee_{i=1}^{\infty} (A_{N_i})$, where $(A_{N_i})'s$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Now (A_{N_i}) is fuzzy neutrosophic nowhere dense set implies that $((A_{N_i})^-)^+ = 0$. Since $(A_{N_i})^+ \leq ((A_{N_i})^-)^+$ implies that $(A_{N_i})^+ = 0$. Hence $\bigvee_{i=1}^{\infty} (A_{N_i}) = 1$, where $(A_{N_i})^+ = 0$ and therefore (X, τ_N) fuzzy neutrosophic almost resolvable space. □

Proposition 4.19. *If the fuzzy neutrosophic topological space (X, τ_N) is a fuzzy neutrosophic almost resolvable space, then $\bigwedge_{i=1}^{\infty} (B_{N_i}) = 0$, where (B_{N_i})'s are fuzzy neutrosophic dense sets in (X, τ_N).*

Proof. Since the fuzzy neutrosophic topological space (X, τ_N) is a fuzzy neutrosophic almost resolvable space, we have $\bigvee_{i=1}^{\infty} (A_{N_i}) = 1$, where $(A_{N_i})^+ = 0$ in (X, τ_N). Now $(A_{N_i})^+ = 0$ implies that $1 - (A_{N_i})^+ = 1 - 0 = 1$ Then $(1 - A_{N_i})^- = 1$. Also $1 - \bigvee_{i=1}^{\infty} (A_{N_i}) = 0$ implies that $\bigwedge_{i=1}^{\infty} (1 - A_{N_i}) = 0$. Let $1 - A_{N_i} = B_{N_i}$. Hence $\bigwedge_{i=1}^{\infty} (B_{N_i}) = 0$, where $(B_{N_i})'s$ are fuzzy neutrosophic dense sets in (X, τ_N). □

Proposition 4.20. *If the fuzzy neutrosophic topological space (X, τ_N) is a fuzzy neutrosophic almost resolvable and fuzzy neutrosophic submaximal space, then $\bigvee_{i=1}^{\infty} (A_{N_i}) = 1$, where $(A_{N_i})'s$ are fuzzy neutrosophic closed sets in (X, τ_N).*

Proof. Since the fuzzy neutrosophic topological space (X, τ_N) is a fuzzy neutrosophic almost resolvable space, by proposition 4.18, we have $\bigwedge_{i=1}^{\infty} (B_{N_i}) = 0$, where $(B_{N_i})'s$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic submaximal space, $(B_{N_i})^- = 1$ implies that $B_{N_i} \in \tau_N$. Then $\bigwedge_{i=1}^{\infty} (B_{N_i}) = 0$ implies that $1 - \bigwedge_{i=1}^{\infty} (B_{N_i}) = 1$

and hence $\bigvee_{i=1}^{\infty}(1-B_{N_i})=1$, where $(1-B_{N_i})'s$ are fuzzy neutrosophic closed sets in (X,τ_N). $(1-B_{N_i})=(A_{N_i})$. Hence $\bigvee_{i=1}^{\infty}(A_{N_i})=1$, where $(A_{N_i})'s$ are fuzzy neutrosophic closed sets in (X,τ_N). □

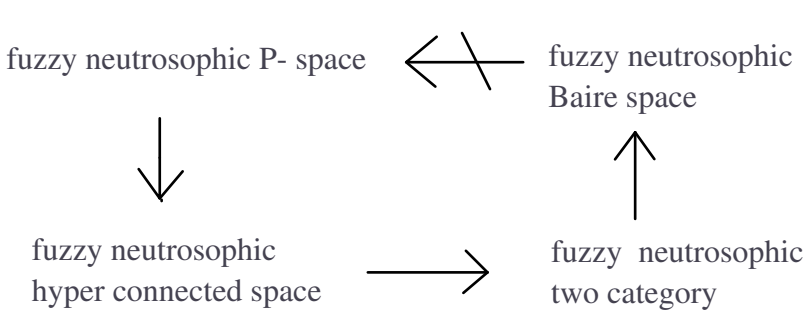
Proposition 4.21. *If the fuzzy neutrosophic topological space (X,τ_N) is a fuzzy neutrosophic irresolvable space, then (X,τ_N) is not a fuzzy neutrosophic almost resolvable space.*

Proof. Suppose that the fuzzy neutrosophic topological space (X,τ_N) is a fuzzy neutrosophic almost resolvable space. Then $\bigvee_{i=1}^{\infty}(A_{N_i})=1$, where $(A_{N_i})^+=0$ in (X,τ_N), implies that $\bigwedge_{i=1}^{\infty}(1-A_{N_i})=0$. Hence there must be atleast two non – zero disjoint fuzzy neutrosophic sets $1-A_{N_i}$ and $1-A_{N_j}$ in (X,τ_N). Then $(1-A_{N_i})+(1-A_{N_j})\leq 1$, which implies that $(1-A_{N_i})\leq A_{N_j}$ and hence $(1-A_{N_i})^-\leq (A_{N_j})^-$. Then $1-(A_{N_i})^+\leq (A_{N_j})^-$. Now $(A_{N_i})^+=0$ implies that $1\leq (A_{N_j})^-$. That is, $(A_{N_j})^-=1$. Also $(A_{N_j})^+$ implies that $1-(A_{N_j})^+=1$ and hence $(1-A_{N_j})^-=1$. Hence (X,τ_N) has a dense fuzzy neutrosophic set A_{N_j} such that $(1-A_{N_j})^-=1$, which means that (X,τ_N) is a fuzzy neutrosophic resolvable space. But this a contradiction to (X,τ_N) being a fuzzy neutrosophic irresolvable space. Therefore (X,τ_N) is not a fuzzy neutrosophic almost resolvable space. □

Proposition 4.22. *If the fuzzy neutrosophic topological space (X,τ_N) is a fuzzy neutrosophic Baire space, then (X,τ_N) is a fuzzy neutrosophic almost irresolvable space.*

Proof. Since (X,τ_N) is a fuzzy neutrosophic Baire space, $(\bigvee_{i=1}^{\infty}(A_{N_i}))^+=0$, where $(A_{N_i})'s$ are fuzzy neutrosophic nowhere dense sets in (X,τ_N). Now A_{N_i} is a fuzzy neutrosophic nowhere dense set implies that $((A_{N_i})^-)^+=0$. Since $(A_{N_i})^+\leq((A_{N_i})^-)^+$, we have $(A_{N_i})^+=0$. Hence $(\bigvee_{i=1}^{\infty}(A_{N_i}))^+=0$, where $(A_{N_i})^+=0$. Suppose that (X,τ_N) is a fuzzy neutrosophic almost resolvable space. Then $\bigvee_{i=1}^{\infty}(A_{N_i})=1$, where $(A_{N_i})^+=0$. Now $(\bigvee_{i=1}^{\infty}(A_{N_i}))^+=(1)^+=1$, which implies that 0=1, a contradiction. Hence we must have $\bigvee_{i=1}^{\infty}(A_{N_i})\neq 1$, where $(A_{N_i})^+=0$. Therefore (X,τ_N) is a fuzzy neutrosophic almost irresolvable space. □

Remark The inter-relations between fuzzy neutrosophic $P-$ space, fuzzy neutrosophic hyperconnected space, fuzzy neutrosophic two category, fuzzy neutrosophic Baire space can be summerized as follows:



5. Fuzzy Neutrosophic Continuous Function And Fuzzy Neutrosophic Directed Baire Spaces

It is observed to the study that fuzzy neutrosophic directed Bairness is not preserved under fuzzy neutrosophic continuous function. For consider the following example.

Example 5.1. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N, C_N, E_N, F_N and P_N defined on X as follows

$A_N = \{\langle a, 0.5, 0.6, 0.7\rangle, \langle b, 0.6, 0.5, 0.6\rangle, \langle c, 0.7, 0.5, 0.6\rangle\}$
$B_N = \{\langle a, 0.8, 0.4, 0.4\rangle, \langle b, 0.4, 0.2, 0.8\rangle, \langle c, 0.2, 0.2, 0.8\rangle\}$
$C_N = \{\langle a, 0.7, 0.5, 0.8\rangle, \langle b, 0.5, 0.7, 0.8\rangle, \langle c, 0.8, 0.8, 0.5\rangle\}$
$E_N = \{\langle a, 0.6, 0.5, 0.6\rangle, \langle b, 0.7, 0.6, 0.7\rangle, \langle c, 0.5, 0.6, 0.5\rangle\}$
$F_N = \{\langle a, 0.4, 0.3, 0.4\rangle, \langle b, 0.2, 0.2, 0.3\rangle, \langle c, 0.8, 0.7, 0.8\rangle\}$
$P_N = \{\langle a, 0.5, 0.5, 0.4\rangle, \langle b, 0.8, 0.7, 0.8\rangle, \langle c, 0.7, 0.7, 0.6\rangle\}$

Then, $\tau_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, A_N \vee C_N, B_N \vee C_N, A_N \wedge B_N, B_N \wedge C_N, A_N \wedge C_N, A_N \vee (B_N \wedge C_N), B_N \vee (A_N \wedge C_N), C_N \vee (A_N \wedge B_N), A_N \vee B_N \vee C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.

Then, $S_N = \{0_N, E_N, F_N, P_N, E_N \vee F_N, E_N \vee P_N, F_N \vee P_N, E_N \wedge P_N, F_N \wedge P_N, E_N \wedge F_N, E_N \vee (F_N \wedge P_N), F_N \vee (E_N \wedge P_N), P_N \wedge (E_N \vee F_N), 1_N\}$ are fuzzy neutrosophic topologies on X.

Now $1 - [C_N \wedge (A_N \vee B_N)]$ and $(1 - B_N \wedge C_N)$ are fuzzy neutrosophic $G_\delta-$ sets and fuzzy neutrosophic nowhere dense sets in (X, τ_N) and $[\{1-[C_N \wedge (A_N \vee B_N)]\} \vee \{1-(B_N \wedge C_N)\}]^+ = 0$ and hence (X, τ_N) is a fuzzy neutrosophic directed Baire space. The fuzzy neutrosophic topological space (X, S_N) is not a fuzzy neutrosophic Baire space, Since there are no fuzzy neutrosophic $G_\delta-$ sets and fuzzy neutrosophic nowhere dense sets in G_{N_i} in (X, S_N) such that $(\bigvee_{i=1}^{\infty}(G_{N_i}))^+ = 0$.
Define a function $f : (X, \tau_N) \implies (X, S_N)$ by $f(a_n) = c_n$, $f(b_n) = a_n$, $f(c_n) = b_n$. Then f is a fuzzy neutrosophic continuous function from the fuzzy neutrosophic directed Baire space

(X, τ_N) onto the fuzzy neutrosophic topological space (X, S_N) which is not a fuzzy neutrosophic directed Baire space.

6. **Conclusion**

In this paper, we have intoduced the concepts of fuzzy neutrosophic directed Baire spaces and fuzzy neutrosophic weakly directed Baire spaces. We have proved that there are spaces which are fuzzy neutrosophic Baire, fuzzy neutrosophic directed Baire, fuzzy neutrosophic weakly directed Baire and fuzzy neutrosophic second category spaces. Also we have proved interrelations between fuzzy neutrosophic Baire space, fuzzy neutrosophic directed Baire space, fuzzy neutrosophic weakly directed Baire space, fuzzy neutrosophic second category space, fuzzy neutrosophic $P-$ space, fuzzy neutrosophic hyperconnected space, fuzzy neutrosophic submaximal space, fuzzy neutrosophic almost resolvable space and fuzzy neutrosophic almost irresolvable spaces. Application to different fields of fuzzy neutrosophic toplogical spaces such as soft computing, artificial intelligence, decision making, pattern recognition, and image processing.

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ON FUZZY NEUTROSOPHIC SEMI-OPEN(CLOSED) SETS IN FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract. In this paper is to define the fuzzy neutrosophic semi-open sets (closed sets) in fuzzy neutrosophic topological spaces. Also, we discuss some of its characterizations of fuzzy neutrosophic semi-open sets (closed sets) and fuzzy neutrosophic semi-interior(closure) are studied. Several examples are given to illustrate the concepts.

Keywords: Fuzzy neutrosophic semi-open set, fuzzy neutrosophic semi-closed set, fuzzy neutrosophic semi-interior, fuzzy neutrosophic semi-closure.

1. Introduction

The concept of fuzzy sets were introduced by L.A. Zadeh in 1965 [8]. Then the fuzzy set theory is extension by many researchers. The important concept of fuzzy topological space was offered by C. L. Chang [4] and from that point forward different ideas in topology have been reached out to fuzzy topological space. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy toplogy has been developed. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [7] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

In this paper, we introduce the concept of fuzzy neutrosophic semi-open (semi-closed) and fuzzy neutrosophic semi-interior(semi-closure) are introduced and several characterizations with examples are studied in fuzzy neutrosophic topological spaces.

2. Preliminaries

Throughout this present paper, X denotes the fuzzy neutrosophic topological spaces. Let A_N be a fuzzy neutrosophic set on X. The fuzzy neutrosophic interior and closure of A_N is denoted as $fn(A_N)^+$, $fn(A_N)^-$ respectively. A fuzzy neutrosophic set A_N is defined to be

fuzzy neutrosophic open set ($fnOS$) if $A_N \leq fn(((A_N)^-)^+)^-$. The complement of a fuzzy neutrosophic open set is called fuzzy neutrosophic closed set ($fnCS$).

Definition 2.1. [3] A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F : X \to [0, 1]$ and $0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3$. With the condition $0 \leq T_{A^*}(x) + I_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2. [3] A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

Definition 2.3. [3] Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then
$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$
$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$

Definition 2.4. [3] The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$.

Definition 2.5. [3] A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0, I_A(x) = 0, F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N.

Definition 2.6. [3] A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N.

Definition 2.7. [3] The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$. The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8. [3] A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X
(i) $0_N, 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$
Satisfying the following axioms.
In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9. [3] The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10. [3] Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by
$int(A) = \cup\{G : G$ is a fuzzy neutrosophic open set in X and $G \subseteq A\ \}$
$cl(A) = \cap\{G : G$ is a fuzzy neutrosophic closed set in X and $A \subseteq G\}$

Theorem 2.11. *[3] Let (X, τ_N) be a fuzzy neutrosophic topological space over X. Then the following properties hold.* $(i)cl(A^c) = (intA)^c$ $(ii)int(A^c) = (clA)^c$

Proposition 2.12. *[3] Let $A_i's$ be fuzzy neutrosophic sets in $X, i \in J$ then* $(i)(\cup A_i)^c = \cap {A_i}^c$ $(ii)(\cap A_i)^c = \cup {A_i}^c$

Proposition 2.13. *[3] Let A and B be fuzzy neutrosophic sets then $A \subseteq B \iff B^c \subseteq A^c$*

Proposition 2.14. *[3] Let A and B be a fuzzy neutrosophic set in X then $(A^c)^c = A$*

Proposition 2.15. *[7] Let (X, τ) be a $FNTS$ and A, B be $FNS's$ in X. Then the following properties hold.*
$a)int(A) \subseteq A, A \subseteq cl(A)$
$b)A \subseteq B \implies int(A) \subseteq int(B), A \subseteq B \implies cl(A) \subseteq cl(B)$
$c)int(int(A)) = int(A), cl(cl(A)) = cl(A)$
$d)int(A \cap B) = int(A) \cap int(B), cl(A \cup B) = cl(A) \cup Cl(B)$
$e)int(1_N) = 1_N, cl(0_N) = 0_N$

3. Fuzzy neutrosophic semi-open sets in fuzzy neutrosophic topological spaces.

In this section, we introduce the concept of fuzzy neutrosophic semi-open(closed) set and also discussed their characterizations.

Definition 3.1. Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X, τ_N). Then A_N is said to be fuzzy neutrosophic semi-open set in (X, τ_N) if there exists a fuzzy neutrosophic open setfnO such that $fnO \leq A_N \leq fncl(fnO)$.

The following theorem is the characterization of fuzzy neutrosophic semi-open set in fuzzy neutrosophic topological space.

Theorem 3.2 A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi-open set if only if $A_N \leq ((A_N)^+)^-$.

Proof. Sufficiency: Let $A_N \leq ((A_N)^+)^-$. Then for $fnO = (A_N)^+$, we have $fnO \leq A_N \leq fncl(fnO)$. *Necessity:* Let A_N be a fuzzy neutrosophic semi-open set in (X, τ_N). Then $fnO \leq A_N \leq fncl(fnO)$ for some fuzzy neutrosophic semi-open set A_N. But $fnO = (A_N)^+$ and thus $fncl(fnO) \leq ((A_N)^+)^-$. Hence $A_N \leq fncl(fnO) \leq ((A_N)^+)^-$. □

Theorem 3.3 Let (X, τ_N) be a fuzzy neutrosophic topological space. Then the union of two fuzzy neutrosophic semi-open sets is a fuzzy neutrosophic semi-open set in the fuzzy neutrosophic topological space (X, τ_N).

Proof. Let A_N and B_N are fuzzy neutrosophic semi-open sets in (X, τ_N). Then $A_N \leq ((A_N)^+)^-$ and $B_N \leq ((B_N)^+)^-$. Therefore $A_N \vee B_N \leq ((A_N)^+)^- \vee ((B_N)^+)^- = ((A_N)^+ \vee (B_N)^+)^- \leq ((A_N \vee B_N)^+)^-$. By proposition 2.15, hence $A_N \vee B_N$ is fuzzy neutrosophic semi-open set in (X, τ_N). □

Theorem 3.4 Let (X, τ_N) be a fuzzy neutrosophic topological space. If $\{A_{N_i}\}_{i\in\Delta}$ is a collection of fuzzy neutrosophic semi-open sets in a fuzzy neutrosophic topological space (X, τ_N). Then $\bigvee_{i\in\Delta} A_{N_i}$ is fuzzy neutrosophic semi-open set in (X, τ_N).

Proof. For each $i \in \Delta$, we have a fuzzy neutrosophic semi-open set fnO_i such that $fnO_i \leq A_{N_i} \leq fncl(fnO_i)$. Then $\bigvee_{i\in\Delta} fnO_i \leq \bigvee_{i\in\Delta} A_{N_i} \leq \bigvee_{i\in\Delta} fncl(fnO_i) \leq fncl(\bigvee_{i\in\Delta} fnO_i)$. Hence let $fnO = \bigvee_{i\in\Delta}(fnO_i)$. □

Remark 3.5 The intersection of any two fuzzy neutrosophic semi-open sets need not be a fuzzy neutrosophic semi-open set in (X, τ_N) as shown by the following example.

Example 3.2. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N and C_N defined on X as follows

$A_N = \{\langle a, 0.5, 0.3, 0.4\rangle, \langle b, 0.5, 0.2, 0.6\rangle, \langle c, 0.4, 0.1, 0.4\rangle\}$
$B_N = \{\langle a, 0.6, 0.1, 0.5\rangle, \langle b, 0.5, 0.8, 0.1\rangle, \langle c, 0.3, 0.2, 0.2\rangle\}$
$C_N = \{\langle a, 0.2, 0.4, 0.8\rangle, \langle b, 0.4, 0.2, 0.4\rangle, \langle c, 0.4, 0.2, 0.3\rangle\}$ Then, $\tau_N = \{0_N, A_N, B_N, C_N, 1_N\}$ is a fuzzy neutrosophic topology on X. Now, we define the two fuzzy neutrosophic semi-open sets as follows:
$E_N = \{\langle a, 0.5, 0.4, 0.8\rangle, \langle b, 0.5, 0.3, 0.5\rangle, \langle c, 0.6, 0.2, 0.4\rangle\}$
$F_N = \{\langle a, 0.5, 0.3, 0.5\rangle, \langle b, 0.5, 0.3, 0.7\rangle, \langle c, 0.4, 0.2, 0.4\rangle\}$.
Here $(E_N)^+ = C_N$ and $((E_N)^+)^- = 1$ and $(F_N)^+ = A_N$ and $((F_N)^+)^- = 1$.
But $E_N \wedge F_N = \{\langle a, 0.5, 0.3, 0.5\rangle, \langle b, 0.5, 0.3, 0.5\rangle, \langle c, 0.4, 0.2, 0.4\rangle\}$ is not a fuzzy neutrosophic semi-open set in (X, τ_N).

Theorem 3.7 Let A_N be a fuzzy neutrosophic semi-open set in the fuzzy neutrosophic topological space (X, τ_N) and suppose $A_N \leq B_N \leq (A_N)^-$. Then B_N is fuzzy neutrosophic semi-open set in (X, τ_N).

Proof. There exists a fuzzy neutrosophic open set fnO such that $fnO \leq A_N \leq fncl(fno)$. Then $fnO \leq B_N$. But $(A_N)^- \leq fncl(fnO)$ and thus $B_N \leq fncl(fnO)$. Hence $fnO \leq B_N \leq fncl(fnO)$ and B_N is a fuzzy neutrosophic semi-open set in (X, τ_N). □

Theorem 3.8 Every fuzzy neutrosophic open set in the fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi-open set in (X, τ_N).

Proof. Let A_N be fuzzy neutrosophic open set in fuzzy neutrosophic topological space (X, τ_N). Then $A_N = (A_N)^+$. Also $(A_N)^+ \leq ((A_N)^+)^-$. This implies that $A_N \leq ((A_N)^+)^-$. Hence by theorem 3.2, A_N is a fuzzy neutrosophic semi-open set in (X, τ_N). □

Remark 3.9 In a fuzzy neutrosophic topological space, every fuzzy neutrosophic semi-open set in (X, τ_N) is need not to be fuzzy neutrosophic open set in (X, τ_N) as shown by the example.

Example 3.3. Let $X = \{a, b, c\}$ and some of the fuzzy neutrosophic semi-open sets are

$A_N = \{\langle a, 0.5, 0.4, 0.2\rangle, \langle b, 0.3, 0.1, 0.2\rangle, \langle c, 0.8, 0.6, 0.9\rangle\}$

$B_N = \{\langle a, 0.4, 0.2, 0.5\rangle, \langle b, 0.2, 0.1, 0.1\rangle, \langle c, 0.8, 0.5, 0.6\rangle\}$

$C_N = \{\langle a, 0.1, 0.6, 0.5\rangle, \langle b, 0.3, 0.1, 0.4\rangle, \langle c, 0.5, 0.9, 0.8\rangle\}$

$D_N = \{\langle a, 0.4, 0.5, 0.3\rangle, \langle b, 0.2, 0.1, 0.6\rangle, \langle c, 0.5, 0.8, 0.7\rangle\}$

$E_N = \{\langle a, 0.6, 0.5, 0.1\rangle, \langle b, 0.1, 0.6, 0.4\rangle, \langle c, 0.8, 0.5, 0.9\rangle\}$

$F_N = \{\langle a, 0.5, 0.4, 0.3\rangle, \langle b, 0.2, 0.3, 0.1\rangle, \langle c, 0.8, 0.7, 0.5\rangle\}$

Then, $\tau_N = \{0_N, A_N, B_N, C_N, D_N, E_N, F_N, 1_N\}$ is a fuzzy neutrosophic topology on X.
Here $A_N, B_N, C_N, D_N, E_N, F_N$ are fuzzy neutrosophic semi-open sets but are not fuzzy neutrosophic open sets in (X, τ_N).

4. Fuzzy Neutrosophic Semi-Closed Sets In Fuzzy Neutrosophic Topological Spaces.

In this section, the concept of fuzzy neutrosophic semi-closed set is introduced and also discussed their properties.

Definition 4.1. Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X, τ_N). Then A_N is said to be fuzzy neutrosophic semi-closed set in (X, τ_N) if there exists a fuzzy neutrosophic closed (fnC) set such that $fnint(fnC) \leq A_N \leq (fnC)$.

Theorem 4.2 A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi-closed set if only if $((A_N)^-)^+ \leq A_N$.

Proof. Sufficiency: Let $((A_N)^-)^+ \leq A_N$. Then for $fnC = (A_N)^-$, we have $(fnC)^+ \leq A_N \leq fnC$. *Necessity:* Let A_N be a fuzzy neutrosophic semi-closed set in (X, τ_N). Then $fn(fnC)^+ \leq A_N \leq fnC$ for some fuzzy neutrosophic semi-closed set fnC. But $fn(A_N)^- \leq fnC$ and thus $((A_N)^-)^+ \leq fn(fnC)^+$. Hence $((A_N)^-)^+ \leq fn(fnC)^+ \leq A_N$. □

Theorem 4.3 Let (X, τ_N) be a fuzzy neutrosophic topological space and A_N be a fuzzy neutrosophic set of (X, τ_N). Then A_N is a fuzzy neutrosophic semi-closed set if and only if complement of $C(A_N)$ is fuzzy neutrosophic semi-open set in (X, τ_N).

Proof. Let A_N be a fuzzy neutrosophic semi-closed set in (X, τ_N). Then by theorem 4.2, $((A_N)^-)^+ \leq A_N$. Taking complement on both sides, $C(A_N) \leq C(((A_N)^-)^+) = (C(A_N)^-)^-$. By using Theorem 2.11, $C(A_N) \leq ((C(A_N))^+)^-$. By Theorem 3.2, $C(A_N)$ is fuzzy neutrosophic semi-open. Conversely let $C(A_N)$ is fuzzy neutrosophic semi-open set. By Theorem 3.2, $C(A_N) \leq ((C(A_N))^+)^-$. Taking complement on both sides, $A_N \geq C((C(A_N))^+)^- = (C(C(A_N))^+)^+$. By using Theorem 2.11, $A_N \geq ((A_N)^-)^+$. By theorem 4.2, A_N is a fuzzy neutrosophic semi-closed set. □

Theorem 4.4 Let (X, τ_N) be a fuzzy neutrosophic topological space. Then intersection of two fuzzy neutrosophic semi-closed sets is a fuzzy neutrosophic semi-closed set in fuzzy neutrosophic topological space (X, τ_N).

Proof. Let A_N and B_N are fuzzy neutrosophic semi-closed sets in (X, τ_N). Then $((A_N)^-)^+ \leq A_N$ and $((B_N)^-)^+ \leq B_N$. Therefore $A_N \wedge B_N \geq ((A_N)^-)^+ \wedge ((B_N)^-)^+ = ((A_N)^- \vee (B_N)^-)^+ \geq ((A_N \vee B_N)^-)^+$ [By proposition 2.15 d)] Hence $A_N \wedge B_N$ is fuzzy neutrosophic semi-closed set in (X, τ_N). □

Theorem 4.4 Let $\{A_{N_i}\}_{i \in \Delta}$ be a collection of fuzzy neutrosophic semi-closed sets in a fuzzy neutrosophic topological space (X, τ_N). Then $\bigwedge_{i \in \Delta} A_{N_i}$ is fuzzy neutrosophic semi-closed set in (X, τ_N).

Proof. For each $i \in \Delta$, we have a fuzzy neutrosophic semi-closed set fnC_i such that $fn(fnC_i)^+ \leq A_{N_i} \leq fnC_i$. Then $fn(\bigwedge_{i \in \Delta} fnC_i)^+ \leq \bigwedge_{i \in \Delta} fn(fnC_i)^+ \leq \bigwedge_{i \in \Delta} A_{N_i} \leq \bigwedge_{i \in \Delta} fnC_i$. Hence let $fnC = \bigwedge_{i \in \Delta} fnC_i$. □

Remark 4.6 The union of any two fuzzy neutrosophic semi-closed sets need not be a fuzzy neutrosophic semi-closed set in (X, τ_N) as shown by the following example.

Example 4.2. Let $X = \{a, b, c\}$ the fuzzy neutrosophic sets A_N, B_N and C_N defined on X as follows

$A_N = \{\langle a, 0.1, 0.5, 0.5\rangle\}$

$B_N = \{\langle a, 0, 0.9, 0.2\rangle\}$

$C_N = \{\langle a, 0.9, 0.2, 0.1\rangle\}$

Then, $\tau_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, A_N \vee C_N, B_N \vee C_N, A_N \wedge B_N, B_N \wedge C_N, A_N \wedge C_N, A_N \vee$

$B_N \vee C_N, 1_N\}$ is a fuzzy neutrosophic topology on X. Now, we define the two fuzzy neutrosophic semi-open sets as follows:
Here $((1-B_N)^+)^- = 1 = (1-B_N)^+ = 0$ and $((1-C_N)^+)^- = 1 = (1-C_N)^+ = 0$ are fuzzy neutrosophic semi-closed sets in (X, τ_N). But $((1-B_N) \vee (1-C_N)^-)^+ = ((B_N \vee C_N)^-)^+ = (1 - B_N \vee C_N)^+ = A_N \neq 0$ is not a fuzzy neutrosophic semi-closed set in (X, τ_N).

Theorem 4.8 Let A_N be fuzzy neutrosophic semi-closed set in the fuzzy neutrosophic topological space (X, τ_N) and suppose $(A_N)^+ \leq B_N \leq A_N$. Then B_N is fuzzy neutrosophic semi-closed set in (X, τ_N).

Proof. There exists a fuzzy neutrosophic semi-closed set (fnC) such that $fn(fnC)^+ \leq A_N \leq fnC$. Then $B_N \leq fnC$. But $fn(fnC)^+ \leq (A_N)^+$ and thus $fn(fnC)^+ \leq B_N$. Hence $fn(fnC)^+ \leq B_N \leq fnC$ and B_N is fuzzy neutrosophic semi-closed set in (X, τ_N). □

Theorem 4.9 Every fuzzy neutrosophic closed set in the fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi-closed set in (X, τ_N).

Proof. Let A_N be fuzzy neutrosophic closed set in fuzzy neutrosophic topological space (X, τ_N). Then $(A_N)^- = A_N$. Also $((A_N)^-)^+ = (A_N)^-$. This implies that $((A_N)^-)^+ \leq A_N$. Hence by theorem 3.2, A_N is a fuzzy neutrosophic semi-closed set in (X, τ_N). □

Remark 4.10 In a fuzzy neutrosophic topological space, every fuzzy neutrosophic semi-closed set in (X, τ_N) is need not to be fuzzy neutrosophic closed set in (X, τ_N). For the following example.

Example 4.3. Let $X = \{a, b, c\}$ some of the fuzzy neutrosophic semi-open sets are
$A_N = \{\langle a, 0.3, 0.6, 0.5\rangle, \langle b, 0.9, 0.7, 0.1\rangle, \langle c, 0.4, 0.6, 0.1\rangle\}$
$B_N = \{\langle a, 0.7, 0.4, 0\rangle, \langle b, 0.9, 0.6, 0.1\rangle, \langle c, 0.8, 0.5, 0.5\rangle\}$
$C_N = \{\langle a, 0.5, 0.4, 0.3\rangle, \langle b, 0.1, 0.3, 0.9\rangle, \langle c, 0.1, 0.4, 0.4\rangle\}$
$D_N = \{\langle a, 0, 0.6, 0.7\rangle, \langle b, 0.1, 0.4, 0.9\rangle, \langle c, 0.5, 0.5, 0.8\rangle\}$
$E_N = \{\langle a, 0.9, 0.4, 0.2\rangle, \langle b, 0.9, 0.2, 0\rangle, \langle c, 0.1, 0.2, 0.3\rangle\}$
Then, $\tau_N = \{0_N, A_N, B_N, C_N, D_N, E_N, 1_N\}$ is a fuzzy neutrosophic topology on X.
Now, $((1-C_N)^-)^+ = (1-C_N)^+ = 0, ((1-D_N)^-)^+ = (1-D_N)^+ = 0, ((1-E_N)^-)^+ = (1-E_N)^+ = 0$. Here the fuzzy neutrosophic semi-closed sets are $1-C_N, 1-D_N, 1-E_N$ but are not a fuzzy neutrosophic closed sets in (X, τ_N).

5. Fuzzy Neutrosophic Semi-Interior In Fuzzy Neutrosophic Topological Spaces.

In this section, we introduce the fuzzy neutrosophic semi-interior and their properties in fuzzy neutrosophic topological space.

Definition 5.1. Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X, τ_N), the fuzzy neutrosophic semi-interior of A_N is the union of all fuzzy neutrosophic semi-open sets of (X, τ_N) contained in A_N.
That is, $fnS(A_N)^+ = \bigvee\{G_N : G_N \text{ is a } fnSO \text{ set in } (X, \tau_N) \text{ and } G_N \leq A_N\}$.

Proposition 5.2 Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have
(i) $fnS(A_N)^+ \leq A_N$
(ii) A_N *is* $fnSO$ *set in* $(X, \tau_N) \iff fnS(A_N)^+ = A_N$
(iii) $fnS(fnS(A_N)^+)^+ = fnS(A_N)^+$
(iv) If $A_N \leq B_N$ then $fnS(A_N)^+ \leq fnS(B_N)^+$.
Proof: (i) follows from definition 5.1 Let A_N be fuzzy neutrosophic semi-open set in (X, τ_N). Then $A_N \leq fnS(A_N)^+$. Conversely assume that $A_N = fnS(A_N)^+$. By using definition 5.1, A_N is a fuzzy neutrosophic semi-open set in (X, τ_N). Thus (ii) proved. By using (ii), $fnS(fnS(A_N)^+)^+ = fnS(A_N)^+$. This proved (iii). Since $A_N \leq B_N$ by using (i), $fnS(A_N)^+ \leq A_N \leq B_N$. Thus $fnS(A_N)^+ \leq B_N$. By (iii), $fnS(fnS(A_N)^+)^+ = fnS(B_N)^+$. Thus $fnS(A_N)^+ \leq fnS(B_N)^+$. This proves (iv).

Theorem 5.2 Let (X, τ_N)be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have
(i) $fnS(A_N \wedge B_N)^+ = fnS(A_N)^+ \wedge fnS(B_N)^+$.
(ii) $fnS(A_N \vee B_N)^+ \geq fnS(A_N)^+ \vee fnS(B_N)^+$.
Proof: Since $A_N \wedge B_N \leq A_N$ and $A_N \wedge B_N \leq B_N$. By using proposition 5.2 (iv), $fnS(A_N \wedge B_N)^+ \leq fnS(A_N)^+$ and $fnS(A_N \wedge B_N)^+ \leq fn(B_N)^+$. This implies that $fnS(A_N \wedge B_N)^+ = fnS(A_N)^+ \wedge fnS(B_N)^+ \implies (1)$. By using proposition 5.2 (i), $fnS(A_N)^+ \leq A_N$ and $fnS(B_N)^+ \leq B_N$. This implies that $fnS(A_N)^+ \wedge fnS(B_N)^+ \leq A_N \wedge B_N$. Now applying proposition 5.2(iv), $fnS(fnS(A_N)^+ \wedge fnS(B_N)^+)^+ \leq fnS(A_N \wedge B_N)^+ \implies (2)$. By (1), $fnS(fnS(A_N)^+)^+ \wedge fnS(fnS(B_N)^+)^+ \leq fnS(A_N \wedge B_N)^+$. By proposition 5.2(iii), $fnS(A_N)^+ \wedge fnS(B_N)^+ \leq fnS(A_N \wedge B_N)^+$. From (1) and (2), $fnS(A_N \wedge B_N)^+ = fnS(A_N)^+ \wedge fnS(B_N)^+$. This implies (i).
Since $A_N \leq A_N \vee B_N$ and $B_N \leq A_N \vee B_N$, by using proposition 5.2 (iv), $fnS(A_N)^+ \leq fnS(A_N \vee B_N)^+$ and $fnS(B_N)^+ \leq fnS(A_N \vee B_N)^+$. This implies that $fnS(A_N)^+ \vee fnS(B_N)^+ \leq fnS(A_N \vee B_N)^+$. Hence (ii). The following example shows that the equality need not hold in Theorem 5.3(ii).

Example 5.2. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets are A_N, B_N, C_N, D_N as follows

$A_N = \{\langle a, 0.7, 0.4, 0.1\rangle, \langle b, 0.6, 0.2, 0.5\rangle, \langle c, 0.3, 0.7, 0.9\rangle\}$
$B_N = \{\langle a, 0.1, 0.6, 0.4\rangle, \langle b, 0.7, 0.7, 0.2\rangle, \langle c, 0.9, 0.5, 0.1\rangle\}$

$C_N = \{\langle a, 0.4, 0.7, 0.1\rangle, \langle b, 0.7, 0.7, 0.2\rangle, \langle c, 0.9, 0.7, 0.1\rangle\}$
$D_N = \{\langle a, 0.3, 0.5, 0.1\rangle, \langle b, 0.4, 0.5, 0.2\rangle, \langle c, 0.8, 0.4, 0.1\rangle\}$
Then, $\tau_N = \{0_N, A_N, B_N, C_N, D_N, E_N, 1_N\}$ is a fuzzy neutrosophic topology on X. Now, we consider the two fuzzy neutrosophic sets as follows:
$E_N = \{\langle a, 0.7, 0.6, 0.2\rangle, \langle b, 0.7, 0.6, 0.3\rangle, \langle c, 0.9, 0.5, 0.2\rangle\}$.
$F_N = \{\langle a, 0.4, 0.6, 0.2\rangle, \langle b, 0.6, 0.7, 0.2\rangle, \langle c, 0.1, 0.7, 0.1\rangle\}$.
Then $(E_N)^+ = D_N$ and $(F_N)^+ = D_N$. This implies that $(E_N)^+ \vee (F_N)^+ = D_N$. Now $(E_N \vee F_N)^+ = C_N$. Then $(I_N)^+ \vee (H_N)^+ \neq (I_N \vee H_N)^+$.

6. **Fuzzy Neutrosophic Semi-Closure In Fuzzy Neutrosophic Topological Spaces**

In this section, we introduce the concept of fuzzy neutrosophic semi-closure and their properties in fuzzy neutrosophic topological space.

Definition 6.1. Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X, τ_N), the fuzzy neutrosophic semi-closure of A_N [$fnScl(A_N)$ for short] is the intersection of all fuzzy neutrosophic semi-closed sets of (X, τ_N) contained in A_N. That is, $fnS(A_N)^- = \wedge\{K_N : K_N \; is \; a fnSC \; set \; in \; (X, \tau_N) \; and \; K_N \geq A_N\}$.

Proposition 6.2 Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a (X, τ_N),
(i) $C(fnS(A_N)^+) \leq fnS(C(A_N))^-$,
(ii) $C(fnS(A_N)^-) \leq fnS(C(A_N))^+$.
Proof: (i) It follows from the definition 5.1, $fnS(A_N)^+ = \bigvee\{G_N : G_N \; is \; a \; fnSO \; set \; in(X, \tau_N) \; and \; G_N \leq A_N\}$. Taking complement on both sides, $C(fnS(A_N)^+) = C(\bigvee\{G_N : G_N \; is \; a \; fnSO \; set \; in(X, \tau_N) \; and \; G_N \leq A_N\}) = \bigwedge\{C(G_N) : C(G_N) \; is \; a \; fnSO \; set \; in(X, \tau_N) \; and \; C(A_N) \leq C(G_N)\}$.Replacing $C(G_N)$ by K_N, we get $C(fnS(A_N)^+) = \bigwedge\{K_N : K_N \; is \; a \; fnSC \; set \; in \; (X, \tau_N) \; and \; K_N \geq C(A_N)\}$. By definition 6.1, $C(fnS(A_N)^+) = fnS(C(A_N))^-$. This proves (i). By using (i), $C(fnS(C(A_N))^+) = fnS(C(C(A_N)))^- = fnS(A_N)^-$. Taking complement on both sides, we get $fnS(C(A_N))^+ = C(fnS(A_N)^-)$. Hence proved (ii).

Proposition 6.3 Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have
(i) $A_N \leq fnS(A_N)^-$
(ii) $A_N \; is \; fnSC \; set \; in(X, \tau_N) \iff fnS(A_N)^- = A_N$
(iii)$fnS(fnS(A_N)^-)^- = fns(A_N)^-$
(iv) $If A_N \leq B_N$ then $fns(A_N)^- \leq fns(B_N)^-$.
Proof: (i) follows from definition 5.1 Let A_N be fuzzy neutrosophic semi-closed set in (X, τ_N).

By using proposition 4.3, $C(A_N)$ be fuzzy neutrosophic semi-open set in (X, τ_N). By proposition 6.2 (ii), $fnS(C(A_N))^+ = C(A_N) \iff C(fnS(A_N)^-) = C(A_N) \iff fnS(A_N)^- = A_N$. Thus (ii) proved.

By using (ii), $fnS(fnS(A_N)^-)^- = fnS(A_N)^-$. This proved (iii).

Since $A_N \leq B_N$, $C(A_N) \leq C(B_N)$. By proposition 5.2 (iv), $fnS(C(B_N))^+ \leq fnS(C(A_N))^+$. Taking complement on both sides, $C(fnS(C(B_N))^+) \geq C(fnS(C(A_N))^+)$. By proposition 6.2 (ii), $fnS(A_N)^- \leq fnS(B_N)^-$. This proves (iv).

Proposition 6.4 Let A_N be a fuzzy neutrosophic set in a fuzzy neutrosophic topological space (X, τ_N). Then $fn(A_N)^+ \leq fnS(A_N)^+ \leq A_N \leq fnS(A_N)^- \leq fn(A_N)^-$.

Proof:

It follows from the definitions of corresponding operators.

Proposition 6.5 Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have

(i) $fnS(A_N \vee B_N)^- = fnS(A_N)^- \vee fnS(B_N)^-$ and

(ii)$fnS(A_N \wedge B_N)^- \leq fnS(A_N)^- \wedge fnS(B_N)^-$.

Proof: Since $fnS(A_N \vee B_N)^- = fnS(C(C(A_N \vee B_N)))^-$, by using proposition 6.2(i), $fnS(A_N \vee B_N)^- = C(fnS(C(A_N \vee B_N))^+) = C(fnS(C(A_N) \wedge C(B_N))^+$. Again using proposition 5.3 (i), $fnS(A_N \vee B_N)^- = C(fnS(C(A_N))^+ \wedge fnS(C(B_N))^+) = fnS(C(C(A_N)) \vee fnS(C(C(B_N)))^- = fnS(A_N)^- \vee fnS(B_N)^-$. This proved (i).

Since $A_N \wedge B_N \leq A_N$ and $A_N \wedge B_N \leq B_N$. By using proposition 6.3 (iv), $fnS(A_N \wedge B_N)^- \leq fnS(A_N)^-$ and $fnS(A_N \wedge B_N)^- \leq fnS(B_N)^-$. This implies that $fnS(A_N \wedge B_N)^- \leq fnS(A_N)^- \wedge fnS(B_N)^-$. This proves (ii).

Example 6.2. Let $X = \{a, b, c\}$, and the fuzzy neutrosophic sets A_N, B_N, C_N, D_N, E_N defined on X as follows

$A_N = \{\langle a, 0.6, 0.5, 0.4\rangle, \langle b, 0.5, 0.5, 0.6\rangle, \langle c, 0.6, 0.7, 0.7\rangle\}$

$B_N = \{\langle a, 0.7, 0.7, 0.6\rangle, \langle b, 0.6, 0.8, 0.7\rangle, \langle c, 0.7, 0.7, 0.8\rangle\}$

$C_N = \{\langle a, 0.6, 0.5, 0.7\rangle, \langle b, 0.7, 0.8, 0.6\rangle, \langle c, 0.8, 0.7, 0.7\rangle\}$

$D_N = \{\langle a, 0.7, 0.8, 0.8\rangle, \langle b, 0.8, 0.7, 0.8\rangle, \langle c, 0.6, 0.8, 0.7\rangle\}$

$E_N = \{\langle a, 0.3, 0.4, 0.3\rangle, \langle b, 0.2, 0.3, 0.3\rangle, \langle c, 0.3, 0.3, 0.3\rangle\}$

Then, $\tau_N = \{0_N, A_N, B_N, C_N, D_N, 1_N\}$ is a fuzzy neutrosophic topology on X. Now, we consider the two fuzzy neutrosophic sets as follows:

$I_N = \{\langle a, 0.3, 0.4, 0.3\rangle, \langle b, 0.4, 0.5, 0.2\rangle, \langle c, 0.2, 0.5, 0.3\rangle\}$.

$H_N = \{\langle a, 0.2, 0.4, 0.2\rangle, \langle b, 0.1, 0.3, 0.2\rangle, \langle c, 0.4, 0.2, 0.1\rangle\}$.

Then $(I_N)^- = A_N$ and $(H_N)^- = A_N$. This implies that $(I_N)^- \wedge (H_N)^- = A_N$. Now $(I_N \wedge H_N)^- = E_N$. Then $(I_N)^- \wedge (H_N)^- \neq (I_N \wedge H_N)^-$.

Proposition 6.7 Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have
(i) $fnS(A_N)^- \geq A_N \vee fnS(fnS(A_N)^+)^-$
, (ii) $fnS(A_N)^+ \leq A_N \wedge fnS(fnS(A_N)^-)^+$,
(iii) $fn(fnS(A_N)^-)^+ \leq fn(fn(A_N)^-)^+$,
(iv)$fn(fnS(A_N)^-)^+ \geq fn(fnS(fnS(A_N)^+)^-)^+$

Proof: By using proposition 6.3 (i), $A_N \leq fnS(A_N)^- \implies$ (1). Again By using proposition 5.2 (i), $fnS(A_N)^+ \leq A_N$. Again By using proposition 5.2 (i), $fnS(A_N)^+ \leq A_N$. Then $fnS(fnS(A_N)^+)^- \leq fnS(A_N)^- \implies$ (2). By (1) and (2) we have, $A_N \vee fnS(fnS(A_N)^+) - \leq fnS(A_N)^-$. This proves (i).
By proposition 5.2 (i), $fnS(A_N)^+ \leq A_N \implies$ (1). Again by using proposition 6.3 (i), $A_N \leq fnS(A_N)^-$. Then $fnS(A_N)^+ \leq fnS(fnS(A_N)^-)^+ \implies$ (2). From (1) and (2) we have $fnS(A_N)^+ \leq A_N \wedge fnS(fnS(A_N)^-)^+$. This proves (ii).
By proposition 6.4, $fnS(A_N)^- \leq fn(A_N)^-$. We get $fn(fnS(A_N)^-)^+ \leq fn(fn(A_N)^-)^+$. Hence
(iii). By (i), $fnS(A_N)^- \geq A_N \vee fnS(fnS(A_N)^+)^-$. We have $fn(fnS(A_N)^-)^+ \geq fn(A_N \vee fnS(fnS(A_N)^+)^-)^+$. Since $fn(A_N \vee B_N)^+ \geq fn(A_N)^+ \vee fn(B_N)^+, fn(fnS(A_N)^-)^+ \geq fn(A_N)^+ \vee fn(fnS(fnS(A_N)^+)^-)^+ \geq fn(fnS(fnS(A_N)^+)^-)^+$. Hence proves (iv).

7. Conclusion

In this paper, we introduce the concept of fuzzy neutrosophic semi-open (semi-closed) and fuzzy neutrosophic semi-interior(semi-closure) are introduced and several characterizations with examples are studied in fuzzy neutrosophic topological spaces. Application to different fields of fuzzy neutrosophic toplogical spaces such as soft computing, artificial intelligence, decision making, pattern recognition, and image processing.

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FUZZY NEUTROSOPHIC SEMI-BAIRE SPACES

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Abstract. In this paper, we introduce the concept of semi-Baire spaces in fuzzy neutrosophic topological spaces. Also we define the fuzzy neutrosophic semi-nowhere dense, fuzzy neutrosophic semi-one category and fuzzy neutrosophic semi-two category sets. Some of its characterizations of fuzzy neutrosophic semi-Baire spaces are also studied. Several examples are given to illustrate the concepts.

Keywords: Fuzzy neutrosophic semi-open set, fuzzy neutrosophic semi-nowhere dense set, fuzzy neutrosophic semi-one category, fuzzy neutrosophic semi-two category and fuzzy neutrosophic semi-Baire spaces.

1. Introduction

The concept of fuzzy sets were introduced by L.A. Zadeh in 1965 [11]. Then the fuzzy set theory is extension by many researchers. The important concept of fuzzy topological space was offered by C. L. Chang [3] and from that point forward different ideas in topology have been reached out to fuzzy topological space. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy toplogy has been developed. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [10] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

In this paper, we introduce the concept fuzzy neutrosophic semi-nowhere dense, fuzzy neutrosophic semi-one category, fuzzy neutrosophic semi-two category sets and fuzzy neutrosophic semi-Baire spaces are introduced and several characterizations with examples are studied in fuzzy neutrosophic topological spaces.

2. Preliminaries

Throughout this present paper, X denotes the fuzzy neutrosophic topological spaces. Let A_N be a fuzzy neutrosophic set on X. The fuzzy neutrosophic interior and closure of A_N is denoted as $fn(A_N)^+$, $fn(A_N)^-$ respectively. A fuzzy neutrosophic set A_N is defined to be

fuzzy neutrosophic open set ($fnOS$) if $A_N \leq fn(((A_N)^-)^+)^-$. The complement of a $fnOS$ is called fuzzy neutrosophic closed set ($fnCS$).

Definition 2.1. [1] A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F : X \to [0, 1]$ and $0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3$. With the condition $0 \leq T_{A^*}(x) + I_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2. [1] A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

Definition 2.3. [1] Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then
$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$
$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$

Definition 2.4. [1] The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$.

Definition 2.5. [1] A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0, I_A(x) = 0, F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N.

Definition 2.6. [1] A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N.

Definition 2.7. [1] The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$. The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8. [1] A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X
(i) $0_N, 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$
Satisfying the following axioms.
In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as $fnOS$in X.

Definition 2.9. [1] The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called $fnCS$in X.

Definition 2.10. [1] Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by
$int(A) = \cup\{G : G \text{ is a } fnOS\text{in } X \text{ and } G \subseteq A\ \}$
$cl(A) = \cap\{G : G \text{ is a } fnCS\text{in } X \text{ and } A \subseteq G\}$

Definition 2.11. [4] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic dense if there exist no $fnCSB_N$ in (X, τ_N) such that $A_N \subset B_N \subset 1_X$. That is, $fn(A_N)^- = 1_N$.

Definition 2.12. [4] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic nowhere dense set if there exist no non-zero $fnOSB_N$ in (X, τ_N) such that $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$.

Definition 2.13. [4] A fuzzy neutrosophic topological space (X, τ_N) is called fuzzy neutrosophic Baire space if $fn(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0_N$, where A_{N_i}'s are fuzzy neutrosophic nowhere dense sets in (X, τ_N).

Definition 2.14. [5] Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X, τ_N). Then A_N is said to be fuzzy neutrosophic semi-open set in (X, τ_N) if there exists a fuzzy neutrosophic open setfnO such that $fnO \leq A_N \leq fncl(fnO)$.

Definition 2.15. [5] Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X, τ_N). Then A_N is said to be fuzzy neutrosophic semi-closed set in (X, τ_N) if there exists a fuzzy neutrosophic closed (fnC) set such that $fnint(fnC) \leq A_N \leq (fnC)$.

Definition 2.16. [5] Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X, τ_N), the fuzzy neutrosophic semi-interior of A_N is the union of all fuzzy neutrosophic semi-open sets of (X, τ_N) contained in A_N.
That is, $fnS(A_N)^+ = \bigvee\{G_N : G_N \textit{ is a } fnSO \textit{ set in } (X, \tau_N) \textit{ and } G_N \leq A_N\}$.

Definition 2.17. [5] Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X, τ_N), the fuzzy neutrosophic semi-closure of A_N [$fnScl(A_N)$ for short] is the intersection of all fuzzy neutrosophic semi-closed sets of (X, τ_N) contained in A_N.
That is, $fnS(A_N)^- = \wedge\{K_N : K_N \textit{ is a } fnSC \textit{ set in } (X, \tau_N) \textit{ and } K_N \geq A_N\}$.

3. **Fuzzy neutrosophic semi-Nowhere Dense Sets.**

Definition 3.1. A fuzzy neutrosophic set A_N in fuzzy neutrosophic topological space (X, τ_N) is called fuzzy neutrosophic semi- (briefly as $fn.S$) nowhere dense if there exists no non-zero fuzzy neutrosophic semi-open set $B_N \leq fn.S(A_N)^-$. That is $fn.S.((A_N)^-)^+ = 0$.

Example 3.2. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N, C_N and D_N defined on X as follows

$A_N = \{\langle a, 0.3, 0.4, 0.4\rangle, \langle b, 0.6, 0.2, 0.5\rangle\}$

$B_N = \{\langle a, 0.2, 0.5, 0.7\rangle, \langle b, 0.5, 0.4, 0.1\rangle\}$ Then consider the family $\tau_N = \{0_N, A_N, B_N, A_N \vee B_N, A_N \wedge B_N, 1_N\}$ is a fuzzy neutrosophic topology on X. Thus (X, τ_N) is a fuzzy neutrosophic topological space. Now $1 - A_N, 1 - B_N, 1 - A_N \vee B_N$ are fuzzy neutrosophic semi- nowhere dense sets in (X, τ_N) .

Proposition 3.3. *If A_N is a fuzzy neutrosophic semi- nowhere dense set in (X, τ_N), then $1 - A_N$ is a fuzzy neutrosophic semi- dense set in (X, τ_N).*

Proposition 3.4. *Let A_N be a set. If A_N is a fuzzy neutrosophic semi-closed set in (X, τ_N) with $fn.S(A_N)^+ = 0$, then A_N is fuzzy neutrosophic semi- nowhere dense set in (X, τ_N).*

Definition 3.5. Let A_N be a fuzzy neutrosophic semi-one category set in (X, τ_N). Then $1 - A_N$ is called fuzzy neutrosophic semi-residual set in (X, τ_N).

Proposition 3.6. *The complement of a fuzzy neutrosophic semi- nowhere dense set in a fuzzy neutrosophic topological space (X, τ_N) need not be a fuzzy neutrosophic semi- nowhere dense set.*

Proof. For, in example 3.2, $1 - A_N$ is a fuzzy neutrosophic semi- nowhere dense set in (X, τ_N) whereas A_N is not a fuzzy neutrosophic semi- nowhere dense set in (X, τ_N). □

Proposition 3.7. *If A_N and B_N are fuzzy neutrosophic semi- nowhere dense set in a fuzzy neutrosophic topological space (X, τ_N), then $A_N \vee B_N$ need not be a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N).*

Proof. For, in example 3.2, $1 - A_N$ and $1 - B_N$ is a fuzzy neutrosophic semi- nowhere dense set in (X, τ_N). But $1 - A_N \vee 1 - B_N$ implies that $fn.S((1 - A_N \vee 1 - B_N)^-)^+ \neq 0$. Therefore union of fuzzy neutrosophic semi- nowhere dense sets need not be a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). □

Proposition 3.8. *If the fuzzy neutrosophic sets A_N and B_N are fuzzy neutrosophic semi-nowhere dense set in a fuzzy neutrosophic topological space (X, τ_N), then $A_N \wedge B_N$ is a fuzzy neutrosophic semi- nowhere dense set in (X, τ_N).*

Proof. Let the fuzzy sets A_N and B_N are fuzzy neutrosophic semi- nowhere dense set in (X, τ_N). Now $fn.S((A_N \wedge B_N)^-)^+ \leq fn.S((A_N)^-)^+ \wedge fn.S((B_N)^-)^+ = 0 \wedge 0$ (since

$fn.S((A_N)^-)^+ = 0$ and $fn.S((B_N)^-)^+ = 0$). That is, $fn.S((A_N \wedge B_N)^-)^+ = 0$. Hence $A_N \wedge B_N$ is a fuzzy neutrosophic semi- nowhere dense set in (X, τ_N). □

Proposition 3.9. *If A_N is a fuzzy neutrosophic semi- nowhere dense set in fuzzy neutrosophic topological space (X, τ_N), then $fn.S(A_N)^+ = 0$.*

Proof. Let A_N be a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). Then, we have $fn.S((A_N)^-)^+ = 0$. Now $A_N \leq fn.S(A_N)^-$ implies that $fn.S(A_N)^+ \leq fn.S((A_N)^-)^+ = 0$. Hence, we have $fn.S(A_N)^+ = 0$. □

Proposition 3.10. *If A_N is a fuzzy neutrosophic semi- nowhere dense set in fuzzy neutrosophic topological space (X, τ_N), then $fn.S((A_N)^-)^+ = 0$.*

Proof. Let A_N be a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). Then, we have $fn.S((A_N)^-)^+ = 0$ and $fn.S(A_N)^+ = 0$. Now $A_N = fn.S(A_N)^-$, A_N is a fuzzy neutrosophic semi- closed set in (X, τ_N), implies that $fn.S((A_N)^-)^+ = fn.S(A_N)^+$. Hence, we have $fn.S((A_N)^-)^+ = 0$. □

Proposition 3.11. *If A_N is a fuzzy neutrosophic semi- nowhere dense set and B_N is any fuzzy neutrosophic set in a fuzzy neutrosophic topological space (X, τ_N), then $(A_N \wedge B_N)$ is a fuzzy neutrosophic semi- nowhere dense set in (X, τ_N).*

Proof. Let A_N be a fuzzy neutrosophic semi- nowhere dense set in (X, τ_N). Then, $fn.S.((A_N)^-)^+ = 0$. Now $fn.S.((A_N \wedge B_N)^-)^+ \leq fn.S.((A_N)^-)^+ \wedge fn.S.((B_N)^-)^+ \leq 0 \wedge fn.S.((B_N)^-)^+ = 0$. That is, $fn.S.((A_N \wedge B_N)^-)^+$. Hence $(A_N \wedge B_N)$ is a fuzzy neutrosophic semi- nowhere dense set in (X, τ_N). □

Definition 3.12. A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic semi-dense if there exist no $fnCS$ B_N in (X, τ_N) such that $A_N < B_N < 1$. That is, $fn.S(A_N)^- = 1$.

Proposition 3.13. *If A_N is a fuzzy neutrosophic semi-dense and fuzzy neutrosophic semi-open set in fuzzy neutrosophic topological space (X, τ_N) and if $B_N \leq 1 - A_N$, then B_N is a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N).*

Proof. Let A_N be a fuzzy neutrosophic semi-dense set in (X, τ_N) such that $fn.S(A_N)^- = 1$ and $fn.S(A_N)^+ = (A_N)^+$. Now $B_N \leq 1 - A_N$ implies that $fn.S(B_N)^- \leq fn.S(1 - A_N)^-$. Then $fn.S(B_N)^- \leq 1 - fn.S(A_N)^+ = 1 - A_N$. Hence $fn.S(B_N)^- \leq (1 - A_N)$, which implies that

$fn.S.((A_N)^-)^+ \leq fn.S(1-A_N)^+ = 1-fn.S(A_N)^- = 1-1 = 0$. That is, $fn.S(((B_N)^-)^+) = 0$. Therefore B_N is a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). □

Proposition 3.14. *If A_N is a fuzzy neutrosophic semi-nowhere dense set in a fuzzy neutrosophic topological space (X, τ_N), then $1 - A_N$ is a fuzzy neutrosophic semi-dense set in (X, τ_N).*

Proof. Let A_N be a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). Then $fn.S.((A_N)^-)^+ = 0$. Now $B_N \leq fn.S(A_N)^-$ implies that $fn.S(A_N)^+ \leq fn.S.((A_N)^-)^+ = 0$. Then $fn.S(A_N)^+ = 0$ and $fn.S(1 - A_N)^- = 1 - fn.S(A_N)^+ = 1 - 0 = 1$ and hence $1 - A_N$ is a fuzzy neutrosophic semi-dense set in (X, τ_N). □

Proposition 3.15. *If A_N is a fuzzy neutrosophic semi-nowhere dense set in a fuzzy neutrosophic topological space (X, τ_N), then $fn.S(A_N)^-$ is also a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N).*

Proof. Let A_N be a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). Then $fn.S.((A_N)^-)^+ = 0$. Now $fn.S.((A_N)^-)^+ = fn.S(A_N)^-$. Hence $fn.S(((A_N)^-)^-)^+ = fn.S.((A_N)^-)^+ = 0$. Therefore $fn.S(A_N)^-$ is also a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). □

Proposition 3.16. *If A_N is a fuzzy neutrosophic semi-nowhere dense set in a fuzzy neutrosophic topological space (X, τ_N), then $1 - fn.S(A_N)^-$ is a fuzzy neutrosophic semi- dense set in (X, τ_N).*

Proof. Let A_N be a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). Then by proposition 3.15, $fn.S(A_N)^-$ is a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). Also by proposition 3.14, $1 - fn.S(A_N)^-$ is a fuzzy neutrosophic semi- dense set in (X, τ_N). □

Proposition 3.17. *Let A_N be a fuzzy neutrosophic semi-dense set in fuzzy neutrosophic topological space (X, τ_N). If B_N is any fuzzy neutrosophic set in (X, τ_N), then B_N is a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N) if and only if $A_N \wedge B_N$ is a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N).*

Proof. Let B_N be a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). Now $fn.S.((B_N)^-)^+ = 0$. Now, $fn.S((A_N \wedge B_N)^-)^+ \leq fn.S.((A_N)^-)^+ \wedge fn.S.((B_N)^-)^+ \leq fn.S.((A_N)^-)^+ \wedge 0 = 0$. That is, $fn.S((A_N \wedge B_N)^-)^+ = 0$. Therefore $A_N \wedge B_N$ is a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N).

Conversely, let $A_N \wedge B_N$ is a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N).Then $fn.S((A_N \wedge B_N)^-)^+ = 0$ Implies that $fn.S.((A_N)^-)^+ \wedge fn.S.((B_N)^-)^+ = 0$. Since A_N is a fuzzy neutrosophic semi- dense set in (X, τ_N), $fn.S(A_N)^- = 1$. Then, $fn.S(1)^+ \wedge fn.S.((B_N)^-)^+ = 0$. That is, $(1) \wedge fn.S.((B_N)^-)^+ = 0$. Hence $fn.S.((B_N)^-)^+ = 0$, which means that B_N is a fuzzy neutrosophic semi-nowhere dense set in (X, τ_N). □

4. **Fuzzy Neutrosophic Semi-Baire Spaces**

Definition 4.1. Let (X, τ_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set A_N in (X, τ_N) is called fuzzy neutrosophic semi-one category set if $A_N = \bigvee_{i=1}^{\infty}(A_{N_i})$, where A_{N_i}'s, are fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). Any other fuzzy neutrosophic set in (X, τ_N) is said to be of fuzzy neutrosophic semi-two category.

Example 4.2. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N, C_N and D_N defined on X as follows

$A_N = \{\langle a, 0.7, 0.7, 0.3\rangle, \langle b, 0.7, 0.7, 0.3\rangle, \langle c, 0.6, 0.6, 0.4\rangle\}$
$B_N = \{\langle a, 0.7, 0.7, 0.4\rangle, \langle b, 0.7, 0.7, 0.4\rangle, \langle c, 0.5, 0.6, 0.4\rangle\}$
$C_N = \{\langle a, 0.2, 0.2, 0.7\rangle, \langle b, 0.2, 0.2, 0.7\rangle, \langle c, 0.2, 0.2, 0.5\rangle\}$
$D_N = \{\langle a, 0.3, 0.3, 0.7\rangle, \langle b, 0.2, 0.2, 0.7\rangle, \langle c, 0.3, 0.3, 0.6\rangle\}$ Then consider the family $\tau_N = \{0_N, A_N, B_N, 1_N\}$ is a fuzzy neutrosophic topology on X. Thus (X, τ_N) is a fuzzy neutrosophic topological space. Now $1 - A_N, 1 - B_N, C_N, D_N$ are fuzzy neutrosophic semi- nowhere dense sets and $[1 - A_N \vee 1 - B_N \vee C_N \vee D_N] = 1 - A_N$ is fuzzy neutrosophic semi-one category set in (X, τ_N).

Definition 4.3. Let A_N be a fuzzy neutrosophic semi-one category set in (X, τ_N). Then $1 - A_N$ is called fuzzy neutrosophic semi-residual set in (X, τ_N).

Proposition 4.4. *If A_N is a fuzzy neutrosophic semi-one category set in (X, τ_N) then $1 - A_N = \bigwedge_{i=1}^{\infty}(B_{N_i})$, where $fn(B_{N_i})^- = 1$.*

Proof. Let A_N be a fuzzy neutrosophic semi-one category set in (X, τ_N). Then $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). Now $1 - A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$ Let $B_{N_i} = 1 - A_{N_i}$. Then $1 - A_N = \bigwedge_{i=1}^{\infty} B_{N_i}$. Since A_{N_i}'s is a fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). Then, by proposition 3.10, we have $1 - A_{N_i}$'s is a fuzzy neutrosophic semi-dense set in (X, τ_N). Hence $fn.S(B_{N_i})^- = fn.S(1 - A_{N_i}) = 1$. Therefore we have $1 - A_N = \bigwedge_{i=1}^{\infty} B_{N_i}$, where $fn.S(B_{N_i})^- = 1$. □

Definition 4.5. A fuzzy neutrosophic topological space (X, τ_N) is called fuzzy neutrosophic semi-one category space if the fuzzy neutrosophic set 1 is a fuzzy neutrosophic semi-one category set in (X, τ_N). That is $1 = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s, are fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). Otherwise (X, τ_N) will be called a fuzzy neutrosophic semi-two category space.

Proposition 4.6. *If A_N is a fuzzy neutrosophic semi-two category set in fuzzy neutrosophic topological space (X, τ_N) then $fn.S(A_N)^+ = 0$.*

Proof. Let A_N be a fuzzy neutrosophic semi-two category set in (X, τ_N) . Then we have $fn.S(A_N)^- = A_N$. Now $fn.S.((A_N)^-)^+ = fn.S(A_N)^+$ and $fn.S(A_N)^+ = 0$, implies that $fn.S.((A_N)^-)^+ = 0$. Hence A_N is a fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). □

Definition 4.7. A fuzzy neutrosophic topological space (X, τ_N) is called fuzzy neutrosophic semi-Baire space if $fn.S(\bigvee_{i=1}^{\infty}(A_{N_i}))^+$, where A_{N_i}'s, are fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N).

Example 4.8. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N, C_N and D_N defined on X as follows

$A_N = \{\langle a, 0.6, 0.6, 0.3\rangle, \langle b, 0.5, 0.6, 0.4\rangle, \langle c, 0.6, 0.6, 0.3\rangle\}$
$B_N = \{\langle a, 0.6, 0.6, 0.3\rangle, \langle b, 0.5, 0.5, 0.7\rangle, \langle c, 0.5, 0.5, 0.3\rangle\}$
$C_N = \{\langle a, 0.3, 0.3, 0.7\rangle, \langle b, 0.3, 0.4, 0.7\rangle, \langle c, 0.4, 0.3, 0.5\rangle\}$
$D_N = \{\langle a, 0.3, 0.3, 0.7\rangle, \langle b, 0.3, 0.4, 0.7\rangle, \langle c, 0.3, 0.4, 0.7\rangle\}$
Then consider the family $\tau_N = \{0_N, A_N, B_N, 1_N\}$ is a fuzzy neutrosophic topology on X. Thus (X, τ_N) is a fuzzy neutrosophic topological space. Now $1-A_N, 1-B_N, C_N, D_N$ are fuzzy neutrosophic semi- nowhere dense sets and $fn.S[1-A_N \vee 1-B_N \vee C_N \vee D_N] = fn.S(1-B_N)^+ = 0$ is fuzzy neutrosophic semi-Baire space.

Example 4.9. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N and B_N defined on X as follows

$A_N = \{\langle a, 0.5, 0.3, 0.4\rangle, \langle b, 0.2, 0.6, 0.5\rangle\}$
$B_N = \{\langle a, 0.6, 0.2, 0.7\rangle, \langle b, 0.3, 0.5, 0.1\rangle\}$
Then consider the family $\tau_N = \{0_N, A_N, B_N, 1_N\}$ is a fuzzy neutrosophic topology on X. Thus (X, τ_N) is a fuzzy neutrosophic topological space. Now $1 - A_N, 1 - B_N, 1 - A_N \vee B_N$ are fuzzy neutrosophic semi- nowhere dense sets and $fn.S(1 - A_N \vee 1 - B_N \vee (1 - A_N \vee B_N))^+ = fn.S(1 - A_N \vee B_N)^+ \neq 0$ in (X, τ_N).

Proposition 4.10. *If* $fn.S(\bigvee_{i=1}^{\infty}(A_{N_i}))^+$, *where* $fn.S(A_{N_i})^+ = 0$ *and* A_{N_i}*'s are fuzzy neutrosophic semi-closed sets in a fuzzy neutrosophic topological space* (X, τ_N), *then* (X, τ_N) *is a fuzzy neutrosophic semi-Baire space.*

Proof. Let A_{N_i}'s is a fuzzy neutrosophic semi-closed sets in (X, τ_N). Since $fn.S(A_{N_i})^+ = 0$. By proposition (4.2), A_{N_i} is a fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). Therefore $fn.S(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0$, where A_{N_i}'s are fuzzy neutrosophic semi-nowhere dense sets in(X, τ_N). Hence (X, τ_N) is a fuzzy neutrosophic semi-Baire space. □

Proposition 4.11. *If* $fn.S(\bigwedge_{i=1}^{\infty}(A_{N_i}))^- = 1$, *where* A_{N_i}*'s are fuzzy neutrosophic semi-dense and fuzzy neutrosophic semi-open sets in fuzzy neutrosophic topological space* (X, τ_N), *then* (X, τ_N) *is a fuzzy neutrosophic semi-Baire space.*

Proof. Let A_{N_i}'s be fuzzy neutrosophic semi-dense sets in (X, τ_N). Then $fn.S(\bigwedge_{i=1}^{\infty}(A_{N_i}))^- = 1$, which implies that $1 - fn.S(\bigwedge_{i=1}^{\infty}(A_{N_i}))^- = 0$. That is $fn.S(1 - \bigwedge_{i=1}^{\infty}(A_{N_i}))^+ = 0$, implies that $fn.S(1 - \bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0$. Since A_{N_i}'s be fuzzy neutrosophic semi-dense, $fn.S(A_{N_i})^- = 1$. Hence $fn.S(1 - A_{N_i})^+ = 1 - fn(A_{N_i})^- = 0$. Consequently $fn.S(1 - \bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 1$, where $fn.S(1 - A_{N_i})^+ = 0$ and A_{N_i}'s be fuzzy neutrosophic semi-closed sets in (X, τ_N) . By proposition 4.3, (X, τ_N) is a fuzzy neutrosophic semi-Baire space. □

Proposition 4.12. *Let* (X, τ_N) *be a fuzzy neutrosophic topological space. Then the following are equivalent.*
(i) (X, τ_N) *is a fuzzy neutrosophic semi-Baire space.*
(ii) $fn.S(A_N)^+ = 0$, *for every fuzzy neutrosophic semi-one category set* A_N *in* (X, τ_N).
(iii) $fn.S(B_N)^- = 1$, *for every fuzzy neutrosophic semi-residual set* B_N *in* (X, τ_N).

Proof. $(i) \implies (ii)$
Let A_N be a fuzzy neutrosophic semi-one category set in (X, τ_N). Then $A_N = \bigvee_{i=1}^{\infty}(A_{N_i})$, where A_{N_i}'s are fuzzy neutrosophic semi- nowhere dense sets in (X, τ_N). Now $fn.S(A_N)^+ = fn.S(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0$. Since (X, τ_N) is a fuzzy neutrosophic semi-Baire space. Therefore $fn.S(A_N)^+ = 0$.

$(ii) \implies (iii)$
Let B_N be a fuzzy neutrosophic semi-residual set in (X, τ_N). Then $1 - B_N$ is a fuzzy neutrosophic one category set in (X, τ_N). By hypothesis, $fn.S(1 - B_N)^+ = 0$ which implies that $1 - fn.S(B_N)^- = 0$. Hence $fn.S(B_N)^- = 1$.

$(iii) \implies (i)$

Let A_N be a fuzzy neutrosophic semi-one category set in (X, τ_N). Then $A_N = \bigvee_{i=1}^{\infty}(A_{N_i})$, where A_{N_i}' s are fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). Now A_N is a fuzzy neutrosophic semi-one category set implies that $1 - A_N$ is a fuzzy neutrosophic semi-residual set in (X, τ_N). By hypothesis, we have $fn.S(1-A_N)^- = 1$ which implies that $1-fn.S(A_N)^- = 0$. Hence $fn.S(A_N)^+ = 0$. That is, $fn.S(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0$, where A_{N_i}' s are fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). Hence (X, τ_N) is a fuzzy neutrosophic semi-Baire space. □

Proposition 4.13. *If a fuzzy neutrosophic topological space (X, τ_N) is a fuzzy neutrosophic semi-Baire space, then (X, τ_N) is a fuzzy neutrosophic semi-two category space.*

Proof. Let (X, τ_N) be a fuzzy neutrosophic semi-Baire space. Then $fn.S(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0$, where A_{N_i} is a fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). Then $\bigvee_{i=1}^{\infty}(A_{N_i}) \neq 1$. (suppose, $\bigvee_{i=1}^{\infty}(A_{N_i}) = 1$, implies that $fn.S(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = fn.S(1)^+ = 0$, which implies that $0 = 1$, a contradiction). Hence (X, τ_N) is a fuzzy neutrosophic semi-two category space. □

Remark 4.14. The converse of the above proposition need not be true. A fuzzy neutrosophic semi-two category space need not be fuzzy neutrosophic semi-Baire space.

Example 4.15. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N and B_N defined on X as follows

$A_N = \{\langle a, 0.4, 0.5, 0.4\rangle, \langle b, 0.6, 0.2, 0.6\rangle\}$

$B_N = \{\langle a, 0.3, 0.6, 0.7\rangle, \langle b, 0.5, 0.4, 0.2\rangle\}$

Then consider the family $\tau_N = \{0_N, A_N, B_N, A_N \vee B_N, A_N \wedge B_N, 1_N\}$ is a fuzzy neutrosophic topology on X. Thus (X, τ_N) is a fuzzy neutrosophic topological space. Now $1-A_N, 1-B_N, 1-A_N \vee B_N$ are fuzzy neutrosophic semi- nowhere dense sets and $(1-A_N \vee 1-B_N \vee (1-A_N \vee B_N)) = (1 - A_N \wedge B_N) \neq 1$ and and $fn.S(1 - A_N \wedge B_N)^+ \neq 0$. Hence the fuzzy neutrosophic semi-two category space need not be fuzzy neutrosophic semi-Baire space in (X, τ_N).

Proposition 4.16. *If a fuzzy neutrosophic topological space (X, τ_N) is a fuzzy neutrosophic semi-Baire space, then no non-zero fuzzy neutrosophic semi-open set in (X, τ_N) is a fuzzy neutrosophic semi-one category set in (X, τ_N).*

Proof. Suppose that A_N is a non-zero fuzzy neutrosophic semi-open set in (X, τ_N) such that $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}' s is a fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). Then

we have $fn.S(A_N)^+ = fn.S(\bigvee_{i=1}^{\infty} A_{N_i})^+$. Since A_N is a non-zero fuzzy neutrosophic semi-open set in (X, τ_N), $fn.S(A_N)^+ = A_N$. Then $fn.S(\bigvee_{i=1}^{\infty} A_{N_i})^+ = A_N \neq 0$. But this a contradiction to (X, τ_N) being a fuzzy neutrosophic semi-Baire space, in which $fn.S(\bigvee_{i=1}^{\infty} A_{N_i})^+ = 0$, where A_{N_i}'s is a fuzzy neutrosophic semi-nowhere dense sets in (X, τ_N). Hence we have $A_N \neq \bigvee_{i=1}^{\infty} A_{N_i}$,. Therefore no non-zero fuzzy neutrosophic semi-open set in (X, τ_N) is a fuzzy neutrosophic semi-one category set in (X, τ_N). □

Proposition 4.17. *A fuzzy neutrosophic semi-Baire space is a fuzzy neutrosophic Baire space. For consider the following example:*

Example 4.18. Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N, C_N and D_N defined on X as follows

$A_N = \{\langle a, 0.5, 0.6, 0.3\rangle, \langle b, 0.5, 0.6, 0.3\rangle, \langle c, 0.6, 0.5, 0.4\rangle\}$
$B_N = \{\langle a, 0.6, 0.5, 0.3\rangle, \langle b, 0.6, 0.6, 0.3\rangle, \langle c, 0.6, 0.6, 0.4\rangle\}$
$C_N = \{\langle a, 0.3, 0.3, 0.5\rangle, \langle b, 0.3, 0.4, 0.7\rangle, \langle c, 0.4, 0.3, 0.7\rangle\}$
$D_N = \{\langle a, 0.3, 0.3, 0.7\rangle, \langle b, 0.3, 0.3, 0.6\rangle, \langle c, 0.3, 0.3, 0.6\rangle\}$

Then consider the family $\tau_N = \{0_N, A_N, B_N, 1_N\}$ is a fuzzy neutrosophic topology on X. Thus (X, τ_N) is a fuzzy neutrosophic topological space. Now $1 - A_N, 1 - B_N, C_N, D_N$ are fuzzy neutrosophic semi- nowhere dense sets and $fn.S[1 - A_N \vee 1 - B_N \vee C_N \vee D_N]^+ = fn.S(1 - B_N)^+ = 0$. Hence the fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi-Baire space.

Here the sets $1 - A_N, 1 - B_N, C_N, D_N$ are fuzzy neutrosophic nowhere dense sets and $fn[1 - A_N \vee 1 - B_N \vee C_N \vee D_N]^+ = fn(1 - B_N)^+ = 0$. Hence the fuzzy neutrosophic semi-Baire space is a fuzzy neutrosophic Baire space.

5. Conclusions

In this paper, we introduced the concept of fuzzy neutrosophic semi-Baire spaces in fuzzy neutrosophic topological spaces. Also we define fuzzy neutrosophic semi-nowhere dense, fuzzy neutrosophic semi-residual set, fuzzy neutrosophic semi-one category set, fuzzy neutrosophic semi-two category sets, fuzzy neutrosophic semi-one category space, fuzzy neutrosophic semi-two category space. Some of its characterizations of fuzzy neutrosophic semi-Baire spaces are also studied. This shall be extended in the future research studies.

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FUZZY NEUTROSOPHIC DENSE SETS AND BAIRE SPACES

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Abstract:

This paper introduces the concept of fuz. neu. den. sets in fuz. neu. top. sp.s. Also, we obtain several charact. of fuz. neu. den. sets in fuz. neu. Baire sp.s are established. Connection. between fuz. neu. den. sets and fuz. neu. Baire sp., fuz. neu. den. Baire sp.s are studied. Several properties are also discussed with illustration and suitable examples.

Keywords:

Fuz. neu. den. set, Fuz. neu. Baire sp., Fuz. neu. den. Baire sp..

1. Introduction:

The concept of fuz. sets were first initiate by Zadeh. L. A [11] in 1965, in which he proposed that the traditional notion of set, in which an element is either a member or not a member, should be generalized to allow for degree of membership. This gives rise to the expansion of fuz. set theory, which has since been applied to a large assortment of fields, involve control theory, image processing, and artificial intelligence.

Neu. sets were first initiate by Florentin Smarandache [7] in 1955, which is a overview of the concepts of fuz. sets and intuitionistic fuz. sets. Neu. sets allow for consideration of indeterminacy, neutrality, and inconsistency in a given sp.. In 2017, Veereswari [10] initiate fuz. neu. top. sp.s, which combines the ideas of fuz. sets and neu. sets to introduce a more general framework for studying top. sp.s.

Since then, researches have been working on developing the theory of fuz. neu. top. sp.s and its application to diverse fields such as soft computing, artificial intelligence, decision making, pattern recognition, and image processing.

2. Preliminaries:

Throughout this present paper, X denotes the fuzzy neutrosophic topological spaces. Let A_N be a fuzzy neutrosophic set on X. The fuzzy neutrosophic interior and closure of A_N is denoted as $fn(A_N)^+$, $fn(A_N)^-$ respectively. A fuzzy neutrosophic set A_N is defined to be fuzzy neutrosophic open set $(fnOS)$ if $A_N \leq fn\,(((A_N)^-)^+)^-$. The complement of a fuzzy neutrosophic open set is called fuzzy neutrosophic closed set $(fnCS)$.

Definition 2.1 [2]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ x \in X$ where $T, I, F: X \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

With the condition $0 \leq T_{A^*}(x) + \mathrm{I}_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2 [2]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$.

Definition 2.3 [2]:

Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$$

$$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$$

Definition 2.4 [2]:

The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.5 [2]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0$, $I_A(x) = 0$, $F_A(x) = 1\ for\ all\ x \in X$. It is denoted by 0_N.

Definition 2.6 [2]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1$, $I_A(x) = 1$, $F_A(x) = 0\ for\ all\ x \in X$. It is denoted by 1_N.

Definition 2.7 [2]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as

$A^C = \langle x, T_{A^C}(x), \mathrm{I}_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8 [1]:

A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X

$(i)\ \ 0_N, 1_N \in \tau$

$(ii)\ A_1 \cap A_2 \in \tau\ \ for\ any\ A_1, A_2 \in \tau$

$(iii)\ \cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i : i \in J\}\ \in \tau$

Satisfying the following axioms.

In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any Fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [1]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10 [1]:

Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$int(A) = \cup \{G : G \text{ is a fuzzy neutrosophic open set in } X \text{ and } G \subseteq A\}$$

$$cl(A) = \cap \{G : G \text{ is a fuzzy neutrosophic closed set in } X \text{ and } A \subseteq G\}$$

Definition 2.11 [4]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $F_\sigma -$ set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$.

Definition 2.12 [4]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $G_\delta -$ set in (X, τ_N) if $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$.

Definition 2.13 [4]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic semi-open if $A_N \leq fn(\,((A_N)^+)^-)$. The complement of A_N in (X, τ_N) is called a fuzzy neutrosophic semi-closed set in (X, τ_N).

Definition 2.14 [4]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic dense if there exist no fuzzy neutrosophic closed set B_N in (X, τ_N) such that $A_N \subset B_N \subset 1_X$. That is, $fn\,(A_N)^- = 1_N$.

Definition 2.15 [4]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic nowhere dense set if there exist no non-zero fuzzy neutrosophic open set B_N in (X, τ_N) such that $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$.

Definition 2.16 [4]:

Let (X, τ_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set A_N in (X, τ_N) is called fuzzy neutrosophic one category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where ${A_N}_i{}'$s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Any other fuzzy neutrosophic set in (X, τ_N) is said to be of fuzzy neutrosophic two category.

Definition 2.17 [4]:

A fuzzy neutrosophic topological space (X, τ_N) is called fuzzy neutrosophic one category space if the fuzzy neutrosophic set 1_X is a fuzzy neutrosophic one category set in (X, τ_N). That is $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$, where

$A_{N_i}{}'$s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Otherwise (X, τ_N) will be called a fuzzy neutrosophic two category space.

Definition 2.18 [4]:

Let A_N be a fuzzy neutrosophic first category set in (X, τ_N). Then $\overline{A_N}$ is called fuzzy neutrosophic residual set in (X, τ_N).

3. On Fuzzy Neutrosophic Dense Sets and Baire Spaces

Definition 3.1:

Let (M, τ_N) be a fuz. neu. top. sp.. A fuz. neu. set A_N in (M, τ_N) is called fuz. neu. Baire den. set if B_N is a fuz. $G_\delta -$ set in (M, τ_N), $A_N \wedge B_N$ is a fuz. neu. two caty. set in (M, τ_N).

Eg; 3.1:

Let $M = \{a, b, c\}$ and consider the family $\tau_N = \{0_{N,} 1_N, A_N, B_N, C_N, E_N, G_N\}$ where

$A_N = \{\langle a, 0.3,0.3,0.5\rangle, \langle b, 0.6,0.6,0.5\rangle, \langle c, 0.6,0.6,0.5\rangle\}$

$B_N = \{\langle a, 0.6,0.6,0.6\rangle, \langle b, 0.3,0.3,0.3\rangle, \langle c, 0.6,0.6,0.6\rangle\}$

$C_N = \{\langle a, 0.3,0.3,0.4\rangle, \langle b, 0.7,0.7,0.4\rangle, \langle c, 0.3,0.3,0.4\rangle\}$

$E_N = \{\langle a, 0.3,0.3,0.3\rangle, \langle b, 0.3,0.3,0.3\rangle, \langle c, 0.7,0.6,0.6\rangle\}$

$G_N = \{\langle a, 0.4,0.3,0.3\rangle, \langle b, 0.3,0.2,0.4\rangle, \langle c, 0.6,0.6,0.5\rangle\}$

Thus (M, τ_N) is a fuz. neu. top. sp.s. Now, $\overline{A_N}, \overline{B_N}$ and C_N, are fuz. neu. nowh. den. sets in (M, τ_N). Now, $\overline{A_N}, \overline{B_N}$ and C_N, are fuz. neu. one caty. sets in (M, τ_N). Now, the fuz. neu. sets E_N & A_N, G_N & B_N are not fuz. neu. one caty. sets in (M, τ_N) and hence fuz. neu. two caty. sets in (M, τ_N). Thus E_N & G_N are fuz. neu. Baire den. set in (M, τ_N).

Pro. 3.1:

If A_N is a fuz. neu. Baire den. set in a fuz. neu. top. sp. (M, τ_N), then for a fuz. $G_\delta -$ set B_N in (X, τ_N), $A_N \wedge B_N = \bigvee_{i=1}^{\infty}(C_{N_i})$, where $(C_{N_i})'s$ are fuz. neu. nowhere den. set in (M, τ_N).

Proof:

Let A_N be a fuz. neu. Baire den. set in a fuz. neu. top. sp. (M, τ_N), then for a fuz. $G_\delta -$ set in B_N in (M, τ_N), $A_N \wedge B_N$ is a fuz. neu. two caty. set in (M, τ_N) and hence $A_N \wedge B_N$ is not a fuz. neu. one caty. set in (M, τ_N). Thus $A_N \wedge B_N = \bigvee_{i=1}^{\infty}(C_{N_i})$, where $(C_{N_i})'s$ are fuz. neu. nowhere den. set in (M, τ_N).

Pro. 3.2:

If A_N is a fuz. neu. Baire den. set in a fuz. neu. top. sp. (M, τ_N), then for a fuz. neu. $G_\delta -$ set B_N in (M, τ_N), $1 - (A_N \wedge B_N) = \bigwedge_{i=1}^{\infty} E_{N_i}$, where $fn\,(E_{N_i})^- = 1_N$.

Proof:

Let A_N be a fuz. neu. Baire den. set in a fuz. neu. top. sp. (M, τ_N). Then by Pro. (3.1), $A_N \wedge B_N = \bigvee_{i=1}^{\infty}(C_{N_i})$, where $(C_{N_i})'s$ are fuz. neu. nowhere den. set in (M, τ_N). Then $1 - (A_N \wedge B_N) = 1 - \bigvee_{i=1}^{\infty}(C_{N_i}) = \bigwedge_{i=1}^{\infty}(1 - C_{N_i})$ in (M, τ_N). Since, C_{N_i} is a fuz. neu. nowhere den. set in (M, τ_N), $fn((C_{N_i}^{-})^{+}) = 0$. Since $fn(C_{N_i}^{+}) \leq fn((C_{N_i}^{-}))^{+}$ in (M, τ_N), $fn(C_{N_i}^{+}) = 0$ and hence $1 - fn(C_{N_i}^{+}) = 1$ in (M, τ_N). This $\Rightarrow fn(1 - C_{N_i})^{-} = 1$. Thus $1 - (A_N \wedge B_N) = \bigwedge_{i=1}^{\infty}(1 - C_{N_i})$, where $(1 - C_{N_i})'s$ are fuz. neu. nowhere den. set in (M, τ_N). Let $E_{N_i} = 1 - C_{N_i}$. Then let $1 - (A_N \wedge B_N) = \bigwedge_{i=1}^{\infty}(E_{N_i})$, where $(E_{N_i})'s$ are fuz. neu. nowhere den. set in (M, τ_N).

Pro. 3.3:

If A_N is a fuz. neu. Baire den. set in a fuz. neu. top. sp. (M, τ_N), then there exist a fuz. $G_\delta -$ set B_N in (X, τ_N), fuz. neu. two caty. set C_N s.t $C_N \leq A_N$ in (M, τ_N).

Proof:

Let A_N be a fuz. neu. Baire den. set in (M, τ_N), then for a fuz. $G_\delta -$ set in B_N in (M, τ_N), $A_N \wedge B_N$ is a fuz. two caty. set in (M, τ_N). Clearly $A_N \wedge$

$B_N \leq A_N$. Let $A_N \wedge B_N = C_N$. Thus C_N is a fuz. neu. two caty. set in (M, τ_N) s.t $C_N \leq A_N$ in (M, τ_N).

Pro. 3.4:

If A_N is a fuz. neu. Baire den. set in (M, τ_N), then each fuz. neu. two caty. set is a fuz. neu. den. set in a fuz. neu. topological (M, τ_N), then A_N is a fuz. neu. den. set in (M, τ_N).

Proof:

Let A_N be a fuz. neu. Baire den. set in (M, τ_N), then by Pro. (3.3), ∃ a fuz. neu. two caty. set $C_N \leq A_N$ in (M, τ_N) and hence $fn(C_N)^- = fn(A_N)^+$, in (M, τ_N). By hypothesis, $fn(C_N)^- = 1$ in (M, τ_N). Then, $1 \leq fn(A_N)^-$. That is, $fn(A_N)^- = 1$ in (M, τ_N). Hence A_N is a fuz. neu. den. set in (M, τ_N).

Definition 3.2:

A fuz. neu. top. sp. (M, τ_N) is called a fuz. neu. submaximal sp. if for each fuz. neu. set A_N in (M, τ_N) s.t $fn(A_N)^- = 1$, $A_N \in \tau_N$ in (M, τ_N).

Definition 3.3:

A fuz. neu. top. sp. (M, τ_N) is called a fuz. neu. $P-$ sp. if countable intersection of fuz. neu. op. sets in (M, τ_N) is fuz. neu. open. That is, every non zero fuz. neu. $G_\delta -$ set in (M, τ_N) is a fuz. neu. op. set in (M, τ_N).

Definition 3.4:

If the fuz. neu. top. sp. (M, τ_N) is a fuz. neu. Baire sp. then no non-zero fuz. neu. op. set is a fuz. neu. one caty. set in (M, τ_N) .

Definition 3.5:

If A_N is a fuz. neu. nowhere den. set in a fuz. neu. submaximal sp. (M, τ_N), then A_N is a fuz. neu. cld. set in (M, τ_N).

Definition 3.6:

If A_N is a fuz. neu. $G_\delta -$ set in a fuz. neu. Baire and fuz. neu. $P -$ sp. (M, τ_N), then A_N is a fuz. neu. two caty. set in (M, τ_N).

Pro. 3.5:

If each fuz. neu. $G_\delta -$ set is a fuz. neu. Baire den. set in a fuz. neu. submaximal sp. (M, τ_N), then (M, τ_N) is a fuz. neu. $P -$ sp..

Proof:

Let A_N be a fuz. neu. $G_\delta -$ set in a fuz. neu. submaximal sp. (M, τ_N). Then by the hypothesis, $A_N \wedge B_N$ is a fuz. neu. Baire den. set in (M, τ_N). Since (M, τ_N) is a fuz. neu. submaximal sp., the fuz. neu. Baire den. set $A_N \wedge B_N$ in (M, τ_N), is a fuz. neu. op. set in (M, τ_N). That is, every fuz. neu. $G_\delta -$ set in (M, τ_N) is a fuz. neu. op. set in (M, τ_N). Therefore (M, τ_N) is a fuz. neu. $P -$ sp..

Pro. 3.6:

If A_N is a fuz. neu. Baire den. set in a fuz. neu. submaximal sp. (M, τ_N), then for fuz. neu. op. set B_N in (M, τ_N), $A_N \wedge B_N$ is not a fuz. neu. $F_\sigma -$ set in (M, τ_N).

Proof:

Let A_N be a fuz. neu. Baire den. set in (M, τ_N). Then by Pro. (3.1), for a fuz. neu. op. set B_N in (M, τ_N), $A_N \wedge B_N = \bigvee_{i=1}^{\infty}(C_{N_i})$, where $(C_{N_i})'s$ are fuz. neu. nowh. den. sets in (M, τ_N). Since (M, τ_N) is a fuz. neu. submaximal sp., by definition (3.5), the fuz. neu. nowh. den. sets $(C_{N_i})'s$ are fuz. neu. closed sets in (M, τ_N). Then $\bigvee_{i=1}^{\infty}(C_{N_i})$ is a fuz. neu. $F_\sigma -$ set in (M, τ_N). Thus $A_N \wedge B_N = \bigvee_{i=1}^{\infty}(C_{N_i}), \Rightarrow A_N \wedge B_N$ is not a fuz. neu. $F_\sigma -$ set in (M, τ_N).

Pro. 3.7:

If A_N is a fuz. neu. set in a fuz. neu. Baire and fuz. neu. $P -$ sp. (M, τ_N) s.t $A_N \wedge B_N$ is a fuz. neu. $G_\delta -$ set, for a fuz. neu. op. set B_N in (M, τ_N), then A_N is a fuz. neu. Baire den. set in (M, τ_N).

Proof:

Let A_N be a fuz. neu. set in (M, τ_N) s.t $A_N \wedge B_N$ is a fuz. neu. $G_\delta -$ set, for a fuz. neu. op. set B_N in (M, τ_N). Since (M, τ_N) is a fuz. neu. $P -$ sp.,

the fuz. neu. $G_\delta -$ set $A_N \wedge B_N$ is a fuz. neu. op. set B_N in (M, τ_N). Also since (M, τ_N) is a fuz. neu. Baire sp. by definition (3.4), the non-zero fuz. neu. op. set $A_N \wedge B_N$ is not a fuz. neu. one caty. set in (M, τ_N) and hence $A_N \wedge B_N$ is a fuz. neu. two caty. set in (M, τ_N). This $\Rightarrow A_N$ is a fuz. neu. Baire den. set in (M, τ_N).

Pro. 3.8:

If A_N is a fuz. neu. $G_\delta -$ set in a fuz. neu. Baire and fuz. neu. $P -$ sp. (M, τ_N) s.t $A_N \leq B_N$, for fuz. neu. op. set B_N in (M, τ_N), then A_N is a fuz. neu. Baire den. set in (M, τ_N).

Proof:

Let A_N be a fuz. neu. $G_\delta -$ set s.t $A_N \leq B_N$, for a fuz. neu. op. set B_N in (M, τ_N). Since (M, τ_N) is a fuz. neu. Baire and fuz. neu. $P -$ sp. by definition (3.6), the fuz. neu. $G_\delta -$ set is a fuz. neu. two caty. set in (M, τ_N). Now $A_N \wedge B_N = A_N \Rightarrow A_N \wedge B_N$ is a fuz. neu. two caty. set in (M, τ_N) and hence A_N is a fuz. neu. Baire den. set in (M, τ_N).

Pro. 3.9:

If A_N is a fuz. neu. $F_\sigma -$ set in a fuz. neu. Baire and fuz. neu. $P -$ sp. (M, τ_N) s.t $A_N \geq B_N$, for fuz. neu. cld. set B_N in (M, τ_N), then $1 - A_N$ is a fuz. neu. Baire den. set in (M, τ_N).

Proof:

Let A_N be a fuz. neu. $F_\sigma -$ set s.t $A_N \geq B_N$, for a fuz. neu. cld. setB_N in (M, τ_N). Now $1 - A_N$ is a fuz. neu. $G_\delta -$ set s.t $1 - A_N \leq 1 - B_N$, where $1 - B_N$ is a fuz. neu. op. set in (M, τ_N). Then by Pro. (3.8), $1 - A_N$ is a fuz. neu. Baire den. set in (M, τ_N).

Pro. 3.10:

If A_N is a fuz. neu. op. set in a fuz. neu. Baire sp. (M, τ_N), then A_N is a fuz. neu. Baire den. set in (M, τ_N).

Proof:

Let A_N be a fuz. neu. op. set in (M, τ_N). Then for a fuz. neu. op. set B_N in (M, τ_N), $A_N \wedge B_N$ is a fuz. neu. op. set in (M, τ_N). Since (M, τ_N) is a fuz. neu. Baire sp. by definition (3.4), $A_N \wedge B_N$ is not a fuz. neu. one caty. set in (M, τ_N) and hence $A_N \wedge B_N$ is a fuz. neu. two caty. set in (M, τ_N). Thus A_N is a fuz. neu. Baire den. set in (M, τ_N).

Pro. 3.11:

If A_N is a fuz. neu. $G_\delta -$ set in a fuz. neu. Baire and fuz. neu. $P -$ sp. (M, τ_N), then A_N is a fuz. neu. Baire den. set in (M, τ_N).

Proof:

Let A_N be a fuz. neu. $G_\delta -$ set in (M, τ_N). Since (M, τ_N) is a fuz. neu. $P -$ sp., the fuz. neu. $G_\delta -$ set A_N is a fuz. neu. op. set in (M, τ_N). Since

(M, τ_N) is a fuz. neu. Baire sp., by Pro. (3.10), A_N is a fuz. neu. Baire den. set in (M, τ_N).

Shortcut keywords:

S.No	KEYWORDS	SHORTCUTS
1	fuzzy	fuz.
2	neutrosophic	neu.
3	category	caty.
4	such that	s.t
5	closed set	cld. set
6	open set	op. set
7	Nowhere dense sets	Nowh.den.sets
8	implies that	⇒
9	Proposition	Pro.
10	Submaximal space	Submax. sp.
11	(X, τ_n)	(M, τ_n)
12	topological space	top.sp.
13	implies that	⇒
14	dense sets	den. sets
15	Example	Eg;
16	residual	resd.
17	universex	U of X
18	characterizations	charact.
19	relations	connection.
20	introduced	initate
21	Lofti A Zadeh	Zadeh. L. A

Conclusion:

In this paper, the concept of fuz. neu. den. sets in fuz. neu. top. sp.s and its properties are discussed. Also, we obtained several charact. of fuz. neu. den. sets in fuz. neu. Baire sp.s. Connection. between fuz. neu. den. sets and fuz. neu. Baire sp., fuz. neu. den. Baire sp.s are studied. Some of its charact. and examples of fuz. neu. Baire sp.s are established. This shall be extended in the future research studies.

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$C-$ Almost $P-$ Spaces and Other Spaces on Fuzzy Neutrosophic Topological Spaces

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ABSTRACT: In this paper, we define fuzzy neutrosophic $C-$ almost $P-$ spaces and analyze some new properties of these spaces. Also, we discuss the other spaces of fuzzy neutrosophic topological spaces.

KEYWORDS: Fuzzy neutrosophic regular closed(open) set, Fuzzy neutrosophic $F_\sigma-$ set, Fuzzy neutrosophic $G_\delta-$ set, Fuzzy neutrosophic $C-$ almost $P-$ space, Fuzzy neutrosophic $P-$space, Fuzzy neutrosophic hyper-connected space, Fuzzy neutrosophic extremally disconnected space, Fuzzy neutrosophic basically disconnected space, Fuzzy neutrosophic resolvable space, Fuzzy neutrosophic submaximal space, Fuzzy neutrosophic almost resolvable space.

AMS subject classification: 54A40, 03E72

I. INTRODUCTION

The concept of fuzzy sets was introduced by L.A. Zadeh in 1965 [8]. Then the fuzzy set theory is extension by many researchers. The important concept of fuzzy topological space was offered by C. L. Chang [2] and from that point forward different ideas in topology have been reached out to fuzzy topological space. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [7] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields.

In this paper, we define a new concept of fuzzy neutrosophic $C-$ almost $P-$ space and we also discussed some new properties and examples based on this concept. Also, we introduced other spaces of fuzzy neutrosophic topological spaces and examples are investigated.

II. PRELIMINARIES

Throughout the present paper, X denote the fuzzy neutrosophic topological spaces. Let A_N be a fuzzy neutrosophic set on X. The fuzzy neutrosophic interior and closure of A_N is denoted by $fn(A_N)^+$, $fn(A_N)^-$ respectively. A fuzzy neutrosophic set A_N is defined to be fuzzy neutrosophic open set $(fnOS)$ if $A_N \leq fn\,(((A_N)^-)^+)^-$. The complement of a fuzzy neutrosophic open set is called fuzzy neutrosophic closed set $(fnCS)$.

Definition 2.1 [1]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $x \in X$ where $T, I, F: X \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

With the condition $0 \leq T_{A^*}(x) + \mathrm{I}_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2 [1]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$.

Definition 2.3 [1]:

Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$

$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$

Definition 2.4 [1]:

The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.5 [1]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1\ for\ all\ x\ \in X.$ It is denoted by 0_N.

Definition 2.6 [1]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0\ for\ all\ x\ \in X.$ It is denoted by 1_N.

Definition 2.7 [1]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), \mathrm{I}_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8 [1]:

A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X

$(i)\ \ 0_N, 1_N \in \tau$

$(ii)\ A_1 \cap A_2 \in \tau\ \ for\ any\ A_1, A_2 \in \tau$

$(iii)\ \cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i : i \in J\}\ \in \tau$

Satisfying the following axioms.

In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any Fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [1]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10 [1]:

Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$int(A) = \cup \{G : G\ is\ a\ fuzzy\ neutrosophic\ open\ set\ in\ X\ and\ G \subseteq A\}$

$cl(A) = \cap \{G : G\ is\ a\ fuzzy\ neutrosophic\ closed\ set\ in\ X\ and\ A \subseteq G\}$

Definition 2.11[3]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $F_\sigma -$ set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$.

Definition 2.12[3]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $G_\delta -$ set in (X, τ_N) if $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$.

Definition 2.13[3]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic semi-open if $A_N \leq fn(\ ((A_N)^+)^-)$. The complement of A_N in (X, τ_N) is called a fuzzy neutrosophic semi-closed set in (X, τ_N).

Definition 2.14[3]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic dense if there exist no fuzzy neutrosophic closed set B_N in (X, τ_N) such that $A_N \subset B_N \subset 1_X$. That is, $fn\ (A_N)^- = 1_N$.

Definition 2.15[3]:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic nowhere dense set if there exist no non-zero fuzzy neutrosophic open set B_N in (X, τ_N) such that $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$.

Definition 2.16[3]:

Let (X, τ_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set A_N in (X, τ_N) is called fuzzy neutrosophic first category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Any other fuzzy neutrosophic set in (X, τ_N) is said to be of fuzzy neutrosophic second category.

Definition 2.17[3]:

A fuzzy neutrosophic topological space (X, τ_N) is called fuzzy neutrosophic first category space if the fuzzy neutrosophic set 1_X is a fuzzy neutrosophic first category set in (X, τ_N). That is $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$, where ${A_{N_i}}'s$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Otherwise (X, τ_N) will be called a fuzzy neutrosophic second category space.

Definition 2.18[3]:

Let A_N be a fuzzy neutrosophic first category set in (X, τ_N). Then $\overline{A_N}$ is called fuzzy neutrosophic residual set in (X, τ_N).

III. Fuzzy Neutrosophic $C-$almost $P-$space

Definition 3.1:

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic regular – open if $A_N = ((A_N)^-)^+$ and fuzzy neutrosophic regular – closed if $A_N = ((A_N)^+)^-$.

Definition 3.2:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic $C-$almost $P-$space if for every fuzzy neutrosophic regular- closed set is a fuzzy neutrosophic $G_\delta-$set A_N in (X, τ_N), $(A_N)^- \neq 0$.

Example3.1:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets $A_N, B_N \; and \; C_N$ defined on X as follows:

A_N: $\{\langle a, 0.5, 0.6, 0.6\rangle, \langle b, 0.5, 0.5, 0.5\rangle, \langle c, 0.5, 0.6, 0.6\rangle\}$

B_N: $\{\langle a, 0.5, 0.6, 0.5\rangle, \langle b, 0.6, 0.6, 0.6\rangle, \langle c, 0.5, 0.5, 0.5\rangle\}$

C_N: $\{\langle a, 0.6, 0.5, 0.5\rangle, \langle b, 0.4, 0.4, 0.4\rangle, \langle c, 0.5, 0.5, 0.6\rangle\}$

Then $\tau_N = \{0_N, A_N, B_N, C_N, \; A_N \vee B_N, \; A_N \vee C_N, \; B_N \vee C_N, \; A_N \wedge B_N, \; A_N \wedge C_N, \; C_N \vee (A_N \wedge B_N), \; A_N \vee B_N \vee C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.

The fuzzy neutrosophic regular closed sets in (X, τ_N) are $1 - B_N, 1 - (A_N \wedge B_N)$, and $1 - (A_N \wedge C_N)$.

Then $1 - B_N, 1 - (A_N \wedge B_N)$, and $1 - (A_N \wedge C_N)$ are fuzzy neutrosophic $G_\delta-$sets in (X, τ_N).

Thus the fuzzy neutrosophic regular closed sets $1 - B_N, 1 - (A_N \wedge B_N)$, $1 - (A_N \wedge C_N)$ are fuzzy neutrosophic $G_\delta-$sets and also $(1 - B_N)^+ \neq 0$, $(1 - A_N \wedge B_N)^+ \neq 0, (1 - A_N \wedge C_N)^+ \neq 0$in (X, τ_N).Hence (X, τ_N) is a fuzzy neutrosophic $C-$almost $P-$space.

Definition 3.3:

A fuzzy neutrosophic somewhere dense set if there exists a non - zero fuzzy neutrosophic open B_N in (X, τ_N) such that $B_N < (A_N)^-$.That is $((A_N)^-)^+ \neq 0$ in (X, τ_N).

Proposition 3.1:

If A_N is a fuzzy neutrosophic $F_\sigma-$set in a fuzzy neutrosophic $C-$almost $P-$space (X, τ_N) then $(A_N)^- \neq 1$.

Proof:

Let A_N be a fuzzy neutrosophic $F_\sigma-$set in a fuzzy neutrosophic $C-$almost $P-$space (X, τ_N). Then,$1 - A_N$ is a fuzzy neutrosophic regular closed and fuzzy neutrosophic $G_\delta-$set in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic $C-$almost $P-$space, for the fuzzy neutrosophic $G_\delta-$set $(1 - A_N)$, we have $(1 - A_N)^+ \neq 0$. This implies that $1 - (A_N)^- \neq 0$ and hence we have $(A_N)^- \neq 1$.

Proposition 3.2:

If each non-zero fuzzy neutrosophic $G_\delta-$set is a fuzzy neutrosophic regular closed set in a fuzzy neutrosophic topological space (X, τ_N), then (X, τ_N) is a fuzzyneutrosophic $C-$almost $P-$space.

Proof:

Let $A_N$be a non-zerofuzzy neutrosophic $G_\delta-$set in (X, τ_N) such that $((A_N)^-)^+ = A_N$.We claim that $(A_N)^+ \neq 0$. Assume the contrary. Then $(A_N)^+ \neq 0$ will imply that $((A_N)^-)^+ = (0)^- = 0$ and hence we will have$A_N = 0$, a contradiction to $A_N$beings a non-zero fuzzy neutrosophic $G_\delta-$set in (X, τ_N).Hence, we must have$(A_N)^+ \neq 0$,for a fuzzy neutrosophic $G_\delta-$set $A_N$in (X, τ_N) andtherefore(X, τ_N) is a fuzzy neutrosophic $C-$almost $P-$space.

Proposition 3.3:

If each non-zero fuzzy neutrosophic $G_\delta-$set is a fuzzy neutrosophic semi – open set in a fuzzy neutrosophic topological space (X, τ_N), then (X, τ_N) is a fuzzyneutrosophic $C-$almost $P-$space.

Proof:

Let $A_N$be a non-zero fuzzy neutrosophic $G_\delta-$set in (X, τ_N) such that $A_N \leq ((A_N)^+)^-$. We claim that $(A_N)^+ \neq 0$. Assume the contrary we will have $A_N = 0$, a contradiction to A_N beings a non-zero fuzzy

neutrosophic $G_\delta -$ set in (X, τ_N). Hence, we must have $(A_N)^+ \neq 0$, for a fuzzy neutrosophic $G_\delta -$ set A_N in (X, τ_N) and therefore (X, τ_N) is a fuzzy neutrosophic $C -$ almost$P -$ space.

Proposition 3.4:

If $A_N$is a fuzzy neutrosophic one category set in a fuzzy neutrosophic $C -$ almost $P -$ space in (X, τ_N), then $(A_N)^+ \neq 1$ in (X, τ_N).

Proof:

Let A_N be a fuzzy neutrosophic one category set in (X, τ_N). Then $A_N = \bigvee_{i=1}^{\infty}(A_{N_i})$, where$(A_{N_i})'s$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Now $A_{N_i} \leq \left(A_{N_i}\right)^-$, implies that $\bigvee_{i=1}^{\infty} A_{N_i} \leq \bigvee_{i=1}^{\infty}\left(A_{N_i}\right)^-$ and hence we have $[\bigvee_{i=1}^{\infty}(A_{N_i})]^- \leq [\bigvee_{i=1}^{\infty}(A_{N_i})^-]^-$. Then $[\bigvee_{i=1}^{\infty}(A_{N_i})]^- \leq [\bigvee_{i=1}^{\infty}(A_{N_i})^-]^- \rightarrow (1)$

Now $\bigvee_{i=1}^{\infty}\left(A_{N_i}\right)^-$ is a fuzzy neutrosophic $F_\sigma -$ set in (X, τ_N). Since (X, τ_N) is a fuzzy neutrosophic $C -$ almost $P -$ space, by proposition 3.1 $[\bigvee_{i=1}^{\infty}(A_{N_i})^-]^- \neq 1 \rightarrow (2)$.

Now we claim that A_N is not a fuzzy neutrosophic dense set in (X, τ_N). Assume the contrary. Suppose that A_N is a fuzzy neutrosophic nowhere dense set, then $(A_N)^- = 1$, implies from (1), $1 \leq [\bigvee_{i=1}^{\infty}(A_{N_i})^-]^-$. That is $[\bigvee_{i=1}^{\infty}(A_{N_i})^-]^- = 1$, a contradiction to (2), hence we must have $(A_N)^- \neq 1$ in (X, τ_N).

Proposition 3.5:

If each non-zero fuzzy neutrosophic one category set is a fuzzy neutrosophic dense set in a fuzzy neutrosophic topological space(X, τ_N), then (X, τ_N) is not a fuzzy neutrosophic $C -$ almost $P -$ space.

Proof:

Let A_N be a fuzzy neutrosophic one category set in (X, τ_N) such that $(A_N)^- = 1$.Then $A_N = \bigvee_{i=1}^{\infty}(A_{N_i})$, where $(A_{N_i})'s$ are fuzzy neutrosophic nowhere dense sets in (X, τ_N). Now $1 - \left(A_{N_i}\right)^-{}'s$ are fuzzy neutrosophic open set in (X, τ_N). Let $B_N = \bigwedge_{i=1}^{\infty}[1 - \left(A_{N_i}\right)^-]$. Then $B_N$fuzzy neutrosophic $G_\delta -$ set in (X, τ_N). Now $B_N = \bigwedge_{i=1}^{\infty}[1 - \left(A_{N_i}\right)^-] = 1 - [\bigvee_{i=1}^{\infty}(A_{N_i})]^- \leq 1 - [\bigvee_{i=1}^{\infty}(A_{N_i})] = 1 - A_{N_i}$. That is, $B_N = 1 - A_N$. Then $(B_N)^+ \leq (1 - A_N)^+$ and hence $(B_N)^+ \leq 1 - (A_N)^- = 1 - 1 = 0_N$. Hence, for the fuzzy neutrosophic $G_\delta -$ set B_N in (X, τ_N), $(B_N)^+ = 0$. Therefore (X, τ_N) is not a fuzzyneutrosophic $C -$ almost$P -$ space.

Proposition 3.6:

If $C_N$is a fuzzy neutrosophic residual set in a fuzzy neutrosophic $C -$ almost $P -$ space in (X, τ_N), then $(C_N)^+ \neq 0$ in(X, τ_N).

Proof:

Let C_N be a fuzzy neutrosophic residual set in (X, τ_N). Then $1 - C_N$ is a fuzzy neutrosophic first category set in (X, τ_N) and hence by proposition 3.4, $(1 - C_N)^- \neq 1$ in (X, τ_N). Therefore $1 - (C_N)^+ = (1 - C_N)^- \neq 1$. Therefore $(C_N)^+ \neq 0$ in(X, τ_N).

Proposition 3.7:

If $B_N$is a fuzzy neutrosophic residual set in a fuzzy neutrosophic $C -$ almost $P -$ space in (X, τ_N), then B_N is not a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proof:

Let $B_N$be a fuzzy neutrosophic residual set in a fuzzy neutrosophic$C -$ almost $P -$ space in (X, τ_N), then by proposition 3.6, $(B_N)^+ \neq 0$,in(X, τ_N). We claim that $((B_N)^-)^+ \neq 0$. Assume that contrary. Then $((B_N)^-)^+ = 0$ and $(B_N)^+ \leq ((B_N)^-)^+$, will imply that $(B_N)^+ = 0$, a contradiction. Hence B_N is not a fuzzy neutrosophic nowhere dense set in (X, τ_N).

Proposition 3.8:

If A_N is a fuzzy neutrosophic semi – open setin a fuzzy neutrosophic topological space (X, τ_N), then $(A_N)^-$ is a fuzzy neutrosophic regular closed set in (X, τ_N).

Proof:

If A_N is a fuzzy neutrosophic semi – open set in (X, τ_N) then $A_N \leq ((A_N)^+)^-$ and hence

$$\begin{aligned}(A_N)^- &\leq (((A_N)^+)^-)^- \\ &= ((A_N)^+)^- \\ &\leq [(A_N{}^-)^+]^- \rightarrow (1)\end{aligned}$$

Also, we have $[(A_N)^-]^+ \leq [(A_N)^-]^- = (A_N)^- \rightarrow (2)$

From (1) and (2),

$[((A_N)^-)^+]^- = (A_N)^-$ and hence $(A_N)^-$ is a fuzzy neutrosophic regular closed set in (X, τ_N).

Proposition 3.9:

If A_N is a fuzzy neutrosophic regular open set in a fuzzy neutrosophic $C-$almost $P-$space, then A_N is a fuzzy neutrosophic $F_\sigma-$set in a (X,τ_N).

Proof:

Let A_N be a fuzzy neutrosophic regular open set in (X,τ_N). Then, $1-A_N$ is a fuzzy neutrosophic regular closed set in (X,τ_N). Since (X,τ_N) is a fuzzy neutrosophic $C-$almost $P-$space, $1-A_N$ is a fuzzy neutrosophic $G_\delta-$set in (X,τ_N). Then, A_N is a fuzzy neutrosophic $F_\sigma-$set in a (X,τ_N).

Definition

A fuzzy neutrosophic set A_N in fuzzy neutrosophic topological space (X,τ_N) is called a fuzzy neutrosophic $\sigma-$nowhere dense set if A_N is a fuzzy neutrosophic $F_\sigma-$set in a (X,τ_N) such that $(A_N)^+=0$.

Proposition 3.10:

If A_N is a fuzzy neutrosophic regular open set in a fuzzy neutrosophic $C-$almost $P-$space, then A_N is not a fuzzy neutrosophic $\sigma-$nowhere dense set in (X,τ_N).

Proof:

Let A_N be a fuzzy neutrosophic regular open set in (X,τ_N). Then, $[(A_N)^-]^+=A_N$, in (X,τ_N) and $[(A_N)^-]^+=(A_N)^+$. Then $[(A_N)^-]^+=(A_N)^+$ and $(A_N)^+=0$, in (X,τ_N). Since (X,τ_N) is a fuzzy neutrosophic $C-$almost $P-$space, by proposition 3.1, A_N is a fuzzy neutrosophic $F_\sigma-$set in a (X,τ_N). Thus A_N is a fuzzy neutrosophic $F_\sigma-$set with $(A_N)^+\neq 0$ in (X,τ_N). Hence A_N is not a fuzzy neutrosophic $\sigma-$nowhere dense set in (X,τ_N).

Proposition 3.11:

If A_N is a fuzzy neutrosophic closed set in a fuzzy neutrosophic $C-$almost $P-$space with $(A_N)^+\neq 0$, then $(A_N)^+$ is a fuzzy neutrosophic $F_\sigma-$set in a (X,τ_N).

Proof:

Let A_N be a fuzzy neutrosophic closed set in (X,τ_N) with $(A_N)^+\neq 0$. Then, $1-A_N$ is a fuzzy neutrosophic open set in (X,τ_N) such that $(A_N)^-\neq 1$. Since (X,τ_N) is a fuzzy neutrosophic $C-$almost $P-$space, $(1-A_N)^-$is a fuzzy neutrosophic $G_\delta-$set in (X,τ_N) and thus $1-(A_N)^+$ is a fuzzy neutrosophic $G_\delta-$set in (X,τ_N). Hence, $(A_N)^+$ is a fuzzy neutrosophic $F_\sigma-$set in a (X,τ_N).

IV. Other Spaces on Fuzzy Neutrosophic Topological Spaces

Definition 4.1:

A fuzzy neutrosophic topological space (X,τ_N) is called a fuzzy neutrosophic $P-$space if each fuzzy neutrosophic $G_\delta-$set A_N in (X,τ_N) is fuzzy neutrosophic open set in (X,τ_N).

Example 4.1:

Let $X=\{a,b,c\}$ and the fuzzy neutrosophic sets $A_N, B_N \text{ and } C_N$ defined on X as follows:

A_N: $\{\langle a,0.5,0.5,0.6\rangle,\langle b,0.6,0.6,0.4\rangle,\langle c,0.6,0.5,0.6\rangle\}$

B_N: $\{\langle a,0.4,0.5,0.5\rangle,\langle b,0.5,0.5,0.5\rangle,\langle c,0.6,0.4,0.5\rangle\}$

C_N: $\{\langle a,0.6,0.5,0.6\rangle,\langle b,0.5,0.5,0.6\rangle,\langle c,0.5,0.4,0.6\rangle\}$

Then $\tau_N=\{0_N,A_N,B_N,C_N,\ A_N\vee B_N,\ A_N\vee C_N,\ B_N\vee C_N,\ A_N\wedge B_N,\ A_N\wedge C_N,\ B_N\wedge C_N,(B_N\wedge C_N)\wedge(A_N\vee B_N),1_N\}$ is a fuzzy neutrosophic topology on X.

$(A_N)^+=A_N$, $(B_N)^+=B_N$, $(C_N)^+=C_N$, $A_N\vee B_N,\ B_N\vee C_N, A_N\vee C_N,\ A_N\wedge B_N,\ B_N\wedge C_N,\ \ A_N\wedge C_N,$ are fuzzy neutrosophic open set in (X,τ_N).

Now the fuzzy neutrosophic sets $B_N\wedge C_N\ \wedge(A_N\vee B_N)=B_N\wedge C_N$, $(B_N\vee C_N)\wedge(A_N\vee C_N)\wedge(A_N\wedge B_N)=A_N\wedge B_N$ are fuzzy neutrosophic $G_\delta-$sets in (X,τ_N) shows that fuzzy neutrosophic topological space (X,τ_N) is a fuzzy neutrosophic $P-$space.

Definition 4.2:

A fuzzy neutrosophic topological space (X,τ_N) is called a fuzzy neutrosophic hyperconnected space if every non – null fuzzy neutrosophic open subset of (X,τ_N) is fuzzy neutrosophic dense in (X,τ_N).

Example 4.2:

Let $X=\{a,b,c\}$ and the fuzzy neutrosophic sets $A_N, B_N \text{ and } C_N$ defined on X as follows:

A_N: $\{\langle a,0.5,0.5,0.6\rangle,\langle b,0.6,0.5,0.5\rangle,\langle c,0.6,0.5,0.5\rangle\}$

B_N: $\{\langle a,0.5,0.6,0.5\rangle,\langle b,0.7,0.6,0.5\rangle,\langle c,0.6,0.6,0.4\rangle\}$

C_N: $\{\langle a,0.4,0.5,0.4\rangle,\langle b,0.5,0.6,0.5\rangle,\langle c,0.6,0.4,0.6\rangle\}$

Then $\tau_N = \{0_N, A_N, B_N, C_N,\ A_N \vee B_N,\ A_N \vee C_N,\ B_N \vee C_N,\ A_N \wedge B_N,\ A_N \wedge C_N,\ B_N \wedge C_N,\ A_N \vee B_N \vee C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.

$(A_N)^- = 1, (B_N)^- = 1, (C_N)^- = 1,$
$(A_N)^+ = A_N, (B_N)^+ = B_N, (C_N)^+ = C_N,$
$(A_N \vee B_N)^+ = A_N \vee B_N, (B_N \vee C_N)^+ = B_N \vee C_N, (A_N \vee C_N)^+ = A_N \vee C_N,$
$(A_N \wedge B_N)^+ = A_N \wedge B_N, (B_N \wedge C_N)^+ = B_N \wedge C_N, (A_N \wedge C_N)^+ = A_N \wedge C_N,$
$(A_N \vee B_N \vee C_N)^+ = A_N \vee B_N \vee C_N$

If every fuzzy neutrosophic open set is fuzzy neutrosophic dense set. Therefore, fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic hyperconnected space.

Definition 4.3:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic extremally disconnected space if the closure of every fuzzy neutrosophic open set of (X, τ_N) is fuzzy neutrosophic open in (X, τ_N).

Example 4.3:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets $A_N, B_N, C_N, D_N,$ and E_N defined on X as follows:
A_N: $\{\langle a, 0.5,0.4,0.5\rangle, \langle b, 0.3,0.3,0.4\rangle, \langle c, 0.4,0.3,0.4\rangle\}$
B_N: $\{\langle a, 0.5,0.6,0.7\rangle, \langle b, 0.7,0.6,0.5\rangle, \langle c, 0.6,0.5,0.5\rangle\}$
C_N: $\{\langle a, 0.3,0.7,0.3\rangle, \langle b, 0.7,0.3,0.2\rangle, \langle c, 0.2,0.3,0.2\rangle\}$
D_N: $\{\langle a, 0.5,0.6,0.4\rangle, \langle b, 0.7,0.6,0.7\rangle, \langle c, 0.4,0.5,0.4\rangle\}$
E_N: $\{\langle a, 0.3,0.2,0.3\rangle, \langle b, 0.3,0.2,0.2\rangle, \langle c, 0.2,0.3,0.3\rangle\}$
Then $\tau_N = \{0_N, A_N, B_N, C_N,\ D_N, E_N, 1_N\}$ is a fuzzy neutrosophic topology on X.

$(A_N)^+ = A_N \Rightarrow (A_N)^- = 1 - B_N = A_N$
$(B_N)^+ = B_N \Rightarrow (B_N)^- = 1 - B_N = A_N$
$(C_N)^+ = C_N \Rightarrow (C_N)^- = 1 - B_N = A_N$
$(D_N)^+ = D_N \Rightarrow (D_N)^- = 1 - B_N = A_N$
$(E_N)^+ = E_N \Rightarrow (E_N)^- = 1 - B_N = A_N$

The closure of every fuzzy neutrosophic open set of (X, τ_N) is fuzzy neutrosophic open set in (X, τ_N).

Definition 4.4:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic basically disconnected space if the closure of every fuzzy neutrosophic open $F_\sigma -$ set of (X, τ_N) is fuzzy neutrosophic open in (X, τ_N).

Example 4.4:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N *and* C_N defined on X as follows:
A_N: $\{\langle a, 0.5,0.5,0.6\rangle, \langle b, 0.7,0.5,0.5\rangle, \langle c, 0.5,0.7,0.6\rangle\}$
B_N: $\{\langle a, 0.5,0.8,0.5\rangle, \langle b, 0.4,0.8,0.5\rangle, \langle c, 0.8,0.4,0.4\rangle\}$
C_N: $\{\langle a, 0.7,0.6,0.7\rangle, \langle b, 0.6,0.5,0.5\rangle, \langle c, 0.5,0.5,0.6\rangle\}$
Then $\tau_N = \{0_N, A_N, B_N, C_N,\ A_N \vee B_N,\ A_N \vee C_N,\ B_N \vee C_N,\ A_N \wedge B_N,\ A_N \wedge C_N,\ B_N \wedge C_N,\ 1_N\}$ is a fuzzy neutrosophic topology on X.
Now, the fuzzy neutrosophic $F_\sigma -$ set $D_N = (1 - A_N \wedge B_N) \vee (1 - B_N \wedge C_N) = A_N \vee C_N$.
Implies that $(D_N)^- = 1 - A_N \wedge B_N = A_N \wedge C_N$ is open set in (X, τ_N). Therefore (X, τ_N) is fuzzy neutrosophic basically disconnected space.

Definition 4.5:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic resolvable space if there exists a fuzzy neutrosophic dense set A_N in (X, τ_N) such that $(1 - A_N)^- = 1$. Otherwise, (X, τ_N) is called a fuzzy neutrosophic irresolvable space.

Example 3.5:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets A_N, B_N, C_N *and* D_N defined on X as follows:
A_N: $\{\langle a, 0.4,0.7,0.4\rangle, \langle b, 0.7,0.4,0.5\rangle, \langle c, 0.4,0.4,0.4\rangle\}$
B_N: $\{\langle a, 0.6,0.8,0.6\rangle, \langle b, 0.1,0.6,0.1\rangle, \langle c, 0.5,0.1,0.2\rangle\}$
C_N: $\{\langle a, 0.2,0.3,0.8\rangle, \langle b, 0.3,0.1,0.2\rangle, \langle c, 0.1,0.2,0.1\rangle\}$
D_N: $\{\langle a, 0.1,0.7,0.4\rangle, \langle b, 0.7,0.4,0.7\rangle, \langle c, 0.4,0.1,0.1\rangle\}$
Then $\tau_N = \{0_N, A_N, B_N, C_N,\ D_N, A_N \vee B_N,\ C_N \vee D_N,\ B_N \vee C_N, A_N \vee D_N,\ A_N \wedge B_N,\ A_N \wedge D_N,\ B_N \wedge C_N,\ C_N \wedge D_N, 1_N\}$ is a fuzzy neutrosophic topology on X.
$(A_N)^- = 1, (1 - A_N)^- = 1,$

$(B_N)^- = 1, (1 - B_N)^- = 1$
$(C_N)^- = 1, (1 - C_N)^- = 1,$
$(D_N)^- = 1, (1 - D_N)^- = 1$
This implies that (X, τ_N) is fuzzy neutrosophic resolvable space.

Definition 4.6:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic submaximal space if for each fuzzy neutrosophic set A_N in (X, τ_N) such that $(A_N)^- = 1$ then $A_N \in \tau_N$ in (X, τ_N). That is, (X, τ_N) is a fuzzy neutrosophic submaximal space if each fuzzy neutrosophic dense set in (X, τ_N) is a fuzzy neutrosophic open set in (X, τ_N).

Example 4.6:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets $A_N, B_N \; and \; C_N$ defined on X as follows:
A_N: $\{\langle a, 0.5,0.6,0.5\rangle, \langle b, 0.4,0.4,0.4\rangle, \langle c, 0.5,0.5,0.4\rangle\}$
B_N: $\{\langle a, 0.4,0.5,0.4\rangle, \langle b, 0.5,0.4,0.4\rangle, \langle c, 0.5,0.5,0.4\rangle\}$
C_N: $\{\langle a, 0.6,0.6,0.5\rangle, \langle b, 0.5,0.6,0.5\rangle, \langle c, 0.5,0.6,0.4\rangle\}$
Then $\tau_N = \{0_N, A_N, B_N, C_N, \; A_N \vee B_N, \; A_N \vee C_N, \; B_N \vee C_N, \; A_N \wedge B_N, \; A_N \wedge C_N, \; B_N \wedge C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.
$(C_N)^- = 1, (C_N)^+ = C_N, (A_N \vee C_N)^- = 1$
$(A_N)^- = 1, (A_N)^+ = A_N.$
Therefore (X, τ_N) is fuzzy neutrosophic submaximal space.

Definition 4.7:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic strongly irresolvable space if $((A_N)^-)^+ = 1$, for each fuzzy neutrosophic dense set A_N in (X, τ_N).

Example 4.7:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets $A_N \; and \; B_N$ defined on X as follows:
A_N: $\{\langle a, 0.6,0.5,0.6\rangle, \langle b, 0.6,0.6,0.5\rangle, \langle c, 0.5,0.5,0.5\rangle\}$
B_N: $\{\langle a, 0.5,0.7,0.7\rangle, \langle b, 0.7,0.5,0.7\rangle, \langle c, 0.5,0.5,0.5\rangle\}$
Then $\tau_N = \{0_N, A_N, B_N, C_N, \; A_N \vee B_N, \; A_N \wedge B_N, 1_N\}$ is a fuzzy neutrosophic topology on X.
$(A_N \wedge B_N)^- = 1, ((A_N \wedge B_N)^-)^+ = 1.$
Now, the fuzzy neutrosophic set $A_N \wedge B_N$ is a fuzzy neutrosophic dense set in (X, τ_N). Implies that $((A_N \wedge B_N)^-)^+ = 1$.Therefore, (X, τ_N) is a fuzzy neutrosophic strongly irresolvable space.

Definition 4.8:

A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic almost resolvable space if $\vee_{i=1}^{\infty}(A_{N_i}) = 1$,where $(A_{N_i})'s$ in (X, τ_N) are such that $\left(A_{N_i}\right)^+ = 0$. Otherwise, (X, τ_N) is called a fuzzy neutrosophic almost irresolvable space.

Example 4.8:

Let $X = \{a, b, c\}$ and the fuzzy neutrosophic sets $A_N, B_N \; and \; C_N$ defined on X as follows:
A_N: $\{\langle a, 0.1,0.7,0.3\rangle, \langle b, 0.3,0.7,0.7\rangle, \langle c, 0.7,0.1,0.1\rangle\}$
B_N: $\{\langle a, 0.4,0.1,0.4\rangle, \langle b, 0.1,0.6,0.6\rangle, \langle c, 0.6,0.4,0.1\rangle\}$
C_N: $\{\langle a, 0.5,0.6,0.5\rangle, \langle b, 0.6,0.1,0.1\rangle, \langle c, 0.1,0.5,0.6\rangle\}$
$\tau_N = \{0_N, A_N, B_N, C_N, \; A_N \vee B_N, \; A_N \vee C_N, \; B_N \vee C_N, \; A_N \wedge B_N, \; A_N \wedge C_N, \; B_N \wedge C_N, 1_N\}$ is a fuzzy neutrosophic topology on X.
Then $(A_N)^+ = 0, (B_N)^+ = 0, (C_N)^+ = 0$ and $\{(A_N) \vee (B_N) \vee (C_N)\} = 1.$
Hence (X, τ_N) is a fuzzy neutrosophic almost resolvable space.

V. CONCLUSION

In this paper, the concept of a new class of spaces called them fuzzy neutrosophic $C -$ almost $P -$spaces and Other spaces on fuzzy neutrosophic topological spaces are defined. Some of its characterizations and examples of fuzzy neutrosophic $C -$ almost $P -$spaces and other spaces on fuzzy neutrosophic topological spaces are also studied. Application to different fields of fuzzy neutrosophic topological spaces such as soft computing, artificial intelligence, decision making, pattern recognition, and image processing. This shall be extended in the future research studies.

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Neutrosophic σ-Baire Spaces on Fuzzy Setting

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Abstract

In this paper the concept of neutrosophic σ- Baire Spaces on fuzzy setting are introduced and several characterizations of fuzzy neutrosophic σ-Baire space are studied. Several examples are given to illustrate the concepts introduced in this paper.

Keywords: Fuzzy neutrosophic F_σ-set, fuzzy neutrosophic G_δ- set, fuzzy neutrosophic σ- nowhere dense set, fuzzy neutrosophic σ-first and second category and fuzzy neutrosophic σ- Baire Spaces.

1 Introduction

The fuzzy idea was invaded all branches of science as far back as the presentation of fuzzy sets by L. A. zadeh [11]. The important concept of fuzzy topological space was offered by C.L. Chang [3]. The idea of fuzzy σ- Baire Spaces was introduced by G. Thangaraj and E. Poongothai [8]. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [10] introduced fuzzy neutrosophic topological spaces. The idea of fuzzy neutrosophic Baire spaces was introduced by E. Poongothai and E. Padmavathi [4].

In this paper, we introduce new concepts of fuzzy neutrosophic σ- nowhere dense sets and fuzzy neutrosophic σ-Baire spaces are discussed. Also we introduce new characterizations and examples are based on this concept.

2 Preliminaries

Definition 2.1 [2] A fy. neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ x \in X$ where $T, I, F : X \to [0,1]$ and $0 \leqslant T_A(x) + I_A(x), F_A(x) \leqslant 3$.
With the condition $0 \leqslant T_{A^*}(x) + I_{A^*}(x), F_{A^*}(x) \leqslant 2$

Definition 2.2 [2] A fy. neutrosophic set A is a subset of a fy. neutrosophic set B(i.e.,) $A \subseteq B$ for all x if $T_A(x) \leqslant T_B(x),\ I_A(x) \leqslant I_B(x),\ F_A(x) \geqslant F_B(x)$

Definition 2.3 [2] Let X be a non-empty set, and
$A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fy. neutrosophic sets. Then

$$\begin{aligned} A \cup B &= \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle \\ A \cap B &= \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle \end{aligned}$$

Definition 2.4 [2] The difference between two fy. neutrosophic sets A and B is defined as $A \backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$

Definition 2.5 [2] A fy. neutrosophic set A over the universe X is said to be null or empty fy. neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N

Definition 2.6 [2] A fy. neutrosophic set A over the universe X is said to be absolute (universe) fy. neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N

Definition 2.7 [2] The complement of a fy. neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x),\ I_{A^C}(x) = 1 - I_A(x),\ F_{A^C}(x) = T_A(x)$. The complement of fy. neutrosophic set A an also be defined as $A^C = 1_N - A$.

Definition 2.8 [2] A fy. neutrosophic topology on a non-empty set X is a τ of fy. neutrosophic sets in X satisfying the following axioms.

(i) $0_N,\ 1_N \in \tau$

(ii) $A_1 \cap A_2 \in \tau$ for any $A_1,\ A_2 \in \tau$

(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$

In this case the pair (X, τ) is called fy. neutrosophic top. space and any fy. neutrosophic set in τ is known as fy. neutrosophic open set in X

Definition 2.9 [1] The complement A^C of a fy. neutrosophic set A in a fy. neutrosophic top. space (X, τ) is called fy. neutrosophic closed set in X.

Definition 2.10 [1] Let (X, τ) be a fy. neutrosophic top. space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fy. neutrosophic set in X. Then the closure and interior of A are defined by

$$\begin{aligned} int(A) &= \cup\{G : G \; is \; a \; fuzzy \; neutrosophic \; open \; set \; in X \; and \; G \subseteq A\} \\ cl(A) &= \cap\{G : G \; is \; a \; fuzzy \; neutrosophic \; closed \; set \; in X \; and \; A \subseteq G\} \end{aligned}$$

Definition 2.11 [1] Let (X, τ) be a fuzzy neutrosophic topological space over X. Then (i) $cl(A^C) = (int\ A)^C$, (ii) $int(A^C) = (cl\ A)^C$

Definition 2.12 [4] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic F_σ-set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$

Definition 2.13 [4] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic G_δ-set in (P, τ_N) if $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$

Definition 2.14 [4] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic dense if there exist no $fnCS$ B_N in (P, τ_N) s.t $A_N \subset B_N \subset 1_X$. That is $fn(A_N)^- = 1_N$

Definition 2.15 [4] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic nowh. dense set if there exist no non zero $fnOS$ B_N in (P, τ_N) s.t $B_N \subset fn(A_N)^-$. That is $fn(((A_N)^-)^+) = 0_N$

Definition 2.16 [4] Let (P, τ_N) be a fy. neutrosophic top. space. A fy. neutrosophic set A_N in (P, τ_N) is called fy. neutrosophic one category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Any other fy. neutrosophic set in (P, τ_N) is said to be of fy. neutrosophic two category.

Definition 2.17 [4] A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic one category space if the fy. neutrosophic set 1_X is a fy. neutrosophic one category set in (P, τ_N). That is $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fy. neutrosophic two category space.

3 Fuzzy Neutrosophic σ- Nowhere Dense Sets

Definition 3.1 *Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic Set λ_N in (X_N, T_N) is called a fuzzy neutrosophic σ-nowhere dense set if λ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N) such that $int(\lambda_N) = 0$*

Example 3.1 Let $X = \{a, b, c\}$. The fuzzy neutrosophic sets λ_N, μ_N and γ_N are defined on X_N as follows

$$\lambda_N = \{\langle a, 0.3, 0.4, 0.5\rangle, \langle b, 0.7, 0.6, 0.5\rangle, \langle c, 0.4, 0.5, 0.5\rangle\}$$

$$\mu_N = \{\langle a, 0.5, 0.4, 0.3\rangle, \langle b, 0.4, 0.4, 0.4\rangle, \langle c, 0.8, 0.4, 0.3\rangle\}$$

$$\gamma_N = \{\langle a, 0.2, 0.4, 0.5\rangle, \langle b, 0.4, 0.3, 0.4\rangle, \langle c, 0.6, 0.5, 0.4\rangle\}$$

Then $T_N = \{0_N, \lambda_N, \mu_N, \gamma_N, \lambda_N \vee \mu_N, \lambda_N \vee \gamma_N, \lambda_N \wedge \mu_N, \lambda_N \wedge \gamma_N, \gamma_N \vee (\lambda_N \wedge \mu_N), 1_N\}$ is clearly a fuzzy neutrosophic topology on X. Now, Consider the fuzzy neutrosophic set $\delta_N = [1 - (\lambda_N \vee \mu_N)] \vee [1 - (\lambda_N \vee \gamma_N)]$ in (X_N, T_N). Then δ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N) and $int(\lambda_N) = 0$.

Hence δ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N).

The fuzzy neutrosophic set $\beta_N = (1 - \lambda_N) \vee (1 - \mu_N) \vee (1 - \gamma_N)$ is a fuzzy neutrosophic F_σ-set in (X_N, T_N) and $int(\lambda_N) \neq 0$.

Hence β_N is not a fuzzy neutrosophic σ- nowhere dense set in (X_N, T_N).

Proposition 3.1 *In a fuzzy neutrosophic topological space (X_N, T_N) a fuzzy neutrosophic set λ_N is fuzzy neutrosophic σ-nowhere dense in (X_N, T_N) if and only if $1 - \lambda_N$ fuzzy neutrosophic dense and fuzzy neutrosophic G_δ-set in (X_N, T_N).*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic topological Space. Let λ_N be a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Then $\lambda_N = \bigvee_1^\infty(\lambda_{N_i})$. Where $1 - \lambda_N \in T_N$ for $i \in I$ and $int(\lambda_N) = 0$. Then $1 - int(\lambda_N) = 1 - 0 = 1$.

Implies that $(1-\lambda_N) = 1$ Also $(1-\lambda_N) = 1 - \bigvee_1^{\infty}(\lambda_{N_i}) = \bigwedge_1^{\infty}(\lambda_{1-N_i})$. Where $1-\lambda_{N_i} \in T_N$ for $i \in I$. Hence we have $I-\lambda_N$ is a fuzzy neutrosophic dense and fuzzy G_δ- set in (X_N, T_N). Conversely, Let λ_N be a fuzzy dense and fuzzy G_δ-set in (X_N, T_N). Then $\lambda_N = \bigwedge_{i=1}^{\infty}(\lambda_{N_i})$ Where $\lambda_{N_i}, \in T_N$ for $i \in I$. Now, $(1-\lambda_N) = 1 - \bigwedge_{i=1}^{\infty}(\lambda_{N_i}) = \bigvee_{i=1}^{\infty}(1-\lambda_{N_i})$. Hence $1-\lambda_N$ is a F_σ-set in (X_N, T_N) and $int(1-\lambda_N) = 1 - cl(\lambda_N) = 1-1 = 0$. [Since λ_N is a fuzzy neutrosophic dense.] Therefore $1-\lambda_N$ is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N)

Proposition 3.2 *If λ_N is a fuzzy neutrosophic dense set in (X_N, T_N) such that $\mu_N \leqslant (1-\lambda_N)$, Where μ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N) then μ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N).*

Proof. Let λ be a fuzzy neutrosophic dense set in (X_N, T_N) such that $\mu_N \leqslant (1-\lambda_N)$. Now, $\mu_N \leqslant (1-\lambda_N)$ implies that $int(\mu_N) \leqslant int(1-\lambda_N) = 1 - cl(\lambda_N) = 1-1 = 0$. Hence $int(\mu_N) = 0$. Therefore μ_N is a fuzzy neutrosophic σ - nowhere dense set in (X_N, T_N).

Proposition 3.3 *If λ_N is a fuzzy neutrosophic F_σ-set and fuzzy neutrosophic nowhere dense set in (X_N, T_N) then λ_N is a fuzzy neutrosophic σ-nowhere dense Set in (X_N, T_N).*

Proof. Given that λ_N is a fuzzy neutrosophic F_σ-set and fuzzy neutrosophic nowhere dense set in (X_N, T_N). Now $\lambda_N \leqslant cl(\lambda_N)$ for any fuzzy neutrosophic nowhere set in (X_N, T_N). Then $int(\lambda_N) \leqslant intcl(\lambda_N)$. Since λ_N is a fuzzy neutrosophic nowhere set in (X_N, T_N). $intcl(\lambda_N) = 0$ Hence $int(\lambda_N) = 0$ and λ_N is a fuzzy neutrosophic F_σ set. Implies that λ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N).

Remark 3.1 *If λ_N is a fuzzy neutrosophic F_σ-set and fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N), then λ_N need not be a fuzzy neutrosophic nowhere dense set in (X_N, T_N).*

For consider the following example.

Example 3.2 Let $X_N = \{a, b, c\}$. The fuzzy neutrosophic set λ_N, μ_N and γ_N are defined on λ_N as follows

$\lambda_N : X_N \to [0, 1]$ defined as, $\lambda_N = \{\langle a, 0.8, 0.5, 0.6\rangle, \langle b, 0.7, 0.6, 0.6\rangle, \langle c, 0.6, 0.7, 0.5\rangle\}$

$\mu_N : X_N \to [0, 1]$ defined as, $\lambda_N = \{\langle a, 0.8, 0.9, 0.5\rangle, \langle b, 0.6, 0.8, 0.9\rangle, \langle c, 0.9, 0.8, 0.7\rangle\}$

$\gamma_N : X_N \to [0, 1]$ defined as, $\lambda_N = \{\langle a, 0.7, 0.5, 0.9\rangle, \langle b, 0.5, 0.6, 0.8\rangle, \langle c, 0.7, 0.7, 0.8\rangle\}$

Now $\eta_N = [(1 - \mu_N) \vee (1 - \lambda_N \wedge \gamma_N) \vee (1 - \gamma_N)]$

and $\delta_N = [(1 - \mu_N \vee \mu_N) \vee (1 - \mu_N \vee \gamma_N) \vee (1 - \lambda_N \vee \gamma_N)]$

are fuzzy neutrosophic F_σ-sets in (X_N, T_N).

Also fuzzy neutrosophic $int(\eta_N) = 0$ and fuzzy neutrosophic $int(\delta_N) = 0$.

Therefore η_N and δ_N are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N)

But $\eta_N \vee \delta_N$ is not a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N).

Definition 3.2 *Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set λ_N in (X_N, T_N) is called fuzzy neutrosophic σ- first category if $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$ Where λ_{N_i}'s are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Any other fuzzy neutrosophic set in (X_N, T_N) is said to be fuzzy neutrosophic σ- second category in (X_N, T_N).*

Definition 3.3 *Let λ_N be a fuzzy neutrosophic σ-first category set in (X_N, T_N). Then $1 - \lambda_N$ is called a fuzzy neutrosophic σ-residual set in (X_N, T_N).*

Definition 3.4 *A fuzzy neutrosophic topological space (X_N, T_N) is called fuzzy neutrosophic σ-first category if the fuzzy set 1_{X_N} is a fuzzy neutrosophic σ-first*

category set in (X_N, T_N). *That is* $1_{X_N} = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$ *Where* λ_{N_i}*'s are fuzzy neutrosophic* σ*- nowhere dense sets in* (X_N, T_N). *Otherwise* (X_N, T_N) *will be called a fuzzy neutrosophic* σ*- second category space.*

Definition 3.5 *Let* λ_N *be a fuzzy neutrosophic* σ*-first category set in* (X_N, T_N). *Then* $1 - \lambda_N$ *is called a fuzzy neutrosophic* σ*-residual set in* (X_N, T_N)

Definition 3.6 *A fuzzy neutrosophic topological space* (X_N, T_N) *is called fuzzy neutrosophic* σ*-first category if the fuzzy neutrosophic set* 1_{X_N} *is a fuzzy neutrosophic* σ*- first category set in* (X_N, T_N). *That is* $1_{X_N} = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$ *Where* λ_{N_i}*'s are fuzzy neutrosophic* σ*- nowhere dense sets in* (X_N, T_N). *Otherwise* (X_N, T_N) *will be called a fuzzy neutrosophic* σ*- second category space.*

4 Fuzzy Neutrosophic σ- Baire space

Definition 4.1 *Let* (X_N, T_N) *be a fuzzy neutrosophic topological space then* (X_N, T_N) *is called a fuzzy neutrosophic* σ*-Baire space if* $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0$, *where* λ_{N_i}*'s are fuzzy neutrosophic* σ*- nowhere dense sets in* (X_N, T_N).

Example 4.1 Let $X_N = \{a, b, c\}$. The fuzzy neutrosophic set λ_N, μ_N and γ_N are defined on λ_N as follows

$\lambda_N : X_N \to [0, 1]$ defined as, $\lambda_N = \{\langle a, 0.8, 0.3, 0.5\rangle, \langle b, 0.2, 0.4, 0.7\rangle, \langle c, 0.3, 0.5, 0.6\rangle\}$

$\mu_N : X_N \to [0, 1]$ defined as, $\lambda_N = \{\langle a, 0.7, 0.5, 0.8\rangle, \langle b, 0.3, 0.4, 0.6\rangle, \langle c, 0.8, 0.9, 0.3\rangle\}$

$\gamma_N : X_N \to [0, 1]$ defined as, $\lambda_N = \{\langle a, 0.5, 0.2, 0.8\rangle, \langle b, 0.3, 0.6, 0.4\rangle, \langle c, 0.9, 0.7, 0.3\rangle\}$

Then $T_N = \{0_N, \lambda_N, \mu_N, \gamma_N, \lambda_N \vee \mu_N, \lambda_N \vee \gamma_N, \mu_N \vee \gamma_N, \lambda_N \wedge \mu_N, \lambda_N \wedge \gamma_N, \mu_N \wedge \gamma_N, 1_N\}$ is a fuzzy neutrosophic topology on X.

Now $\delta_N = (1 - \lambda_N \vee \mu_N) \vee (1 - \lambda_N \vee \gamma_N)$ and $int(\delta_N) = 0$

$\psi_N = (1 - \mu_N \vee \gamma_N) \vee (1 - \lambda_N \wedge \mu_N)$ and $int(\psi_N) = 0$

Then δ_N and ψ_N are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N).

And also $int(\delta_N \vee \psi_N) = 0$

Therefore (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.

Proposition 4.1 *Let (X_N, TN) be a fuzzy neutrosophic topological space. Then the following are equivalent*

(1) (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.

(2) $int(\lambda_N) = o$ for every fuzzy neutrosophic σ- first category set λ_N in (X_N, T_N).

(3) $cl(\mu_N) = 1$ for every fuzzy neutrosophic σ-residual set μ_N in (X_N, T_N)

Proof. (1) $\Rightarrow$ (2)

Let λ_N be a fuzzy neutrosophic σ- first category set in (X_N, T_N). Then $\lambda_N = (\bigvee_{i=1}^{\infty}(\lambda_{N_i}))$ where λ_{N_i}'s are fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Then we have $int(\lambda_N) = (\bigvee_{i=1}^{\infty}(\lambda_{N_i}))$. Since (X_N, T_N) is a fuzzy neutrosophic σ-Baire space $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i}))$ Hence $int(\lambda_N) = 0$ for any fuzzy neutrosophic σ- first category set λ_N in (X_N, T_N).

(2) $\Rightarrow$ (3)

Let μ_N be a fuzzy neutrosophic σ- residual set in (X_N, T_N). Then $(1 - \mu_N)$ is a fuzzy neutrosophic σ- first category set in (X_N, T_N). By hypothesis $int(1-\mu_N) = 0$. Then $1 - cl(\lambda_N) = 0$. Hence $cl(\lambda_N) = 1$ for any fuzzy neutrosophic σ- residual set μ_N in (X_N, T_N).

(3) $\Rightarrow$ (1)

Let λ_N be a fuzzy neutrosophic σ- first category set in (X_N, T_N). Then $\lambda_N = (\bigvee_{i=1}^{\infty}(\lambda_{N_i}))$ where λ_{N_i}'s are fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Now λ_N is a fuzzy neutrosophic σ- first category set in (X_N, T_N).

$\Rightarrow 1-\lambda_N$ is a fuzzy neutrosophic σ- residual set in (X_N, T_N). By hypothesis we have $cl(1-\lambda_N) = 1$. Then $1 - int(\lambda_N) = 1$. Hence $int(\lambda_N) = 0$. (i.e) $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0$ where λ_{N_i}'s are fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Hence (X_N, T_N) is a fuzzy neutrosophic σ- Baire space.

Proposition 4.2 *If $cl(\bigwedge_{i=1}^{\infty}(\lambda_{N_i})) = 1$ where λ_{N_i}'s are fuzzy neutrosophic dense G_σ-set in (X_N, T_N). Then (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.*

Proof. Given that $cl(\bigwedge_{i=1}^{\infty}(\lambda_{N_i})) = 1. \Rightarrow \quad 1 - cl(\bigwedge_{i=1}^{\infty}(\lambda_{N_i})) = 0$. Then we have $int(\bigvee_{i=1}^{\infty}(1-\lambda_{N_i})) = 0 \qquad (I)$. Let $\mu_{N_i} = 1-\lambda_{N_i}$. Then $int(\bigvee_{i=1}^{\infty}(\mu_{N_i})) = 0$. Since λ_{N_i} is a fuzzy neutrosophic dense G_σ-set in (X_N, T_N). By proposition 3.1, $1-\lambda_{N_i}$ is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Hence $int(\bigvee_{i=1}^{\infty}(\mu_{N_i})) = 0$ where μ_{N_i}'s are fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Therefore (X_N, T_N) is a fuzzy neutrosophic σ- Baire space.

Proposition 4.3 *If the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic σ-Baire space, then (X_N, T_N) is a fuzzy neutrosophic σ-second category space.*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic σ-Baire space. Then $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0$ where μ_{N_i}'s are fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Then $\bigvee_{i=1}^{\infty}(\lambda_{N_i}) \neq 1_{X_N}$ Otherwise

$$\begin{aligned} \bigvee_{i=1}^{\infty}(\lambda_{N_i}) &= 1_{X_N} \\ int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) &= int 1_{X_N} \\ int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) &= 1_{X_N} \\ 0 &= 1 \end{aligned}$$

This is a contradiction. Hence (X_N, T_N) is a fuzzy neutrosophic σ-second category space. Hence this completes the proof of the proposition.

Proposition 4.4 *If the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic Baire space and if fuzzy neutrosophic nowhere dense sets in (X_N, T_N) are fuzzy neutrosophic F_σ-sets in (X_N, T_N) then (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic Baire space such that every fuzzy neutrosophic nowhere dense set λ_{N_i} is a fuzzy neutrosophic F_σ-set in (X_N, T_N). Then $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0$ where μ_{N_i}'s are fuzzy neutrosophic nowhere dense sets in (X_N, T_N). By proposition 3.3 λ_{N_i} is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Hence $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0$ where μ_{N_i}'s are fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Therefore (X_N, T_N) is a fuzzy neutrosophic σ-Baire space. Hence the proof.

Proposition 4.5 *Let (X_N, T_N) be a fuzzy neutrosophic topological space. If $\bigwedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 0$ where λ_{N_i}'s are fuzzy neutrosophic dense and fuzzy neutrosophic G_δ-set in (X_N, T_N) then (X_N, T_N) is a fuzzy neutrosophic σ-second category space.*

Proof. Given (X_N, T_N) be a fuzzy neutrosophic topological space. Also given $\bigwedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 0$ where λ_{N_i}'s are fuzzy neutrosophic dense and fuzzy neutrosophic G_δ-set in (X_N, T_N). Now $\bigwedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 0$. Implies that $1 - (\bigwedge_{i=1}^{\infty}(\lambda_{N_i})) \neq 1 - 0 = 0$. Then we have $(\bigvee_{i=1}^{\infty}(1-\lambda_{N_i})) \neq 1$ By proposition 1.1. $1-\lambda_{N_i}$ is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Hence $\bigvee_{i=1}^{\infty}(1 - \lambda_{N_i}) \neq 1$ where $1 - \lambda_{N_i}$'s are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Hence (X_N, T_N) is not a fuzzy neutrosophic σ-first category space. Therefore (X_N, T_N) is a fuzzy neutrosophic σ-second category space.

5 Conclusion

In this paper, introduced and discussed fuzzy neutrosophic σ- nowhere dense sets, fuzzy neutrosophic σ- Baire spaces. Some results concerning fuzzy neutrosophic σ-nowhere dense sets and fuzzy neutrosophic σ- Baire Spaces are established.

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A CHARACTERIZATION OF NEUTROSOPHIC σ BAIRE SPACE ON FUZZY SETTING

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Abstract. In this paper we investigate several characterization of F. neu. σ-baire spaces and study conditions under which a fy. neu. top. space we becomes a F. neu. σ-B.S

Keywords: F. neu. σ-nwd set, F. neu. σ-first category space, F. neu. σ- second category space, F. neu. σ-B.S, F. neu. almost resolvable space , F. neu. hyper connected , F. neu. sub maximal space.)

1. Introduction

The fuzzy idea was invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [10]. The important concept of fuzzy topological space was offered by C.L. Chang [2]. The idea of fuzzy σ- Baire Spaces was introduced by G. Thangaraj and E. Poongothai [8]. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [9] introduced Fy. neutrosophic topological spaces. The idea of fy. neutrosophic Baire spaces was introduced by E. Poongothai and E. Padmavathi [3].

In this paper, we discussed and established several characterization of neutrosophic σ-baire space on fuzzy setting. Also, we introduce to study the conditions under witch a neu. top. space becomes a neu. σ-B.S on fuzzy setting.

2. Preliminaries

Definition 2.1. [1] A fy. neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ x \in X$ where $T, I, F : X \to [0,1]$ and $0 \leq T_A(x) + I_A(x), F_A(x) \leq 3$. With the condition $0 \leq T_{A^*}(x) + I_{A^*}(x), F_{A^*}(x) \leq 2$

Definition 2.2. [1] A fy. neutrosophic set A is a subset of a fy. neutrosophic set B(i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x),\ F_A(x) \geq F_B(x)$

Definition 2.3. [1] Let X be a non-empty set, and
$A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fy. neutrosophic sets. Then

$$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$$
$$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$$

Definition 2.4. [1] The difference between two fy. neutrosophic sets A and B is defined as $A \backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$

Definition 2.5. [1] A fy. neutrosophic set A over the universe X is said to be null or empty fy. neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N

Definition 2.6. [1] A fy. neutrosophic set A over the universe X is said to be absolute (universe) fy. neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N

Definition 2.7. [1] The complement of a fy. neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x),\ I_{A^C}(x) = 1 - I_A(x),\ F_{A^C}(x) = T_A(x)$. The complement of fy. neutrosophic set A an also be defined as $A^C = 1_N - A$.

Definition 2.8. [1] A fy. neutrosophic topology on a non-empty set X is a τ of fy. neutrosophic sets in X satisfying the following axioms.

(i) $0_N,\ 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1,\ A_2 \in \tau$
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$

In this case the pair (X, τ) is called fy. neutrosophic top. space and any fy. neutrosophic set in τ is known as fy. neutrosophic open set in X

Definition 2.9. [1] The complement A^C of a fy. neutrosophic set A in a fy. neutrosophic top. space (X, τ) is called fy. neutrosophic closed set in X.

Definition 2.10. [1] Let (X, τ) be a fy. neutrosophic top. space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fy. neutrosophic set in X. Then the closure and I.erior of A are defined by

$I.(A) = \cup\{G : G\ is\ a\ fuzzy\ neutrosophic\ open\ set\ in X\ and\ G \subseteq A\}$

$C.(A) = \cap\{G : G\ is\ a\ fuzzy\ neutrosophic\ closed\ set\ in X\ and\ A \subseteq G\}$

Definition 2.11. [1] Let (X, τ) be a fy. neutrosophic topological space over X. Then $(i)\ C.(A^C) = (I.\ A)^C$, $(ii)\ I.(A^C) = (cl\ A)^C$

Definition 2.12. [5] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic F_σ-set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$

Definition 2.13. [5] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic G_δ-set in (P, τ_N) if $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$

Definition 2.14. [5] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic dense if there exist no $fnCS\ B_N$ in (P, τ_N) s.t $A_N \subset B_N \subset 1_X$. That is $fn(A_N)^- = 1_N$

Definition 2.15. [3] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic nowh. dense set if there exist no non zero $fnOS\ B_N$ in (P, τ_N) s.t $B_N \subset fn(A_N)^-$. That is $fn(((A_N)^-)^+) = 0_N$

Definition 2.16. [3] Let (P, τ_N) be a fy. neutrosophic top. space. A fy. neutrosophic set A_N in (P, τ_N) is called fy. neutrosophic one category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Any other fy. neutrosophic set in (P, τ_N) is said to be of fy. neutrosophic two category.

Definition 2.17. [3] A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic one category space if the fy. neutrosophic set 1_X is a fy. neutrosophic one category set in (P, τ_N). That is $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fy. neutrosophic two category space.

3. A Characterization of Fuzzy Neutrosophic σ Baire Space on Fuzzy Setting

Definition 3.1. Let (X_N, T_N) be a Fy. neutrosophic topological space then (X_N, T_N) is called a F. neutrosophic σ-Baire space if $I.(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0$, where λ_{N_i}'s are F. neutrosophic σ- nowhere dense sets in (X_N, T_N).

Theorem 3.2. *[5] Let (X_N, TN) be a Fy. neutrosophic topological space. Then the following are equivalent*

(1) *(X_N, T_N) is a F. neutrosophic σ-Baire space.*

(2) $I.(\lambda_N) = o$ *for every F. neutrosophic* σ*- first category set* λ_N *in* (X_N, T_N).

(3) $C.(\mu_N) = 1$ *for every F. neutrosophic* σ*-residual set* μ_N *in* (X_N, T_N)

Theorem 3.3. *[5] In a Fy. neutrosophic topological space* (X_N, T_N) *a fy. neutrosophic set* λ_N *is F. neutrosophic* σ*-nowhere dense in* (X_N, T_N) *if and only if* $1 - \lambda_N$ *F. neutrosophic dense and F. neutrosophic* G_δ*-set in* (X_N, T_N).

Theorem 3.4. *[3] If* λ_N *is a F. neu. dense and F. neu.* G_δ*-set in a fy. neu. top. space* (X_N, T_N) *then* $1 - \lambda_N$ *is a F. neu. first category set in* (X_N, T_N).

Proposition 3.5. *If the fuzzy neu. top. space* (X_N, T_N) *is a F. neu.* σ*-Baire space, then* $C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1$*, where the fuzzy sets* (λ_{N_i})*'s* $(i = 1$ *to* $\infty)$ *are F. neu. dense set and F. neu.* G_δ*-set in* (X_N, T_N).

Proof. Let (λ_{N_i})'s $(i = 1$ *to* $\infty)$ be a F. neu. dense set. and F. neu. G_δ-set in (X_N, T_N) By theorem 3.3 $(1 - \lambda_{N_i})$'s are fuzzy neu. σ-nwd sets in (X_N, T_N). Then the fuzzy neu. set $\lambda_N = \vee_{i=1}^{\infty}(1 - \lambda_{N_i})$ is a F. neu. σ-first catogory set in (X_N, T_N). Now $I.(\lambda_N) = I.(\vee_{i=1}^{\infty}(1 - \lambda_{N_i})) = I.(1 - [\wedge_{i=1}^{\infty}(\lambda_{N_i})]) = 1 - C.(1 - [\wedge_{i=1}^{\infty}(\lambda_{N_i})])$. Since (X_N, T_N) is a F. neu. σ-baire space, by theorem 3.1, we have $I.(\lambda_N) = 0$. Then $1 - C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 0$. This implies that $C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1$ □

Proposition 3.6. *If the fy. neu. top. space* (X_N, T_N) *is a F. neu.* σ*-B.S, then* $I.[\vee_{i=1}^{\infty}(1 - \lambda_{N_i})] = 0$*, where the F. neu. sets* $(1 - \lambda_{N_i})$*'s* $(i = 1$ *to* $\infty)$ *are fuzzy first category sets formed from the fuzzy neu. dense and F. neu.* G_δ*-set* λ_{N_i} *in* (X_N, T_N).

Proof. Let the fy. neu. top. space (X_N, T_N) is a F. neu. σ-B.S and the F. neu. sets (λ_{N_i})'s $(i = 1$ *to* $\infty)$ be F. neu. dense and F. neu. G_δ-sets in (X_N, T_N). By prop. 3.5, $C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1$. Then $1 - C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 0$. This implies that $I.[\vee_{i=1}^{\infty}(1 - \lambda_{N_i})] = 0$. Also by theorem 3.4, $(1 - \lambda_{N_i})$'s are F. neu. first category sets in (X_N, T_N). Hence $I.[\vee_{i=1}^{\infty}(1 - \lambda_{N_i})] = 0$, where the fuzzy neu. sets $(1 - \lambda_{N_i})$'s $(i = 1$ *to* $\infty)$ are fuzzy neu. first category sets formed from the F. neu. dense and F. neu. G_δ-set λ_{N_i} in (X_N, T_N). □

Theorem 3.7. *[3] Let* (X_N, T_N) *be a fy. neu. top. space. Then the following are equivalent.*

(1) (X_N, T_N) *is a F. neu. B.S.*

(2) $f_n(A_N)^+ = 0_N$ *for every F. neu. first category set* A_N *in* (X_N, T_N).

(3) $f_n(A_N)^- = 1$*, for every F. neu. residual set* B_N *in* (X_N, T_N).

Proposition 3.8. *If the F. neu. first category sets are formed from the F. neu. dense and F. neu.* G_δ*-sets in a F. neu.* σ*-B.S* (X_N, T_N)*, then* (X_N, T_N)*. is a F. neu. B.S.*

Proof. Let the fy. neu. top. space (X_N, T_N) be a fuzzy neu. σ-B.S. By prop.3.6, $I.[\vee_{i=1}^{\infty}(1-\lambda_{N_i})] = 0$, where the F. neu. sets $(1-\lambda_{N_i})$'s $(i = 1\ to\ \infty)$ are F. neu. first category sets formed from the F. neu. dense and and F. neu. G_δ-sets λ_{N_i} in (X_N, T_N). Now $\vee_{i=1}^{\infty}[I.(1-\lambda_{N_i})] \leq [I.(\vee_{i=1}^{\infty}(1-\lambda_{N_i}))]$. Then we have $\vee_{i=1}^{\infty}[I.(1-\lambda_{N_i})] = 0$. This implies that $I.(1-\lambda_{N_i}) = 0$, where $1-\lambda_{N_i}$ is a fuzzy neu. first category set in (X_N, T_N). By theorem 3.7, (X_N, T_N) is a fu.neu. B.S. □

Theorem 3.9. *[5] A fy. neu. top. space (X_N, T_N) is called F. neu. σ-first category if the F. neu. set 1_{X_N} is a F. neu. σ- first category space in (X_N, T_N). That is $1_{X_N} = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$ Where λ_{N_i}'s are F. neu. σ- nowhere dense sets in (X_N, T_N). Otherwise (X_N, T_N) will be called a F. neu. σ- second category space.*

Proposition 3.10. *If the fy. neu. top. space (X_N, T_N) is a F. neu. σ-first category space, then (X_N, T_N) is not a F. neu. σ-Baire space.*

Proof. Let the fy. neu. top. space (X_N, T_N) is a F. neu. σ-first category space. Then $\vee_{i=1}^{\infty}(\lambda_{N_i}) = 1_{X_N}$, where λ_{N_i}'s are F. neu. σ-nwd sets in (X_N, T_N). Now $I.\vee_{i=1}^{\infty}(\lambda_{N_i}) = I.(1_{X_N}) = 1 \neq 0$. Hence, (X_N, T_N) is not a F. neu.σ-B.S. □

Proposition 3.11. *If $\wedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 0$, where the F. neu. sets (λ_{N_i})'s are F. neu. dense and F. neu. G_δ-set in a fy. neu. top. space (X_N, T_N), then (X_N, T_N) is a F. neu. σ-second category space.*

Proof. Let (λ_{N_i})'s $(i = 1\ to\ \infty)$ are F. neu. dense and F. neu. G_δ-sets in (X_N, T_N). By theorem 3.3, $(1-\lambda_{N_i})$'s are F. neu. σ-nwd sets in (X_N, T_N). Now $\wedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 0$ implies that $1 - \wedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 1$. Then $\vee_{i=1}^{\infty}(1-\lambda_{N_i}) \neq 1$. Hence (X_N, T_N) is not a F. neu. σ-first category space and therefore (X_N, T_N) is a F. neu. σ-second category space. □

Proposition 3.12. *If λ_N is a F. neu. σ-first category set in (X_N, T_N), then there is a F. neu. F_σ-set δ_N in (X_N, T_N) such that $\lambda_N \leq \delta_N$*

Proof. Let λ_N is a F. neu. σ-first category set in (X_N, T_N). Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$ where (λ_{N_i})'s are F. neu. σ-nwd sets in (X_N, T_N). Now $[1 - C.(\lambda_{N_i})]$'s are F. neu. open sets in (X_N, T_N). Then $\mu_N = \wedge_{i=1}^{\infty}[1 - C.(\lambda_{N_i}]$ is a F. neu. G_δ-set in (X_N, T_N) and $1 - \mu_N = 1 - [\wedge_{i=1}^{\infty}(1 - C.(\lambda_{N_i}))] = \vee_{i=1}^{\infty}(C.(\lambda_{N_i}))$. Now $\lambda = \vee_{i=1}^{\infty}(\lambda_{N_i}) \leq \vee_{i=1}^{\infty}(C.(\lambda_{N_i})) = 1 - \mu_N$. That is, $\lambda_N \leq 1 - \mu_N$ and $1 - \mu_N$ is a F. neu. F_σ-set in (X_N, T_N). Let $\delta_N = 1 - \mu_N$. Hence, if λ_N is a F. neu. σ-first category set in (X_N, T_N), then there is a F. neu. F_σ-set δ_N in (X_N, T_N) such that $\lambda_N \leq \delta_N$. □

Proposition 3.13. *If σ_N is a F. neu. σ-residual set in a fy. neu. top. space (X_N, T_N) such that $\eta_N \leq \delta_N$ where η_N is a F. neu. dense and F. neu. G_δ-set in (X_N, T_N), then (X_N, T_N) is a F. neu. σ-B.S.*

Proof. Let σ_N be a F. neu. σ-residual set in a fy. neu. top. space (X_N, T_N). Then $1 - \delta_N$ is a F. neu. σ-first category set in (X_N, T_N). Now by prop.3.12, their is a F. neu. F_σ-set μ_N in (X_N, T_N) such that $1 - \delta_N \leq \mu_N$. This implies that $1 - \mu_N \leq \sigma_N$. Let $\eta_N = 1 - \mu_N$. Then η_N is a F. neu. G_δ-set in (X_N, T_N) and $\eta_N \leq \delta_N$ implies that $C.(\eta_N) \leq C.(\delta_N)$. If $C.(\eta_N) = 1$, then we have $C.(\delta_N) = 1$. Hence, by theorem3.2, (X_N, T_N) is a F. neu. σ-B.S. □

Proposition 3.14. *If the fy. neu. top. space (X_N, T_N) is a F. neu. σ B.S and if $\vee_{i=1}^{\infty}(\lambda_{N_i}) = 1$, then there exists atleast one F. neu. F_σ-set λ_{N_i} such that $I.(\lambda_{N_i}) \neq 0$*

Proof. Suppose that $I.(\lambda_{N_i}) = 0$, for $i = 1to\infty$, where (λ_{N_i})'s are F. neu. σ-nwd sets in (X_N, T_N). Then $\vee_{i=1}^{\infty}(\lambda_{N_i}) = 1$. implies that $I.[\vee_{i=1}^{\infty}(\lambda_{N_i})] = I.[1] = 1 \neq 0$, a contradiction to (X_N, T_N) being a F. neu. σ-B.S. Hence $I.(\lambda_{N_i}) \neq 0$, fort atleast one F. neu. F_σ-set λ_{N_i} in (X_N, T_N). □

Proposition 3.15. *If the fy. neu. top. space (X_N, T_N) is F. neu. σ-B.S, then no non-zero F. neu. open set is a F. neu. σ-first category set in (X_N, T_N).*

Proof. Let λ_N be a non zero F. neu. open set in a F. neu. σ-B.S (X_N, T_N). Suppose that $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where the fuzzy neu. sets (λ_{N_i})'s are F. neu. σ-nwd sets in (X_N, T_N). then $I.(\lambda_{N_i}) = I.(\vee_{i=1}^{\infty}(\lambda_{N_i}))$. Since (X_N, T_N) is a F. neu. σ-B.S, $I.(\vee_{i=1}^{\infty}(\lambda_{N_i})) = 0$. This implies that $I.(\lambda_N) = 0$. Then we will have $\lambda_N = 0$, which is a contradiction. Since $\lambda_N \in T_N$ implies that $I.(\lambda_N) = \lambda_N \neq 0$. Hence no non-zero F. neu. open set is a F. neu. σ-first category set in (X_N, T_N). □

Theorem 3.16. *[4] A fy. neu. top. space (X_N, T_N) is called a F. neu. submaximal space if for each F. neu. set λ_N in (X_N, T_N) such that $C.(\lambda_N) = 1$ then $C.(\lambda_N \in T_N)$ in (X_N, T_N).*

Proposition 3.17. *If the fy. neu. top. space (X_N, T_N) is a F. neu. submaximal space and if λ_N is a F. neu. σ-first category set in (X_N, T_N), then λ_N is a F. neu. first category set in (X_N, T_N)*

Proof. Let $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$ be a F. neu. σ-first category set in (X_N, T_N), where the fuzzy sets (λ_{N_i})'s $(i = 1to\infty)$ are F. neu. σ-nwd sets in (X_N, T_N). Then we have $I.(\lambda_{N_i}) = 0$ and (λ_{N_i})'s $(i = 1to\infty)$ are F. neu. F_σ-sets in (X_N, T_N). Now $I.(\lambda_{N_i}) = 0$, implies that $1 - I.(\lambda_{N_i}) = 1 - 0 = 1$ and hence $C.(1 - \lambda_{N_i}) = 1$. Since (X_N, T_N) is a F. neu. submaximal

space, the F. neu. dence sets $(1-\lambda_{N_i})$'s are F. neu. open sets in (X_N, T_N) and hence λ_{N_i}'s are F. neu. closed sets in (X_N, T_N). Then $C.(\lambda_{N_i}) = \lambda_{N_i}$ and $I.(\lambda_{N_i}) = 0$ implies that $I.C.(\lambda_{N_i}) = I.(\lambda_{N_i}) = 0$. That is (λ_{N_i})'s are F. neu. nwd sets (X_N, T_N). Therefore $\lambda = \vee_{i=1}^{\infty}(\lambda_{N_i})$ is a F. neu. first category set in (X_N, T_N). □

Proposition 3.18. *If the F. neu. top space (X_N, T_N) is a F. neu. σ-B.S and F. neu. submaximal space, then (X_N, T_N) is a F. neu. B.S.*

Proof. Let μ_N be a F. neu. σ-first category set in (X_N, T_N). Since (X_N, T_N) is a F. neu. submaximal space, by prop3.17, μ_N is a F. neu.first category set in (X_N, T_N). Since (X_N, T_N) is a F. neu. σ-B.S, by them3.2 $I.(\mu_N) = 0$. Hence, for the F. neu. first category set μ_N in (X_N, T_N), we have $I.(\mu_N) = 0$. Therefore, by thm.3.7, (X_N, T_N) is a F. neu. B.S. □

Theorem 3.19. *[4] A fy. neu. top. space (X_N, T_N) is called a F. neu.P-space if countable intersection of F. neu. open sets in (X_N, T_N) is F. neu. open. That is, every non-zero F. neu. G_δ-set in (X_N, T_N) is F. neu. open in (X_N, T_N).*

Proposition 3.20. *If the fy. neu. top. space (X_N, T_N) is a F. neu. σ-B.S and F. neu. P-space, then (X_N, T_N) is a F. neu. B.S.*

Proof. Let the fy. neu. top. space (X_N, T_N) be a F. neu. σ-B.S. Then, by prop.3.5, $C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1$, where the F. neu. sets (λ_{N_i})'s $(i = 1to\infty)$ are F. neu. dense and F. neu. G_δ-sets in (X_N, T_N). Now from, $C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1$, we have $1 - C.[\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 0$. This implies that $I.[\vee_{i=1}^{\infty}(1-\lambda_{N_i})] = 0$. Since the F. neu. sets $(\lambda_{N_i}))$'s are F. neu. dense in (X_N, T_N), $C.(\lambda_{N_i}) = 1$. Then we have $1 - C.(\lambda_{N_i}) = 0$. This implies that $I.(1-\lambda_{N_i}) = 0$. This implies that $I.(1-\lambda_{N_i}) = 0$. Also, since (X_N, T_N) is a F. neu. P-space, the non-zero F. neu. G_δ-sets (λ_{N_i})'s in (X_N, T_N), are F. neu. open in (X_N, T_N). Then $(1-\lambda_{N_i})$'s are F. neu. closed sets in (X_N, T_N). Then $C.(1-\lambda_{N_i}) = 1-\lambda_{N_i}$ and $I.(1-\lambda_{N_i}) = 0$ implies that $I.C.(1-\lambda_{N_i}) = I.(1-\lambda_{N_i}) = 0$. That is $(1-\lambda_{N_i})$'s are F. neu. nwd sets in (X_N, T_N). Hence, by thm.3.7, (X_N, T_N) is a F. neu. B.S. □

Theorem 3.21. *[4] A F. neu. top space (X_N, T_N) is called a F. neu. almost resolvable space if $\vee_{i=1}^{\infty}(\lambda_{N_i})] = 0$, where the F. neu. sets (λ_{N_i})'s in (X_N, T_N) are such that $I.(\lambda_{N_i}) = 0$. Otherwise (X_N, T_N) is called a fuzzy neu. almost irresolvable space.*

Proposition 3.22. *If the fy. neu. top. space (X_N, T_N) is a F. neu. almost resolvable space, then (X_N, T_N) is a F. neu. σ-second category space.*

Proof. Let (λ_{N_i})'s $(i = 1to\infty)$ be the F. neu. dense and F. neu. G_δ-sets in (X_N, T_N). Now $C.(\lambda_{N_i}) = 1$ implies that $1 - C.(\lambda_{N_i}) = 0$. That is, $I.(1 - \lambda_{N_i}) = 0$. Since (X_N, T_N) is a F. neu. almost irresolvable space, $\vee_{i=1}^{\infty}(1 - \lambda_{N_i}) \neq 1$, where the F. neu. sets (λ_{N_i})'s in (X_N, T_N) are such that $I.(1 - \lambda_{N_i}) = 0$. Now $I.\vee_{i=1}^{\infty}(1 - \lambda_{N_i}) \neq 1$ implies that $1 - \vee_{i=1}^{\infty}(1 - \lambda_{N_i}) \neq 0$. Hence we have $\wedge_{i=1}^{\infty}(\lambda_{N_i}) \neq 0$, where the F. neu. sets (λ_{N_i})'s are F. neu. dense and F. neu. G_δ-sets in a fy. neu. top. space (X_N, T_N). Thus, by prop.3.11 (X_N, T_N) is a F. neu. σ-second category space. □

Theorem 3.23. *[4] A fy. neu. top. space (X_N, T_N) is called a F. neu. hyperconnected space if every F. neu. open set λ_N is F. neu. dense in (X_N, T_N). That is, $C.(\lambda_N) = 1$ for all $0 \neq \lambda_N \in T_N$*

Theorem 3.24. *If $C.(\wedge_{i=1}^{\infty}(\lambda_{N_i})) = 1$, where λ_{N_i}'s are F. neu. dense and F. neu. G_σ-sets in (X_N, T_N). Then (X_N, T_N) is a F. neu. σ-B.S.*

Proposition 3.25. *If $C.(\wedge_{i=1}^{\infty}(\lambda_{N_i})) = 1$, where the F. neu. sets (λ_{N_i})'s are F. neu. G_δ-sets in a F. neu. hyperconnected and F. neu. P-space (X_N, T_N), then (X_N, T_N) is a F. neu. σ-B.S.*

Proof. Let(λ_{N_i})'s $(i = 1to\infty)$ be the F. neu. G_δ-sets in (X_N, T_N) such that $C.(\wedge_{i=1}^{\infty}(\lambda_{N_i})) = 1$. Since (X_N, T_N) is a F. neu. P-space, the F. neu. G_δ-sets (λ_{N_i})'s in (X_N, T_N), are F. neu. open in (X_N, T_N). Also since (X_N, T_N) is a F. neu. hyperconnected space, the F. neu. open sets (λ_{N_i})'s in (X_N, T_N), are F. neu. dense sets in (X_N, T_N). Hence the F. neu. sets (λ_{N_i})'s $(i = 1\ to\ \infty)$ are F. neu. dense and F. neu. G_δ-sets in (X_N, T_N) and $C.(\wedge_{i=1}^{\infty}(\lambda_{N_i})) = 1$. Hence by thm3.24 (X_N, T_N) is a F. neu. σ-B.S. □

4. Conclusion

In this paper several characterization of F. neutrosophic σ-Baire space and other F. neutrosophic spaces are discussed and established.

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NEUTROSOPHIC BOUNDARY AND NEUTROSOPHIC SEMI-BOUNDARY ON FUZZY SETTING

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Abstract. The aim of this paper is to introduce the concept of fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary by using fuzzy neutrosophic open, fuzzy neutrosophic closed, fuzzy neutrosophic semi-open, fuzzy neutrosophic semi-closed of a fuzzy neutrosophic topological space. Some characterization are discussed. several examples and properties are obtained .

Keywords: Fuzzy neutrosophic open, fuzzy neutrosophic closed, fuzzy neutrosophic semi-open, fuzzzy neutrosophic semi-closed, fuzzy neutrosophic semi-interior, fuzzy neutrosophic semi-closure, fuzzy neutrosophic boundary, fuzzy neutrosophic semi-boundary.

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1. Introduction

After Zadeh's[27] introduction of fuzzy sets, Chang [8] defined and studied the notion of a fuzzy topological space in 1968. Since then, much attention has been paid to generalize the basic concepts of classical topologyin fuzzy setting and thus a modern theory of fuzzy topology has been developed.

In recent years, fuzzy topology has been found to be very useful in solving many practical problems. Du etal [9] fuzzified the very successful 9-intersection Egenhofer model [9,10] for depicting topological relations in geographic information systems (GIS) query.

In [11,12], El-Naschie showed that the notion of fuzzy topology might be relevant to quantum particle physics and quantum gravity in connection with string theory and e^{∞} theory. Tang [25] used a slightly changed version of Chang's fuzzy topological space to model spatial objects for GIS databases and structured query language (SQL) for GIS.

Levine [20] introduced the concepts of semiopen sets and semicontinuous mappings in topological space. Interestingly, his work found applications in the field of digital topology [10]. for example, it was found that digital line is a $T_{\frac{1}{2}}$-space [11], which is a weaker separation axiom based upon semiopen sets. Fuzzy digital topology [24] was introduced by Rosenfeld, which demonstrated the need for the fuzzification of weaker forms of notions of classical topology. Azad [6] carried out this fuzzification in 1981, and presented some general properties of fuzzy spaces. several properties of fuzzy semiopen (resp., fuzzy semiclosed), fuzzy regular open(resp., closed) sets have been discussed. moreover, he defined fuzzy semicontinuous (resp.,semiopen, semiclosed) functions and studied the properties of fuzzy semicontinuous function in product-related spaces. Finally, he defined and characterized fuzzy almost continuous mappings. In this direction much work followed subsequently, for example [19-27].

Pu and Liu [23] defined the notion of fuzzy boundary in fuzzy topological spaces in 1980 and followed Ahmad and Athar [3] studied and discussed their properties. Neutrosophic sets were first introduced by Florentin Smarandache [13] in 1955, which is a generalization of the concepts of fuzzy sets and intuitionistic fuzzy sets. Neutrosophic sets allow for consideration of indeterminacy, neutrality, and inconsistency in a given space. In 2017, Veereswari [26] introduced fuzzy neutrosophic topological spaces. This concept is the solution and representation of the problems with various fields. Furthermore the authors E.Poongothai and E.Padmavathi [22] introduced the concept of fuzzy neutrosophic semi-open(semi-closed) and fuzzy neutrosophic semi-interior(semi-closure) are introduced. In this paper we have to introduce fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary and several properties with suitable examples are examined.

2. Preliminaries

For the basic concepts and notations are given. The following definitions and lemmas are used in studying the properties of fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary.

Definition 2.1. [5] A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F : X \to [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. With the condition $0 \leq T_{A^*}(x) + I_{A^*}(x) + F_{A^*}(x) \leq 2$.

Definition 2.2. [5] A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \leq F_B(x)$.

Definition 2.3. [5] Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$$

$$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$$

Definition 2.4. [5] The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$

Definition 2.5. [5] A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0, I_A(x) = 0, F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N.

Definition 2.6. [5] A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N.

Definition 2.7. [5] The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where

$$T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = (T_A(x).$$

The complement of a fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8. [20] A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X

(i) $0_N, 1_N \in \tau$

(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$

(iii) $\cup A_i \in$

τ for any arbitrary family $\{A_i : i \in J\} \in \tau$. Satisfying the following axioms. In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9. [26] The complement A^C of a fuzzy neutrosophic set in A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10. [26] Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$int(A) = \cup\{G : G \ is \ a \ fuzzy \ neutrosophic \ open \ set \ in \ X \ and \ G \subseteq A\}$$
$$cl(A) = \cap \ \{G : G \ is \ a \ fuzzy \ neutrosophic \ closed \ set \ in \ X \ and \ G \subseteq G\}$$

Definition 2.11. [26] Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X, τ). Then A_N is said to be fuzzy neutrosophic semi-open in (X, τ_N) if there exists a fuzzy neutrosophic open set such that $fn0 \leq A_N \leq fncl(fn0)$.

Lemma 2.12. [22] *Let A_N be any fuzzy neutrosophic set in the fuzzy neutrosophic topological space (X_N, τ_N). Then*

$$\text{(i). } (Sint(A_N))^C = Scl(AC) \text{ and (ii). } (Scl(A))^C = Sint(A^C)$$

Theorem 2.13. [22] *A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi-open set if only if $A_N \leq ((A_N)^+)^-$.*

Theorem 2.14. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then the union of two fuzzy neutrosophic semi-open sets is a fuzzy neutrosophic semi-open set in the fuzzy neutrosophic topological space (X, τ_N).*

Theorem 2.15. [22] *Let A_N be a fuzzy neutrosophic semi-open in the fuzzy neutrosophic topological space (X, τ_N). and suppose $A_N \leq B_N \leq (A_N)^-$. Then B_N is a fuzzy neutrosophic semi-open set in (X, τ_N).*

Definition 2.16. [22] Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X, τ_N).Then A_N is said to be fuzzy neutrosophic semi-closed set in (X, τ_N) if there exists a fuzzy neutrosophic closed (fnC) set such that $fn(fnC)^+ \leq A_N \leq (fnC)$.

Theorem 2.17. [22] *A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X, τ_N) is fuzzy neutrosophic semi-closed set if only if $((A_N)^-)^+$.*

Theorem 2.18. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space and A_N be a fuzzy neutrosophic set of (X, τ_N). Then A_N is a fuzzy neutrosophic semi-closed set if and only if complement of $C(A_N)$ is fuzzy neutrosophic semi-open set in (X, τ_N).*

Theorem 2.19. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then intersection of two fuzzy neutrosophic semi-closed sets is a fuzzy neutrosophic semi-closed set in fuzzy neutrosophic topological space (X, τ_N).*

Definition 2.20. [22] Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X, τ_N), the fuzzy neutrosophic semi-interior of A_N is the union of all fuzzy neutrosophic semi-open sets of (X, τ_N) contained in A_N.
That is, $fnS(A_N)^+ = \vee\{G_N : G_N \; is \; a \; fuzzy \; neutrosophic \; semi-open \; set \; in \; (X, \tau_N) \; and \; G_N \leq A_N\}$

Definition 2.21. [22] Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set $A_N$of (X, τ_N), the fuzzy neutrosophic semi-closure of A_N $[fnS \; cl(A_N) \; for \; short]$ is the intersection of all fuzzy neutrosophic semi-closed sets of (X, τ_N) contained in A_N.
That is, $fnS(A_N)^- = \wedge\{K_N : K_N \; isafnSC \; set \; in \; (X, \tau_N) \; and \; K_N \geq A_N\}$

Theorem 2.22. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we*

have (i) $fnS(A_N \vee B_N)^- = fnS(A_N)^- \vee fnS(B_N)^-$ *and*

(ii) $fnS(A_N \wedge B_N)^- = fnS(A_N)^- \wedge fnS(B_N)^-$

Theorem 2.23. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic sets A_N and B_N of a (X, τ_N),*

(i) $C(fnS(A_N)^+) \leq fnS(C(A_N))^-$,

(ii) $C(fnS(A_N)^-) \leq fnS(C(A_N))^+$

Theorem 2.24. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have*

(i) $A_N \leq fnS(A_N)^-$,

(ii) A_N *is* $fnSC$ *set in* $(X, \tau_N) \Leftrightarrow fnS(A_N)^- = A_N$,

(iii) $fnS(fnS(A_N)^-)^- = fnS(A_N)^-$,

(iv) *If* $A_N \leq B_N$ *then* $fnS(A_N)^- \leq fnS(B_N)^-$.

Theorem 2.25. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have*

(i) $fnS(A_N)^- \geq A_N \vee fnS(fnS(A_N)^+)^-$,

(ii) $fnS(A_N)^+ \leq A_N \wedge fnS(fnS(A_N)^-)^+$,

(iii) $fn(fnS(A_N)^-)^+ \leq fn(fn(A_N)^-)^+$,

(iv) $fn(fnS(A_N)^-)^+ \geq fn(fnS(fnS(A_N)^+)^-)^+$

Theorem 2.26. [22] *Let (X, τ_N) be a fuzzy neutrosophic topological space. Then for any fuzzy neutrosophic sets A_N and B_N of a fuzzy neutrosophic topological space (X, τ_N) we have*

(i) $fnS(A_N \wedge B_N)^+ = fnS(A_N)^+ \wedge fnS(B_N)^+$,

(ii) $fnS(A_N \vee B_N)^+ \geq fnS(A_N)^+ \vee fnS(B_N)^+$.

Lemma 2.27. [26] *Let A_N and B_N be any fuzzy neutrosophic set in the fuzzy neutrosophic topological space (X_N, T_N). Then the following are true.*

(i) $cl(A_N)$ *is a fuzzy neutrosophic closed in* X_N.

(ii) $int(A_N)$ *is a fuzzy neutrosophic open in* X_N.

(iii) $cl(A_N) \leq cl(B_N)$*if* $A_N \leq B_N$.

(iv) $int(Int(A_N)) = int(A_N)$.

(v) $cl(cl(A_N)) = cl(A_N)$.

(vi) $int(A_N \wedge B_N) = int(A_N) \wedge int(B_N)$.

(vii) $int(A_N \vee B_N) = int(A_N) \vee int(B_N)$.

(viii) $cl(A_N \vee B_N) = cl(A_N) \vee cl(B_N)$.

(ix) $cl(A_N \wedge B_N) = cl(A_N) \wedge cl(B_N)$

3. On fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary

Definition 3.1. Let A_N be a fuzzy neutrosophic set in an fuzzy neutrosophic topological space (X_N, T_N). Then the fuzzy neutrosophic boundary of A_N is defined as $Bdy(A_N) = C.(A_N) \wedge C.(1 - A_N)$. Obviously $Bdy(A_N)$ is a fuzzy neutrosophic closed set.

Remark 3.2. In fuzzy neutrosophic topology, we have $A_N \vee Bdy(A_N) \leq C.(A_N)$, for an arbitary fuzzy neutrosophic set A_N in X_N, the equality need not hold as the following example.

Example 3.3. Let $X_N = \{a, b\}$ and $T_N = \{0, A_N, B_N, 1\}$. Then (X_N, T_N) be a fuzzy neutrosophic topological space. Let $A_N = \{a_{0.6}, b_{0.4}\}, B_N\{a_{0.3}, b_{0.4}\}$.Then $C.(A_N) = \{a_{0.7}, b_{0.6}\}$ and $Bdy(A_N) = \{a_{0.6}, b_{0.4}\}$. It follows that $C.(A_N) \neq \{a_{0.6}, b_{0.6}\} = A_N \vee Bdy(A_N)$

Proposition 3.4. *For a fuzzy neutrosophic set* A_N *in a fuzzy neutrosophic topological space.* (X_N, T_N)*, then the following conditions hold.*

(1) $Bdy(A_N) = Bdy(1 - A_N)$

(2) *If* A_N *is fuzzy neutrosophic closed set, then* $Bdy(A_N) \leq (A_N)$

(3) *If* A_N *is fuzzy neutrosophic open set, then* $Bdy(A_N) \leq (1 - A_N)$

(4) *Let* $A_N \leq B_N$ *and* $B_N \in c.(X_N)$ *(*$C.(X_N)$ *denotes the class of fuzzy neutrosophic closed set). Then* $Bdy(A_N) \leq B$*(resp.* $Bdy(A_N \leq 1 - B_N)$*).*

(5) $1 - Bdy(A_N) = I.(A_N) \vee I.(1 - A_N)$

(6) $Bdy(A_N) \leq Bdy(A_N)$

(7) $C.(Bdy(A_N)) \leq Bdy(A_N)$.

Proof : (1). By Definition,3.1, $Bdy(A_N) = C.(A_N) \wedge C.(1 - A_N)$ and $Bdy(1 - A_N) = C.(1 - A_N) \wedge C.(A_N)$. Therefore $Bdy(A_N) = Bdy(1 - A_N)$. Hence (1) proved.

(2). Let A_N be fuzzy neutrosophic closed set. By lemma 2.27, $C.(A_N) = A_N$. $Bdy(A_N) \leq C.(A_N) \wedge C.(1 - A_N) \leq C.(A_N) = A_N$ Hence (2) proved.

(3). Let A_N be fuzzy neutrosophic open set. By lemma 2.27, $I.(A_N) = A_N$. It follows that $Bdy(A_N) \leq C.(1 - A_N) = 1 - (I.(A_N)) = 1 - A_N$. Hence (3) proved.

(4). Let $A_N \leq B_N$. Then by Lemma 2.27, $C.(A_N) \leq C.(B_N)$. Since $B \in c.(X_N)$, we have $C.(B_N) = B_N$. This implies that, $Bdy(A_N) = C.(A_N) \wedge C.(1 - A_N) \leq C.(B_N) \wedge C.(1 - B_N) \leq C.(B_N) = B$. That is, $Bdy(A_N) \leq B_N$. Let $B \in c.(X_N)$. Then $1 - B_N \in c.(X_N)$. Using the above, $Bdy(A_N) \leq 1 - B_N$. Hence (4) proved.

(5). By definition 3.1, $Bdy(A_N) = C.(A_N) \wedge C.(1 - A_N)$

Taking complement on both sides, we get $1 - (Bdy(A_N)) = 1 - (C.(A_N) \wedge C.(1 - A_N)) = (1 - C.(A_N)) \vee 1 - (C.(1 - A_N)) = I.(1 - A_N) \vee I.(A_N)$. Hence (5) proved.

(6). Since $C.(A_N) \leq C.(A_N)$ and $C.(1 - A_N) \leq C.(1 - A_N)$, we have $Bdy(A_N) = C.(A_N) \wedge C.(1 - A_N) \leq C.(A_N) \wedge C.(1 - A_N) = Bdy(A_N)$. Hence (6) proved.

(7). $C.(Bdy(A_N)) = C.(C.(A_N) \wedge C.(1 - A_N)) \leq C.(C.(A_N)) \wedge C.(C.(1 - A_N)) = C.(A_N) \wedge C.(1 - A_N) = Bdy(A_N) \leq Bdy(A_N)$. Hence (7) proved.

Proposition 3.5. *Let A_N be a fuzzy neutrosophic set in an fuzzy neutrosophic topological space. (X_N, T_N). Then*

(1) $Bdy(A_N) = C.(A_N) \wedge (1 - I.(A_N))$

(2) $Bdy(I.(A_N)) \leq Bdy(A_N)$

(3) $Bdy(C.(A_N)) \leq Bdy(A_N)$

(4) $I.(A_N) \leq A_N \wedge 1 - (Bdy(A_N))$

Proof :(1).Since $C.(1-A_N) = 1-I.(A_N)$, we have $Bdy(A_N) = C.(A_N) \wedge C.(1-A_N) = C.(A_N) \wedge 1 - I.(A_N)$. This proves (1).

(2). By definition 3.1, $Bdy(I.(A_N)) = C.(I.(A_N)) \wedge C.(1 - I.(A_N)) = C.(I.(A_N)) \wedge C.(C.(1 - A_N)) = C.(I.(A_N)) \wedge C.(1 - A_N) = C.(I.(A_N)) \wedge 1 - (I.(A_N) \leq C.(A_N) \wedge (1 - I.(A_N)) = Bdy(A_N)$. Hence (2) proved.

(3). $Bdy(C.(A_N)) = C.(C.(A_N)) \wedge C.(1 - C.(A_N)) = C.(C.(A_N)) \wedge 1 - (I.(C.(A_N))) \leq C.(A_N) \wedge (1 - I.(A_N)) = Bdy(A_N)$. Thus proves (3)

(4). $A_N \wedge 1 - (Bdy(A_N)) = A_N \wedge 1 - ((C.(A_N)) \wedge C.(1 - A_N)) = (A_N) \wedge (I.(1 - A_N)) \vee I.(A_N) = A_N \wedge I.(1 - A_N) \vee (A_N \wedge I.(A_N) = A_N \wedge I.(1 - A_N) \vee I.(A_N) \geq I.(A_N)$. Hence (4).

Proposition 3.6. *Let A_N and B_N be a fuzzy neutrosophic sets in an fuzzy neutrosophic topological space (X_N, T_N) then $Bdy(A_N \vee B_N) \leq Bdy(A_N) \vee Bdy(B_N)$*

Proof : We use Lemma 2.27, to prove this, $Bdy(A_N \vee B_N) = C.(A_N \vee B_N) \wedge C.(1 - (A_N \vee B_N)) = C.(A_N \vee B_N) \wedge C.((1-A_N) \wedge (1-B_N)) \leq C.(A_N) \vee C.(B_N) \wedge (C.(1-(A_N) \wedge C.(1 - B_N)) \leq (C.(A_N)) \wedge C.(1 - A_N)) \vee C.(B_N)) \wedge C.(1 - B_N)) = Bdy(A_N) \vee Bdy(B_N)$. Hence the proposition.

Example 3.7. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.3}, b_{0.4}\rangle\}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed sets of T_N is $1 - T_N = \{0, 1, \langle a_{0.7}, b_{0.6}\rangle\}$ and then let$A_N = \{a_{0.3}, b_{0.6}\}, B_N = \{a_{0.4}, b_{0.3}\}$. On computation, we see that $Bdy(A_N) = \{a_{0.4}, b_{0.5}\}$ and $Bdy(B_N) = \{a_{0.6}, b_{0.5}\}$. Now $A_N \vee B_N = \{a_{0.4}, b_{0.6}\}$ and $Bdy(A_N \vee B_N) = \{a_{0.5}, b_{0.5}\}$. This gives that $Bdy(A_N) \vee Bdy(B_N) = \{a_{0.6}, b_{0.5}\} \not\leq Bdy(A_N \vee B_N) = \{a_{0.5}, b_{0.5}\}$.

The following example shows that $Bdy(A_N \wedge B_N) \not\leq Bdy(A_N) \wedge Bdy(B_N)$ and $Bdy(A_N) \wedge Bdy(B_N) \not\leq Bdy(A_N \wedge B_N)$.

Example 3.8. Using the Example 3.7, $A_N = \{a_{0.3}, b_{0.6}\}$ and $B_N = \{a_{0.4}, b_{0.3}\}$. Then $Bdy(A_N) = \{a_{0.4}, b_{0.5}\}$ and $Bdy(B_N) = \{a_{0.6}, b_{0.5}\}$. Now $A_N \wedge B_N = \{a_{0.3}, b_{0.3}\}$ and $Bdy(A_N \wedge B_N) = \{a_{0.5}, b_{0.4}\}$.

This implies that $Bdy(A_N) \wedge Bdy(B_N) = \{a_{0.4}, b_{0.5}\} \not\leq Bdy(A_N \wedge B_N)) = \{a_{0.5}, b_{0.4}\}$ and $Bdy(A_N \wedge B_N) \not\leq Bdy(A_N) \wedge Bdy(B_N)$.

Proposition 3.9. *For any fuzzy neutrosophic sets A_N and B_N in an fuzzy neutrosophic topological space (X_N, T_N), then $Bdy(A_N \wedge B_N) \leq (Bdy(A_N) \wedge C.(B_N)) \vee (Bdy(B_N) \wedge C.(A_N))$*

Proof : We use Lemma 2.27, to prove this $Bdy(A_N \wedge B_N) = C.(A_N \wedge B_N) \wedge C.(1 - (A_N \wedge B_N)) = C.(A_N \wedge B_N) \wedge C.((1 - A_N) \vee (1 - B_N)) \leq (C.(A_N) \wedge C.(B_N)) \wedge (C.(1 - A_N) \vee C.(1 - B_N)) = (C.(A_N) \wedge C.(B_N) \wedge C.(1 - A_N)) \vee (C.(A_N) \wedge C.(B_N) \wedge C.(1 - B_N)) = (Bdy(A_N) \wedge C.(B_N)) \vee (Bdy(B_N) \wedge C.(A_N))$. Hence proved.

Proposition 3.10. *For any fuzzy neutrosophic set A_N in an fuzzy neutrosophic topological space then*
(1). $Bdy(Bdy(A_N)) \leq Bdy(A_N)$ (2). $Bdy(Bdy(Bdy(A_N))) \leq Bdy(Bdy(A_N))$

Proof : We use Lemma 2.27 and Definition 3.1 to prove this $Bdy(Bdy(A_N)) = C.(Bdy(A_N)) \wedge C.(1 - Bdy(A_N)) \leq C.(Bdy(A_N)) = C.(C.(A_N) \wedge C.(1 - A_N)) = C.(C.(A_N)) \wedge C.(C.(1 - A_N)) = C.(A_N) \wedge C.(1 - A_N) = Bdy(A_N)$. This proves (1). $Bdy(Bdy(Bdy(A_N))) = C.(Bdy(Bdy(A_N))) \wedge C.(Bdy(1 - Bdy(A_N))) = Bdy(Bdy(A_N)) \wedge C.(Bdy(1 - Bdy(A_N))) \leq Bdy(Bdy(A_N))$. Hence the proposition.
This reverse inequality in proposition are not true as shown in the following example.

Example 3.11. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.2}, b_{0.2}\rangle\}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed sets of T_N is $1 - T_N = \{0, 1, \langle a_{0.8}, b_{0.8}\rangle\}$. Let $A_N = \{a_{0.3}, b_{0.4}\}$. On computation, we see that $Bdy(A_N) = \{a_{0.7}, b_{0.7}\} \not\leq Bdy(Bdy(A_N)) = \{a_{0.6}, b_{0.7}\}$. Also, calculate $Bdy(Bdy(Bdy(A_N))) = \{a_{0.4}, b_{0.4}\}$. This shows that $Bdy(Bdy(A_N)) = \{a_{0.6}, b_{0.7}\} \not\leq Bdy(Bdy(Bdy(A_N))) = \{a_{0.4}, b_{0.4}\}$.

4. On fuzzy neutrosophic semi-boundary

In this section we define fuzzy neutrosophic semi-boundary and discuss their properties with example.

Definition 4.1. Let A_N be a fuzzy neutrosophic set in an fuzzy neutrosophic topological space (X_N, T_N). then the fuzzy neutrosophic semi-boundary of A_N is defined as $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N)$. Obviously, $SBdy(A_N)$ is a fuzzy neutrosophic semi-closed set.

Remark 4.2. In a fuzzy neutrosophic topology, we have $A_N \vee SBdy(A_N) \leq SC.(A_N)$ for an arbitrary fuzzy neutrosophic set A_N in X_N, the equality need not hlod as shown in the following example.

Example 4.3. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.8}, b_{0.8}\rangle, \langle a_{0.2}, b_{0.2}\rangle, \langle a_{0.3}, b_{0.7}\rangle\}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed sets of T_N is $1-T_N = \{0, 1, \langle a_{0.2}, b_{0.2}\rangle, \langle a_{0.8}, b_{0.8}\rangle, \langle a_{0.7}, b_{0.3}\rangle\}$. Let $A_N = \{a_{0.6}, b_{0.9}\}$. Then $SC.(A_N) = \{a_{0.8}, b_{0.9}\}$ and $SBdy(A_N) = \{a_{0.4}, b_{0.3}\}$. It follows that $SC.(A_N) \neq \{a_{0.6}, b_{0.9}\} = A_N \vee SBdy(A_N)$.

Proposition 4.4. *For a fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X_N, T_N), the following conditions hold.*

(1) $SBdy(A_N) = SBdy(1 - A_N)$.

(2) *If A_N is fuzzy neutrosophic semi-closed, then $SBdy(A_N) \leq A_N$.*

(3) *If A_N is fuzzy neutrosophic semi-open, then $SBdy(A_N) \leq 1 - A_N$.*

(4) *Let $A_N \leq B_N$ and $B_N \in Sc.(X_N)$ (resp.,$B_N \in So.(X_N)$). Then $SBdy(A_N) \leq B_N$ (resp.,$SBdy(A_N) \leq 1 - B_N$), where $Sc.(X_N)$ (resp., $So(X_N)$) denotes the class of family of fuzzy neutrosophic semi-closed (resp., fuzzy neutrosophic semi-open) sets in X_N.*

(5) $(1 - SBdy(A_N)) = SI.(A_N) \vee SI.(1 - A_N)$

(6) $SBdy(A_N) \leq Bdy(A_N)$.

(7) $SC.(SBdy(A_N)) \leq Bdy(A_N)$.

Proof : By definition 4.1, $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N)$ and $SBdy(1 - A_N) = SC.(1 - A_N) \wedge SC.(A_N)$. Therefore $SBdy(A_N) = SBdy(1 - A_N)$. Hence proved (1).

Let A_N be a fuzzy neutrosophic semi-closed. By lemma 2.12 $SC.(A_N) = A_N$. $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N) \leq SC.(A_N) = A_N$. Hence proved (2).

Let A_N be a fuzzy neutrosophic semi-open, by Lemma 2.12 $SI.(A_N) = A_N$. $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N) \leq SC.(1 - A_N) = (1 - SI.(A_N)) = 1 - A_N$. Hence proved (3).

Let $A_N \leq B_N$, Then by Lemma 2.12, $SC.(A_N) \leq SC.(B_N) = B_N$. That is, $SBdy(A_N) \leq B_N$. Let $B_N \in So.(X_N)$. Then $1 - B_N \in Sc.(X_N)$. Then by the above, $SBdy(A_N) \leq 1 - B_N$. Hence proved (4).

By definition 4.1, $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N)$. Taking complement on both sides, we get $(1 - SBdy(A_N)) = (1 - SC.(A_N) \wedge SC.(1 - A_N)) = (1 - SC.(A_N)) \vee (1 - SC.(1 - A_N)) = SI.(1 - A_N) \vee SI.(A_N)$. Hence proved (5).

Since $SC.(A_N) \leq C.(A_N)$ and $SC.(1 - A_N) \leq C.(1 - A_N)$, then we have $SBdy(A_N) = SC.(A_N) \wedge SC.(1 - A_N) \leq C.(A_N) \wedge C.(1 - A_N) = SBdy(A_N)$. Hence proved (6)

$SC.(SBdy(A_N)) = SC.(SC.(A_N) \wedge SC.(1 - A_N)) \leq SC.(SC.(A_N)) \wedge SC.(SC.(1 - A_N)) = SC.(A_N) \wedge SC.(1 - A_N) = SBdy(A_N) \leq Bdy(A_N)$. Thus $SC.(SBdy(A_N)) \leq Bdy(A_N)$. Hence proved (7).

The converse of (2) and (3) and reverse inequalities of (6) and (7) are not true as shown in the following example.

Example 4.5. Let $A_N = \{a_{0.6}, b_{0.9}\}$ and $B_N = \{a_{0.4}, b_{0.1}\}$ in the fuzzy neutrosophic topological space (X_N, T_N) be defined in Example 4.3 then $SBdy(A_N) = \{a_{0.4}, b_{0.3}\} \leq A_N$, but A_N is not fuzzy neutrosophic semi-closed. $SBdy(B_N) = \{a_{0.4}, b_{0.3}\} \leq 1 - B_N$, but B_N is not fuzzy neutrosophic semi-open. Then $Bdy(B_N) = \{a_{0.7}, b_{0.3}\} \not\leq SBdy(B_N) = \{a_{0.4}, b_{0.3}\}$ and $Bdy(B_N) \not\leq SC.(SBdy(A_N)) = \{a_{0.5}, b_{0.3}\}$.

Proposition 4.6. *Let A_N be a fuzzy neutrosophic set in a fuzzy neutrosophic topological space (X_N, T_N). Then*

(1) $SBdy(A_N) = SC.(A_N) \wedge (1 - SI.(A_N))$

(2) $SBdy(SI.(A_N)) \leq SBdy(A_N)$.

(3) $SBdy(SC.(A_N)) \leq SBdy(A_N)$.

(4) $SI.(A_N) \leq A_N \wedge (1 - SBdy(A_N))$

Proof : Since $SC.(1-A_N) = (1-SI.(A_N))$, we have $SBdy(A_N) = SC.(A_N) \wedge SC.(1-A_N) = SC.(A_N) \wedge (1 - SI.(A_N))$. This proves (1).

$SBdy(SI.(A_N)) = SC.(SI.(A_N)) \wedge SC.(1 - SI.(A_N)) = SC.(SI.(A_N)) \wedge SC.(SC.(1 - A_N)) = SC.(SI.(A_N)) \wedge SC.(1 - A_N) = SC.(SI.(A_N)) \wedge (1 - SI.(A_N)) \leq SC.(A_N) \wedge (1 - SI.(A_N)) = SBdy(A_N)$. Hence proves (2).

$SBdy(SC.(A_N)) = SC.(SC.(A_N)) \wedge SC.(1 - SC.(A_N)) = SC.(SC.(A_N)) \wedge (1 - SI.(SC.(A_N))) \leq SC.(A_N) \wedge (1 - SI.(A_N)) = SBdy(A_N)$. Hence proves (3).

$A_N \wedge (1 - SBdy(A_N)) = A_N \wedge SC.(A_N) \wedge (1 - SC.(1 - A_N)) = A_N \wedge (SI.(1 - A_N) \vee SI.(A_N)) = (A_N \wedge SI.(1 - A_N)) \vee (A_N \wedge SI.(A_N)) = (A_N \wedge SI.(1 - A_N)) \vee SI.(A_N) \geq SI.(A_N)$. Hence proves (4).

Proposition 4.7. *Let A_N and B_N be a fuzzy neutrosophic sets in an fuzzy neutrosophic topological space (X_N, T_N). Then $SBdy(A_N \vee B_N) \leq SBdy(A_N) \vee SBdy(B_N)$.*

Proof : We use Theorem 2.22 to prove this $SBdy(A_N \vee B_N) = SC.(A_N \vee B_N) \wedge SC.(1 - (A_N \vee B_N)) = SC.(A_N \vee B_N) \wedge (SC.(1 - A_N) \wedge (1 - B_N)) \leq (SC.(A_N) \vee SC.(B_N)) \wedge (SC.(1 - A_N) \wedge SC.(1 - B_N)) \leq (SC.(A_N) \wedge SC.(1 - A_N)) \vee SC.(B_N) \wedge SC.(1 - B_N) = SBdy(A_N) \vee SBdy(B_N)$. Hence the proposition.

The reverse inequality are not true as shown in the following example.

Example 4.8. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.8}, b_{0.8}\rangle, \langle a_{0.2}, b_{0.1}\rangle, \}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed set of T_N is $1 - T_N = \{0, 1, \langle a_{0.2}, b_{0.2}\rangle, \langle a_{0.8}, b_{0.9}\rangle, \}$. Let $A_N = \{a_{0.4}, b_{0.1}\}$ and $B_N = \{a_{0.3}, b_{0.4}\}$. Then computation, we see that $SBdy(A_N) = \{a_{0.4}, b_{0.3}\}$ and $SBdy(B_N) = \{a_{0.3}, b_{0.6}\}$. Now $A_N \vee B_N = \{a_{0.4}, b_{0.4}\}$ and $SBdy(A_N \vee B_N) = \{a_{0.4}, b_{0.5}\}$. This gives that $SBdy(A_N) \vee SBdy(B_N) = \{a_{0.4}, b_{0.6}\} \not\leq SBdy(A_N \vee B_N) = \{a_{0.4}, b_{0.5}\}$.

The following example shows that $SBdy(A_N \wedge B_N) \not\leq SBdy(A_N) \wedge SBdy(B_N)$ and $SBdy(A_N) \wedge SBdy(B_N) \not\leq SBdy(A_N \wedge B_N)$.

Example 4.9. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.8}, b_{0.6}\rangle, \langle a_{0.2}, b_{0.3}\rangle, \}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed set of T_N is $1 - T_N = \{0, 1, \langle a_{0.2}, b_{0.4}\rangle, \langle a_{0.8}, b_{0.7}\rangle, \}$. Let $A_N = \{a_{0.3}, b_{0.4}\}$ and $B_N = \{a_{0.6}, b_{0.2}\}$. Then the calculations give $SBdy(A_N) = \{a_{0.3}, b_{0.5}\}$ and $SBdy(B_N) = \{a_{0.5}, b_{0.4}\}$. Now $A_N \wedge B_N = \{a_{0.3}, b_{0.2}\}$ and $SBdy(A_N \wedge B_N) = \{a_{0.4}, b_{0.3}\}$. This gives that $SBdy(A_N) \wedge SBdy(B_N) = \{a_{0.3}, b_{0.4}\} \nleq SBdy(A_N \wedge B_N) = \{a_{0.4}, b_{0.3}\}$. and $SBdy(A_N \wedge B_N) \nleq SBdy(A_N) \wedge SBdy(B_N)$.

Proposition 4.10. *For any fuzzy neutrosophic sets A_N and B_N in an fuzzy neutrosophic topological space (X_N, T_N) then $SBdy(A_N \wedge B_N) \leq (SBdy(A_N)) \wedge SC.(B_N)) \vee (SBdy(B_N) \wedge SC.(A_N))$.*

Proof : We use Theorem 2.22 to prove this, $SBdy(A_N \wedge B_N) = SC.(A_N \wedge B_N) \wedge SC.(1 - (A_N \wedge B_N)) = SC.(A_N \wedge B_N) \wedge (SC.(1 - A_N) \vee (1 - B_N)) \leq (SC.(A_N) \wedge SC.(B_N)) \wedge (SC.(1 - A_N) \vee SC.(1 - B_N)) = (SC.(A_N) \wedge SC.(B_N) \wedge SC.(1 - A_N)) \vee (SC.(A_N) \wedge SC.(B_N) \wedge SC.(1 - B_N)) = (SBdy(A_N) \wedge SC.(B_N)) \vee (SBdy(B_N) \wedge SC.(A_N))$. Hence proved.

Remark 4.11. The reverse inequality in the above proposition not true as shown by the following example.

Example 4.12. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.3}, b_{0.2}\rangle, \langle a_{0.6}, b_{0.8}\rangle, \}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed sets of T_N is $1 - T_N = \{0, 1, \langle a_{0.7}, b_{0.8}\rangle, \langle a_{0.4}, b_{0.2}\rangle, \}$. Let $A_N = \{a_{0.4}, b_{0.6}\}$ and $B_N = \{a_{0.5}, b_{0.3}\}$. Then the calculations, we see that $SBdy(A_N) = \{a_{0.5}, b_{0.4}\}, SC.(A_N) = \{a_{0.5}, b_{0.6}\}, SC.(B_N) = \{a_{0.5}, b_{0.4}\}$ and $SBdy(B_N) = \{a_{0.5}, b_{0.4}\}$. Now $A_N \wedge B_N = \{a_{0.4}, b_{0.3}\}$ and $SBdy(A_N \wedge B_N) = \{a_{0.4}, b_{0.4}\}$. This gives that $(SBdy(A_N) \wedge SBdy(B_N)) \vee (SBdy(B_N) \wedge SC.(A_N)) = \{a_{0.5}, b_{0.4}\} \nleq SBdy(A_N \wedge B_N) = \{a_{0.4}, b_{0.4}\}$.

Proposition 4.13. *For any fuzzy neutrosophic set A_N in an fuzzy neutrosophic topological space (X_N, T_N) then*

(1) $SBdy(SBdy(A_N)) \leq SBdy(A_N)$.

(2) $SBdy(SBdy(SBdy(A_N))) \leq SBdy(SBdy(A_N))$.

Proof : $SBdy(SBdy(A_N)) = SC.(SBdy(A_N)) \wedge SC.(1 - SBdy(A_N)) \leq SC.(SBdy(A_N)) = SC.(SC.(A_N) \wedge SC.(1 - A_N)) = SC.(SC.(A_N)) \wedge SC.(SC.(1 - A_N)) = SC.(A_N) \wedge SC.(1 - A_N) = SBdy(A_N)$. This proves (1).
$SBdy(SBdy(SBdy(A_N))) = SC.(SBdy(SBdy(A_N))) \wedge SC.(Bdy(1 - SBdy(A_N))) = SBdy(SBdy(A_N)) \wedge SC.(SBdy(1 - SBdy(A_N))) \leq SBdy(SBdy(A_N))$. Hence proved (2).

Remark 4.14. The reverse inequality in the above proposition are not true as shown by the following example.

Example 4.15. Let $X_N = \{a, b\}$ and $T_N = \{0, 1, \langle a_{0.3}, b_{0.2}\rangle, \langle a_{0.6}, b_{0.8}\rangle, \}$. Then (X_N, T_N) is a fuzzy neutrosophic topological space. The family of all fuzzy neutrosophic closed sets of T_N is $1 - T_N = \{0, 1, \langle a_{0.7}, b_{0.8}\rangle, \langle a_{0.4}, b_{0.2}\rangle\}$. Let $A_N = \{a_{0.4}, b_{0.6}\}$. Then calculations gives $SBdy(A_N) = \{a_{0.5}, b_{0.7}\} \not\leq SBdy(SBdy(A_N)) = \{a_{0.5}, b_{0.6}\}$. It follows that $SBdy(SBdy(A_N)) = \{a_{0.5}, b_{0.6}\} \not\leq SBdy(SBdy(SBdy(A_N))) = \{a_{0.5}, b_{0.5}\}$.

5. Conclusion

In this paper, the concept of fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary deals with fuzzy neutrosophic open, fuzzy neutrosophic closed, fuzzy neutrosophic semi-open, fuzzy neutrosophic semi-closed, fuzzy neutrosophic semi-interior, fuzzy neutrosophic semi-closure are introduced. Further, we examine certain characterizations and examples are discussed the concept of fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary. Also, we investigate their interrelations between fuzzy neutrosophic boundary and fuzzy neutrosophic semi-boundary on fuzzy neutrosophictopological spaces.

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SOME REMARKS ON FUZZY NEUTROSOPHIC σ-BAIRE SPACES AND OTHER FUZZY NEUTROSOPHIC SPACES

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Abstract. In this paper we define some results and examples on fuzzy neutrosophic σ-Baire spaces and other fuzzy neutrosophic spaces. Also we discuss several characterizations of other fuzzy neutrosophic spaces namely fuzzy neutrosophic P-space, fuzzy neutrosophic submaximal space, fuzzy neutrosophic almost resolvable space and fuzzy neutrosophic almost irresolvable space.

Keywords: Fuzzy Neutrosophic dense set; Fuzzy Neutrosophic G_δ-set; Fuzzy Neutrosophic σ-nowhere dense Set; Fuzzy Neutrosophic Baire space; Fuzzy Neutrosophic σ-Baire; Fuzzy Neutrosophic P-space; Fuzzy Neutrosophic Submaximal Space; Fuzzy Neutrosophic almost resolvable Space and Fuzzy Neutrosophic Almost Irresolvable Space.

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1. Introduction

The fuzzy idea was invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [19]. The important concept of fuzzy topological space was offered by C.L. Chang [3]. The idea of fuzzy σ- Baire Spaces was introduced by G. Thangaraj and E. Poongothai [12]. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [18] introduced fuzzy neutrosophic topological spaces. The idea of fuzzy neutrosophic Baire space was introduced by E. Poongothai and E. Padmavathi [7].

In this paper we investigate several characterizations of fuzzy neutrosophic σ-Baire spaces and also study the interrelations between fuzzy neutrosophic σ-Baire spaces, fuzzy neutrosophic Baire space, fuzzy neutrosophic almost resolvable spaces, fuzzy neutrosophic P-space and fuzzy neutrosophic submaximal spaces.

2. **Preliminaries**

Definition 2.1. [2] A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ x \in X$ where $T, I, F : X \to [0,1]$ and $0 \leq T_A(x) + I_A(x), F_A(x) \leq 3$, with the condition $0 \leq T_{A^*}(x) + I_{A^*}(x), F_{A^*}(x) \leq 2$

Definition 2.2. [2] A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B(i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x),\ F_A(x) \geq F_B(x)$

Definition 2.3. [2] Let X be a non-empty set, and
$A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then $A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$
$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$

Definition 2.4. [2] The difference between two fuzzy neutrosophic sets A and B is defined as $A \backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$

Definition 2.5. [2] A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N

Definition 2.6. [2] A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N

Definition 2.7. [2] The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x),\ I_{A^C}(x) = 1 - I_A(x),\ F_{A^C}(x) = T_A(x)$. The complement of fuzzy neutrosophic set A an also be defined as $A^C = 1_N - A$.

Definition 2.8. [2] A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X satisfying the following axioms.

(i) $0_N,\ 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1,\ A_2 \in \tau$
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$

In this case the pair (X, τ) is called fuzzy neutrosophic top. space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X

Definition 2.9. [1] The complement of a fuzzy neutrosophic set A in a fuzzy neutrosophic top. space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10. [1] Let (X, τ) be a fuzzy neutrosophic top. space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and Interior of A are defined by $I.(A) = \cup\{G : G$ is a fuzzy neutrosophic open set in X and $G \subseteq A\} C.(A) = \cap\{G : G$ is a fuzzy neutrosophic closed set in X and $A \subseteq G\}$

Definition 2.11. [1] Let (X, τ) be a fuzzy neutrosophic topological space over X. Then (i) $C.(A^C) = (I.\ A)^C$, (ii) $I.(A^C) = (cl\ A)^C$

Definition 2.12. [7] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic top. space (P, τ_N) is called a fuzzy neutrosophic F_σ-set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$

Definition 2.13. [7] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic top. space (P, τ_N) is called a fuzzy neutrosophic G_δ-set in (P, τ_N) if $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$

Definition 2.14. [7] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic top. space (P, τ_N) is called a fuzzy neutrosophic dense if there exist no $fnCS$ B_N in (P, τ_N) s.t $A_N \subset B_N \subset 1_X$. That is $fn(A_N)^- = 1_N$

Definition 2.15. [7] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic top. space (P, τ_N) is called a fuzzy neutrosophic nowh. dense set if there exist no non zero $fnOS$ B_N in (P, τ_N) s.t $B_N \subset fn(A_N)^-$. That is $fn(((A_N)^-)^+) = 0_N$

Definition 2.16. [7] Let (P, τ_N) be a fuzzy neutrosophic top. space. A fuzzy neutrosophic set A_N in (P, τ_N) is called fuzzy neutrosophic one category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowh. dense sets in (P, τ_N). Any other fuzzy neutrosophic set in (P, τ_N) is said to be of fuzzy neutrosophic two category.

Definition 2.17. [7] A fuzzy neutrosophic top. space (P, τ_N) is called fuzzy neutrosophic one category space if the fuzzy neutrosophic set 1_X is a fuzzy neutrosophic one category set in (P, τ_N). That is $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowh. dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fuzzy neutrosophic two category space.

Definition 2.18. [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space then (X_N, T_N) is called a fuzzy neutrosophic σ-Baire space if $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where λ_{N_i}'s are fuzzy neutrosophic σ- nowhere dense sets in (X_N, T_N).

Theorem 2.19. *[5] In a fuzzy neutrosophic topological space (X_N, T_N) a fuzzy neutrosophic set λ_N is fuzzy neutrosophic σ-nowhere dense in (X_N, T_N) if and only if $1_N - \lambda_N$ fuzzy neutrosophic dense and fuzzy neutrosophic G_δ-set in (X_N, T_N).*

Theorem 2.20. *[5] If λ_N is a fuzzy neutrosophic dense set in (X_N, T_N) such that $\mu_N \leq (1_N - \lambda_N)$, where μ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N) then μ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N).*

Theorem 2.21. *[7] Let (P, τ_N) be a fy. neutrosophic top. space. Then the following are equivalent.*

(1) *(P, τ_N) is a fy. neutrosophic Baire space.*
(2) *$fn(A_N)^+ = 0_N$, fore every fy. neutrosophic one category set A_N in (P, τ_N).*
(3) *$fn(B_N)^+ = 1_N$, for every fy. neutrosophic re. set B_N in (P, τ_N).*

Definition 2.22. [9] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic submaximal space if for each fuzzy neutrosophic set A_N in (X, τ_N) such that $(A_N)^- = 1$ then $A_N \in \tau_N$ in (X, τ_N). That is (X, τ_N) is a fuzzy neutrosophic submaximal space if each fuzzy neutrosophic dense set in (X, τ_N) is a fuzzy neutrosophic open set in (X, τ_N).

Definition 2.23. [9] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic resolvable space if there exist a fuzzy neutrosophic dense set A_N in (X, τ_N) such that $(1 - A_N)^- = 1$. Otherwise, (X, τ_N) is called a fuzzy neutrosophic irresolvable space.

Definition 2.24. [9] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic hyperconnected space if every non-null fuzzy neutrosophic open subset of (X, τ_N) is fuzzy neutrosophic dense in (X, τ_N).

Definition 2.25. [9] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic P-space if each fuzzy neutrosophic G_δ-set in (X, τ_N) is fuzzy neutrosophic open set in (X, τ_N).

Definition 2.26. [9] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic almost resolvable space if $\vee_{i=1}^{\infty}(A_{N_i}) = 1$, where (A_{N_i})'s in (X, τ_N) are such that $(A_{N_i})^+ = 0$, otherwise, (X, τ_N) is called a fuzzy neutrosophic almost irresolvable space.

Theorem 2.27. *[4] If λ_N is a fuzzy neutrosophic σ-first category set in (X_N, T_N), then there is a fuzzy neutrosophic F_σ-set δ_N in (X_N, T_N) such that $\lambda_N \leq \delta_N$*

Theorem 2.28. *[5] Let (X_N, TN) be a fuzzy neutrosophic topological space. Then the following are equivalent*

(1) *(X_N, T_N) is a fuzzy neutrosophic σ-Baire space.*
(2) *$int(\lambda_N) = 0_N$ for every fuzzy neutrosophic σ- first category set λ_N in (X_N, T_N).*
(3) *$cl(\mu_N) = 1_N$ for every fuzzy neutrosophic σ-residual set μ_N in (X_N, T_N)*

3. On Fuzzy Neutrosophic σ-Baire Spaces

Proposition 3.1. *If λ_N is a fuzzy neutrosophic σ-nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N), then λ_N is a fuzzy neutrosophic first category set in (X_N, T_N)*

Proof. Let λ_N be a fuzzy neutrosophic σ-nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N). Then λ_N is a fuzzy neutrosophic F_σ-set such that $int(\lambda_N) = 0_N$. Since λ_N is a fuzzy neutrosophic F_σ-set , $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic closed sets in (X_N, T_N). Now $int(\lambda_N) = 0_N$ implies that $int(\vee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$. But we have $\vee_{i=1}^{\infty} int(\lambda_{N_i}) \leq int(\vee_{i=1}^{\infty}(\lambda_{N_i}))$. This implies that $\vee_{i=1}^{\infty} int(\lambda_{N_i}) \leq 0_N$. That is, $\vee_{i=1}^{\infty} int(\lambda_{N_i}) = 0_N$. Hence we have $int(\lambda_{N_i}) = 0_N$ for each i. Since (λ_{N_i})'s are fuzzy neutrosophic closed, $cl(\lambda_{N_i}) = \lambda_{N_i}$. Then $intcl(\lambda_{N_i}) = int(\lambda_{N_i}) = 0_N$. That is $intcl(\lambda_{N_i}) = 0_N$ for each i. Hence (λ_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X_N, T_N). Therefore $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic nowhere dense sets, implies that λ_N is a fuzzy neutrosophic first category set in (X_N, T_N). □

Proposition 3.2. *If the fuzzy neutrosophic nowhere dense set λ_N is a fuzzy neutrosophic F_σ-set in a fuzzy neutrosophic topological space (X_N, T_N), then λ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N).*

Proof. Let the fuzzy neutrosophic nowhere dense set λ_N be a fuzzy neutrosophic F_σ-set in (X_N, T_N). Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where $1_N - \lambda_{N_i} \in T_N$. Since λ_N is a fuzzy neutrosophic nowhere dense set, we have $intcl(\lambda_N) = 0_N$ in (X_N, T_N) . Now $int(\lambda_N) \leq intcl(\lambda_N)$ implies that $int(\lambda_N) = 0_N$. Hence λ_N is a fuzzy neutrosophic F_σ set such that $int(\lambda_N) = 0_N$. Then λ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). □

Proposition 3.3. *If each fuzzy neutrosophic nowhere dense set λ_N is a fuzzy neutrosophic F_σ-set in a fuzzy neutrosophic topological space (X_N, T_N), then λ_N is a fuzzy neutrosophic first category set in (X_N, T_N).*

Proof. Let each fuzzy neutrosophic nowhere dense setλ_N be a fuzzy neutrosophic F_σ-set in (X_N, T_N). Since λ_N is a fuzzy neutrosophic nowhere dense set in (X_N, T_N), we have $intcl(\lambda_N) = 0_N$. Now $int(\lambda_N) \leq intcl(\lambda_N)$, implies that $int(\lambda_N) \leq 0_N$. That is, $int(\lambda_N) = 0_N$. Hence λ_N is a fuzzy neutrosophic F_σ-set such that $int(\lambda_N) = 0_N$. Then λ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Then by proposition 3.1, λ_N is a fuzzy neutrosophic first category set in (X_N, T_N). □

Proposition 3.4. *If a fuzzy neutrosophic σ-first category set λ_N is a fuzzy neutrosophic closed set in a fuzzy neutrosophic σ-Baire space (X_N, T_N), then λ_N is a fuzzy neutrosophic nowhere dense set in (X_N, T_N).*

Proof. Let $\lambda_N$be a fuzzy neutrosophic σ-first category set in (X_N, T_N). Since (X_N, T_N) ia a fuzzy neutrosophic σ-Baire space, by theorem 2.28, $int(\lambda_N) = 0_N$, by hypothesis, λ_N is fuzzy neutrosophic closed in (X_N, T_N). This implies that $cl(\lambda_N) = \lambda_N$. Then $intcl(\lambda_N) = int(\lambda_N) = 0_N$. That is, $intcl(\lambda_N) = 0_N$ in (X_N, T_N). Hence (X_N, T_N) is a fuzzy neutrosophic nowhere dense set in (X_N, T_N). □

Proposition 3.5. *If a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic nowhere dense set, then (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.*

Proof. Let λ_N be a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic topological space (X_N, T_N) such that $intcl(\lambda_N) = 0_N$. Then $int(\lambda_N) \leq intcl(\lambda_N)$ implies that $int(\lambda_N) \leq 0_N$. Hence for a fuzzy neutrosophic σ-first category set λ_N in (X_N, T_N), we have $int(\lambda_N) = 0_N$. Hence by theorem 2.28 is a fuzzy neutrosophic σ-Baire space. □

Proposition 3.6. *If λ_N is a fuzzy neutrosophic σ-nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N), then there exist a fuzzy neutrosophic F_σ-set δ_N in (X_N, T_N) such that $\lambda_N \leq \delta_N$.*

Proof. Let λ_N be a fuzzy neutrosophic σ-nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N). Then by proposition 3.1, λ_N is a fuzzy neutrosophic first category set in (X_N, T_N). Then by theorem 2.27, there is a fuzzy neutrosophic F_σ-set in (X_N, T_N) such that $\lambda_N \leq \delta_N$. □

Proposition 3.7. *If $cl(\wedge_{i=1}^{N}(\lambda_{N_i})) = 1_N$, where (λ_{N_i})'s are fuzzy neutrosophic dense and fuzzy neutrosophic G_δ-sets in a fuzzy neutrosophic topological space (X_N, T_N) and then $int(\vee_{i=1}^{N}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N).*

Proof. Let λ_{N_i}'s $(i = 1toN)$ be a fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Then by Theorem 2.19, $1_N - \lambda_{N_i}$'s are fuzzy neutrosophic dense and fuzzy neutrosophic G_δ-sets in (X_N, T_N). Since $cl[\wedge_{i=1}^{N}(1_N - \lambda_{N_i})] = 1_N$. Now, $cl[\wedge_{i=1}^{N}(1_N - \lambda_{N_i})] = 1_N$, implies that $cl[1_N - \vee_{i=1}^{N}(\lambda_{N_i})] = 1_N$. Then we have $1_N - int[\vee_{i=1}^{N}(1_N - \lambda_{N_i})] = 1_N$. This implies that $int[\vee_{i=1}^{N}(\lambda_{N_i})] = 0_N$ □

Proposition 3.8. *If a fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic σ-Baire space, then $cl[\wedge_{i=1}^{N}(1_N - \lambda_{N_i})] = 1_N$, where the fuzzy neutrosophic sets $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic dense and fuzzy neutrosophic G_δ-sets in (X_N, T_N).*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic σ-Baire space. Then $int(\vee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Now $int(\vee_{i=1}^{N}(\lambda_{N_i})) \leq int[\vee_{i=1}^{\infty}(\lambda_{N_i}))$ implies that $int(\vee_{i=1}^{N}(\lambda_{N_i})) = 0_N$. Then $1_N - int(\vee_{i=1}^{N}(\lambda_{N_i})) = 1_N - 0_N = 1_N$. Then we have $cl(1_N - \vee_{i=1}^{N}(\lambda_{N_i})) = 1_N$.This implies that $cl(\wedge_{i=1}^{N}(1_N - \lambda_{N_i})) = 1_N$. Since λ_{N_i}'s $(i = 1toN)$ are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N) By theorem 2.19 $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic dense and fuzzy neutrosophic G_δ-sets in (X_N, T_N). □

4. Fuzzy Neutrosophic σ-First Category Sets and Fuzzy Neutrosophic P-Spaces

Proposition 4.1. *If λ_N is a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic P-space (X_N, T_N), then λ_N is not a fuzzy neutrosophic dense set in (X_N, T_N).*

Proof. Let us assume that the contrary. Suppose that λ_N is a fuzzy neutrosophic σ-first category set in (X_N, T_N) such that $cl(\lambda_N) = 1_N$. Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Since (λ_{N_i})'s are fuzzy neutrosophic σ-nowhere dense sets, (λ_{N_i})'s are fuzzy neutrosophic F_σ- sets and $int(\lambda_{N_i}) = 0_N$ $(i = 1\ to\ \infty)$. Then $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic G_δ-sets in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic P-space, the fuzzy neutrosophic G_δ-sets $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic open sets in (X_N, T_N). Let $\mu_N = \wedge_{i=1}^{\infty}(1_N - \lambda_{N_i})$. Then μ_N is a non-zero fuzzy neutrosophic G_δ-set in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic P-space, μ_N is a fuzzy neutrosophic open set in (X_N, T_N) and

hence $int(\mu_N) = \mu_N$. Now, $\mu_N = 1_N - \vee_{i=1}^{\infty}(\lambda_{N_i}) = 1_N - \lambda_N$ Then we have $int(\mu_N) = int(1_N - \lambda_N) = 1_N - cl(\lambda_N) = 1_N - 1_N = 0_N$. This implies that a contradiction. Hence, in a fuzzy neutrosophic P-space (X_N, T_N), fuzzy neutrosophic σ-first category sets are not fuzzy neutrosophic dense sets in (X_N, T_N). □

Proposition 4.2. *If λ_N is a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic P-space then $int(1_N - \lambda_N) \neq 0_N$*

Proof. Let λ_N be a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic P-space (X_N, T_N). Then by proposition 4.1, λ_N is not a fuzzy neutrosophic dense set. That is, $cl(\lambda_N) \neq 1_N$. Then, $1_N - cl(\lambda_N) \neq 0_N$. This implies that $int(1_N - \lambda_N) \neq 0_N$. □

Remark-1:

In view of the above proposition 4.2, we have the following: In a fuzzy neutrosophic P-space (X_N, T_N) , the interior of a fuzzy neutrosophic σ-residual set is non-zero in (X_N, T_N).

Proposition 4.3. *If λ_N is a fuzzy neutrosophic σ- first category set in a fuzzy neutrosophic P-space (X_N, T_N), then λ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N).*

Proof. If λ_N be a fuzzy neutrosophic σ-first category set in (X_N, T_N). Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Since (λ_{N_i})'s are fuzzy neutrosophic σ-nowhere dense sets, (λ_{N_i})'s are fuzzy neutrosophic F_σ-sets and $int(\lambda_{N_i}) = 0_N (i = 1to\infty)$. Then $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic G_δ-sets in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic P-space, the fuzzy neutrosophic G_δ-sets $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic open sets in (X_N, T_N). Let $\mu_N = \wedge_{i=1}^{\infty}(1_N - \lambda_{N_i})$, then μ_N is a non-zero fuzzy neutrosophic G_δ-set in (X_N, T_N). Then $\mu_N = 1_N - \vee_{i=1}^{\infty}(\lambda_{N_i})$, we have $\mu_N = 1_N - \lambda_N$ and hence $\lambda_N = 1_N - \mu_N$. Since μ_N is a fuzzy neutrosophic G_δ-set in $(X_N, T_N), \lambda_N$ is a fuzzy neutrosophic F_σ-set in (X_N, T_N). Hence in a fuzzy neutrosophic P-space (X_N, T_N), then the fuzzy neutrosophic σ-first category sets are fuzzy neutrosophic F_σ-sets in (X_N, T_N). □

Proposition 4.4. *If (X_N, T_N) is a fuzzy neutrosophic σ-Baire and fuzzy neutrosophic P-space, then each fuzzy neutrosophic σ-first category set is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic σ-Baire and fuzzy neutrosophic P-space (X_N, T_N). Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s

are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic σ-Baire space, by theorem 2.28, $int(\lambda_N) = 0_N$. Since (X_N, T_N) is a fuzzy neutrosophic P-space by proposition 4.3, the fuzzy neutrosophic σ-first category set λ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N). Hence λ_N is a fuzzy neutrosophic F_σ-set such that $int(\lambda_N) = 0_N$. Then λ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Hence, if (X_N, T_N) is a fuzzy neutrosophic σ-Baire and fuzzy neutrosophic P-space, then each fuzzy neutrosophic σ-first category set is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). □

Proposition 4.5. *If λ_N is a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic hyperconnected and fuzzy neutrosophic P-space (X_N, T_N), then λ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N)*

Proof. Let λ_N be a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic hyperconnected and fuzzy neutrosophic P-space (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic P-space, by proposition 4.3, λ_N is a fuzzy neutrosophic F_σ-set and hence $1_N - \lambda_N$ is a fuzzy neutrosophic G_δ-set. Since (X_N, T_N) is a fuzzy neutrosophic P-space, $1_N - \lambda_N$ is a fuzzy neutrosophic open in (X_N, T_N). Again since (X_N, T_N) is a fuzzy neutrosophic hyperconnected space the fuzzy neutrosophic open set $1_N - \lambda_N$ is fuzzy neutrosophic dense in (X_N, T_N). That is $cl(1_N - \lambda_N) = 1_N$. Then we have $int(\lambda_N) = 0_N$. Hence λ_N is a fuzzy neutrosophic F_σ-set such that $int(\lambda_N) = 0_N$. Therefore, in a fuzzy neutrosophic hyperconnected and fuzzy neutrosophic P-space (X_N, T_N), the fuzzy neutrosophic σ-first category set λ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). □

Proposition 4.6. *If λ_N is a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic hyperconnected and fuzzy neutrosophic P-space (X_N, T_N), then λ_N is a fuzzy neutrosophic first category set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic hyperconnected and fuzzy neutrosophic P-space (X_N, T_N). Then by proposition 4.5, λ_N is a fuzzy neutrosophic σ-nowhere dense set in (X_N, T_N). Then by proposition 3.1, λ_N is a fuzzy neutrosophic first category set in (X_N, T_N). □

Proposition 4.7. *If (X_N, T_N) is a fuzzy neutrosophic Baire, fuzzy neutrosophic P-space and fuzzy neutrosophic hyperconnected space then (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.*

Proof. Let λ_N be a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic Baire, fuzzy neutrosophic P-space and fuzzy neutrosophic hyperconnected space (X_N, T_N). Then by proposition 4.6, $\lambda_N$is a fuzzy neutrosophic first category set in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic Baire space, by theorem 2.28 $int(\lambda_N) = 0_N$. Hence, for the fuzzy neutrosophic σ-first category set λ_N in (X_N, T_N), we have $int(\lambda_N) = 0_N$ Then by theorem 2.28, implies that (X_N, T_N) is a fuzzy neutrosophic σ-Baire space. □

Proposition 4.8. *If each fuzzy neutrosophic open set is fuzzy neutrosophic dense in a fuzzy neutrosophic Baire and fuzzy neutrosophic P-space (X_N, T_N), then (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.*

Proof. Let λ_N be a fuzzy neutrosophic σ-first category set in a fuzzy neutrosophic Baire and fuzzy neutrosophic P-space (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic P-space, by proposition 4.3, the fuzzy neutrosophic σ-first category set λ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N). Then $1_N - \lambda_N$ is a fuzzy neutrosophic G_δ-set in (X_N, T_N). Again, since (X_N, T_N) is a fuzzy neutrosophic P-space, $1_N - \lambda_N$ is a fuzzy neutrosophic open set in (X_N, T_N). By hypothesis, $1_N - \lambda_N$ is fuzzy neutrosophic dense in (X_N, T_N). That is, $cl(1_N - \lambda_N) = 1_N$. Then $1_N - int(\lambda_N) = 1_N$ and hence $int(\lambda_N) = 0_N$. Hence for fuzzy neutrosophic σ-first category set λ_N in (X_N, T_N), we have $int(\lambda_N) = 0_N$. By theorem 2.28, (X_N, T_N) is a fuzzy neutrosophic σ-Baire space. □

Proposition 4.9. *If λ_N is a fuzzy neutrosophic σ-nowhere dense set in a fuzzy neutrosophic P-space (X_N, T_N) then λ_N is not a fuzzy neutrosophic dense set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic σ-nowhere dense set in a fuzzy neutrosophic P-space (X_N, T_N). Then by proposition 3.6, there exists a fuzzy neutrosophic F_σ-set δ_N in (X_N, T_N) such that $\lambda_N \leq \delta_N$. Then $1_N - \lambda_N \geq 1_N - \delta_N$. Since δ_N is a fuzzy neutrosophic F_σ-set, $1_N - \delta_N$ is a fuzzy neutrosophic G_δ-set in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic P-space $1_N - \delta_N$ is a fuzzy neutrosophic open set in (X_N, T_N). That is $int(1_N - \delta_N) = 1_N$. $- - - -(1)$. Now, $\lambda_N \leq \delta_N$ implies that $1_N - \lambda_N \geq 1_N - \delta_N$. Then $int(1_N - \lambda_N) \geq int(1_N - \delta_N)$ and hence $int(1_N - \lambda_N) \geq 1_N - \delta_N$ from(1).Since $1_N - \delta_N \neq 0_N, int(1_N - \lambda_N) \neq 0_N$. Then $1_N - cl(\lambda_N) \neq 0_N$. This implies that $cl(\lambda_N) \neq 1_N$. Hence if λ_N is a fuzzy neutrosophic σ-nowhere dense set in a fuzzy neutrosophic P-space, then $cl(\lambda_N) \neq 1_N$. Therefore λ_N is not a fuzzy neutrosophic dense set in (X_N, T_N). □

Remark-2:

The inter-relation between fuzzy neutrosophic σ-Baire space, fuzzy neutrosophic almost irresolvable spaces and fuzzy neutrosophic σ-second category spaces can be summarized as follows.

Fuzzy neutrosophic σ-Baire spaces

$\downarrow$

Fuzzy neutrosophic almost irresolvable spaces

$\downarrow$

Fuzzy neutrosophic σ-second category spaces

Proposition 4.10. *If the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic strongly irresolvable fuzzy neutrosophic σ-Baire space and λ_N is a fuzzy neutrosophic σ-first category set in (X_N, T_N) then λ_N is a fuzzy neutrosophic nowhere dense set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic σ-first category set in (X_N, T_N). Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic σ-nowhere dense sets in(X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic σ-Baire space. By theorem 2.28 $int(\lambda_N) = 0_N$ in (X_N, T_N). Then we have $1_N - int(\lambda_N) = 1_N$. This implies that $cl(1_N - \lambda_N) = 1_N$. Since (X_N, T_N) is a fuzzy neutrosophic strongly irresolvable space for the fuzzy neutrosophic dense set $(1_N - \lambda_N)$ in (X_N, T_N), we have $clint(1_N - \lambda_N) = 1_N$. Then $1_N - intcl(\lambda_N) = 1_N$ and hence $intcl(\lambda_N) = 0_N$. Therefore λ_N is a fuzzy neutrosophic nowhere dense set in (X_N, T_N). □

Proposition 4.11. *If λ_{N_i}'s are the fuzzy neutrosophic σ-first category sets in a fuzzy neutrosophic strongly irresolvable, fuzzy neutrosophic σ-Baire space (X_N, T_N), then $\vee_{i=1}^{\infty}(\lambda_{N_i})$ is a fuzzy neutrosophic first category set in (X_N, T_N).*

Proof. Let (λ_{N_i})'s be the fuzzy neutrosophic σ-first category sets in a fuzzy neutrosophic strongly irresolvable, fuzzy neutrosophic σ-Baire space (X_N, T_N). Then by proposition 4.10, (λ_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X_N, T_N) and hence $\vee_{i=1}^{\infty}(\lambda_{N_i})$ is a fuzzy neutrosophic first category set in (X_N, T_N). □

Proposition 4.12. *If (λ_{N_i})'s $(i = 1toN)$ are fuzzy neutrosophic σ-nowhere dense sets in a fuzzy neutrosophic topological space (X_N, T_N) and $int(\vee_{i=1}^{N}(\lambda_{N_i})) = 0_N$ then $cl[\wedge_{i=1}^{N}(\lambda_{N_i})] = 1_N$, where (λ_{N_i})'s are fuzzy neutrosophic dense and fuzzy neutrosophic G_δ-sets in (X_N, T_N).*

Proof. Let (λ_{N_i})'s $(i = 1\ to\ N)$ be fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Then (λ_{N_i})'s are fuzzy neutrosophic F_σ-sets with $int(\lambda_{N_i}) = 0_N (i = 1 to N)$. Now, $1_N - int(\lambda_{N_i}) = 1_N$, then we have $cl(1_N - \lambda_{N_i}) = 1_N$, that is $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic dense sets in (X_N, T_N). Since (λ_{N_i})'s are fuzzy neutrosophic F_σ-sets, $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic G_δ-sets in (X_N, T_N). Hence $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic dense and fuzzy neutrosophic G_δ-sets in (X_N, T_N). Now, $cl(\wedge_{i=1}^{N}(\lambda_{N_i})) = cl(1_N - [\vee_{i=1}^{N}(\lambda_{N_i})]) = 1_N - int[\vee_{i=1}^{N}(\lambda_{N_i})] = 1_N - 0_N = 1_N$ in (X_N, T_N). □

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ON FUZZY NEUTROSOPHIC PRE σ-BAIRE SPACES

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ABSTRACT
In this paper, the concepts of fuzzy neutrosophic pre σ-nowhere dense set, fuzzy neutrosophic pre σ-first category set and fuzzy neutrosophic pre σ-second category set in fuzzy neutrosophic topological spaces are introduced and studied. By means of fuzzy neutrosophic pre σ-nowhere dense sets, the concept of fuzzy neutrosophic pre σ-Baire space is defined and several characterizations of fuzzy neutrosophic pre σ-Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.
Keywords*:* Fuzzy neutrosophic pre-open(closed) set, fuzzy neutrosophic pre Fσ-set, fuzzy neutrosophic pre $G_\delta -$-set, fuzzy neutrosophic pre dense, fuzzy neutrosophic pre residual, fuzzy neutrosophic nowhere dense set, fuzzy neutrosophic pre σ-first and second category sets, fuzzy neutrosophic pre σ-Baire space.

I. INTRODUCTION

The fuzzy idea was invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [21]. The important concept of fuzzy topological space was offered by C.L. Chang [3]. The idea of fuzzy σ- Baire Spaces was introduced by G. Thangaraj and E. Poongothai [13]. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [20] introduced fuzzy neutrosophic topological spaces. The idea of fuzzy neutrosophic Baire spaces was introduced by E. Poongothai and E. Padmavathi [10]. In this paper, the concepts of fuzzy neutrosophic pre σ-nowhere dense set, fuzzy neutrosophic pre σ-first category set and fuzzy neutrosophic pre σ-second category set in fuzzy neutrosophic topological spaces are introduced and studied. By means of fuzzy neutrosophic pre σ-nowhere dense sets, the concept of fuzzy neutrosophic pre σ-Baire space is defined and several characterizations of fuzzy neutrosophic pre σ-Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

II. PRELIMINARIES

Definition 2.1 [2] A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = (x, T_A(x), I_A(x), F_A(x)),\ x \in X$ where $T, I, F : X \to [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.2 [2] A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B(i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x),\ F_A(x) \geq F_B(x)$.

Definition 2.3 [2] Let X be a non-empty set, and $A = (x, T_A(x), I_A(x), F_A(x)),\ B = (x, T_B(x), I_B(x), F_B(x))$ be two fuzzy neutrosophic sets. Then

$$A \cup B = (x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x)))$$
$$A \cap B = (x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x)))$$

Definition 2.4 [2] The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = (x, min(T_A(x), F_B(x)), min(I_A(x), 1— I_B(x)), max(F_A(x), T_B(x)))$.

Definition 2.5 [2] A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N.

Definition 2.6 [2] A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N.

Definition 2.7 [2] The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = (x, T_{A^c}(x), I_{A^c}(x), F_{A^c}(x))$ where $T_{A^c}(x) = F_A(x)$, $I_{A^c}(x) = 1 - I_A(x), F_{A^c}(x) = T_A(x)$. The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N —A$.

Definition 2.8 [1] A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X satisfying the following axioms.

(i) $0_N,\ 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1,\ A_2 \in \tau$
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$

In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [1] The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10 [1] Let (X, τ) be a fuzzy neutrosophic topological space and $A = (x, T_A(x), I_A(x), F_A(x))$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$int(A) = \cup\{G : G \text{ is a fuzzy neutrosophic open set in X and } G \subseteq A\}$$

$$cl(A) = \cap\{G : G \text{ is a fuzzy neutrosophic closed set in X and } A \subseteq G\}$$

Definition 2.11 [1] Let (X, τ) be a fuzzy neutrosophic topological space over X. Then the following properties hold, (i) $cl(A^C) = (int\ A)^C$, (ii) $int(A^C) = (cl\ A)^C$.

Definition 2.12 [10] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic F_σ-set if $A_N = \vee_{i=1}^{\infty}(A_{N_i})$, where $\overline{A_{N_i}} \in \tau_N$ ~~for~~ $i \in I$.

Definition 2.13 [10] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic G_δ-set in (P, τ_N) if $A_N = \wedge_{i=1}^{\infty} A_{N_i}$, where $A_{Ni} \in \tau_N$ for $i \in I$.

Definition 2.14 [10] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic dense if there exist no fnCS B_N in (P, τ_N) s.t $A_N \subset B_N \subset 1_X$. That is $fn(A_N)^- = 1_N$.

Definition 2.15 [10] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic nowh. dense set if there exist no non zero fnOS B_N in (P, τ_N) s.t $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$.

Definition 2.16 [10] Let (P, τ_N) be a fy. neutrosophic top. space. A fy. neutrosophic set A_N in (P, τ_N) is called fy. neutrosophic one category set if $A_N = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Any other fy. neutrosophic set in (P, τ_N) is said to be of fy. neutrosophic two category.

Definition 2.17 [10] A fy.neutrosophic top.space (P, τ_N) is called fy. neutrosophic one category space if the fy. neutrosophic set 1_X is a fy. Neutrosophic one category set in (P, τ_N). That is $1_X = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. Neutrosophic nowh. dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fy. neutrosophic two category space.

Definition 2.18 [10] Let A_N be a fy. neutrosophic one category set in (P, τ_N). Then $\overline{A_N}$ is called fy. neutrosophic re. set in (P, τ_N).

Definition 2.19 [10] A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic Baire space if $fn\left(\vee_{i=1}^{\infty}(A_{N_i})\right)^+ = 0_N$, where (A_{N_i})'s are fy. neutrosophic nowh. dense sets in (P, τ_N).

Theorem 2.1 [10] Let (P, τ_N) be a fy. neutrosophic top. space. Then the following are equivalent

(1) (P, τ_N) is a fy. neutrosophic Baire space.
(2) $fn(A_N)^+ = 0_N$, for every fy. neutrosophic one category set A_N in (P, τ_N).
(3) $Fn(B_N)^+ = 1_N$, for every fy. neutrosophic re. set B_N in (P, τ_N).

Definition 2.20 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic Set λ_N in (X_N, T_N) is called a fuzzy neutrosophic σ-nowhere dense set if λ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N) such that $int(\lambda_N) = 0_N$.

Definition 2.21 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set λ_N in (X_N, T_N) is called fuzzy neutrosophic σ- first category set if $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N).Any other fuzzy neutrosophic set in (X_N, T_N) is said to be fuzzy neutrosophic σ- second category sets in (X_N, T_N).

Definition 2.22 [5] Let λ_N be a fuzzy neutrosophic σ-first category set in (X_N, T_N). Then $1_N - \lambda_N$ is called a fuzzy neutrosophic σ-residual set in (X_N, T_N).

Definition 2.23 [5] A fuzzy neutrosophic topological space (X_N, T_N) is called fuzzy neutrosophic σ-first category space if the fuzzy neutrosophic set 1_{X_N} is a fuzzy neutrosophic σ-first category set in (X_N, T_N). That is $1_{X_N} = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic σ- nowhere dense sets in (X_N, T_N). Otherwise (X_N, T_N) will be called a fuzzy neutrosophic σ- second category space.

Definition 2.24 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then (X_N, T_N) is called a fuzzy neutrosophic σ-Baire space if $int(\vee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic σ- nowhere dense sets in (X_N, T_N).

Theorem 2.2 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then the following are equivalent

(1) (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.
(2) $int(\lambda_N) = 0_N$, for every fuzzy neutrosophic σ- first category set λ_N in (X_N, T_N).

(3) $cl(\mu_N) = 1_N$, for every fuzzy neutrosophic σ-residual set μ_N in (X_N, T_N).

Definition 2.25 [12] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic submaximal space if for each fuzzy neutrosophic set A_N in (X, τ_N) such that $(A_N)^- = 1$ then $A_N \in \tau_N$ in (X, τ_N). That is (X, τ_N) is a fuzzy neutrosophic submaximal space if each fuzzy neutrosophic dense set in (X, τ_N) is a fuzzy neutrosophic open set in (X, τ_N).

Definition 2.26 [12] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic resolvable space if there exist a fuzzy neutrosophic dense set A_N in (X, τ_N) such that $(1-A_N)^- = 1$. Otherwise, (X, τ_N) is called a fuzzy neutrosophic irresolvable space.

Definition 2.27 [12] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic hyperconnected space if every non-null fuzzy neutrosophic open subset of (X, τ_N) is fuzzy neutrosophic dense in (X, τ_N).

Definition 2.28 [12] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic P-space if each fuzzy neutrosophic G_δ-set in (X, τ_N) is fuzzy neutrosophic open set in (X, τ_N).

Definition 2.29 [12] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic almost resolvable space if $\vee_{i=1}^{\infty}(A_{N_i}) = 1$, where (A_{N_i})'s in (X, τ_N) are such that $(A_{N_i})^+ = 0$, otherwise, (X, τ_N) is called a fuzzy neutrosophic almost irresolvable space.

Definition 2.30 [4] FNS λ_N in FNTS (X, τ) is called Fuzzy neutrosophic regular-open set (Briefly, FNR-open) if $\lambda_N = FNInt(FNcl(\lambda_N))$

Definition 2.31 [4] FNS λ_N in FNTS (X, τ) is called Fuzzy neutrosophic regular-closed set (Briefly, FNR-closed) if $\lambda_N = FNcl(FNInt(\lambda_N))$.

Definition 2.32 [4] Fuzzy neutrosophic pre-open set (Briefly, FNP-open) if $\lambda_N \subseteq FNInt(FNCl(\lambda_N))$.

Definition 2.33 [4] Fuzzy neutrosophic pre-closed set(Briefly, FNP-closed) if $FNCl(FNInt(\lambda_N)) \wedge (\lambda_N)$.

III FUZZY NEUTROSOPHIC PRE σ-NOWHERE DENSE SETS

Definition 3.1 A fuzzy neutrosophic set λ_N in a fuzzy neutrosophic topological space (X_N, T_N) is called a fuzzy neutrosophic pre F_σ-set in (X_N, T_N) if $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$ where (λ_{N_i})'s are fuzzy neutrosophic pre-closed sets in (X_N, T_N).

Definition 3.2 A fuzzy neutrosophic set λ_N in a fuzzy neutrosophic topological space (X_N, T_N) is called a fuzzy neutrosophic pre G_δ-set in (X_N, T_N) if $\lambda_N = \wedge_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic pre-open sets in (X_N, T_N).

Definition 3.3 A fuzzy neutrosophic set λ_N in a fuzzy neutrosophic topological space (X_N, T_N) is called a fuzzy neutrosophic pre dense if there exist no fuzzy neutrosophic pre-closed set, where μ_N in (X_N, T_N) such that $\lambda_N < \mu_N < 1_N$. That is, $pcl(\lambda_N) = 1_N$ in (X_N, T_N).

Definition 3.4 A fuzzy neutrosophic set λ_N in a fuzzy neutrosophic topological space (X_N, T_N) is called a fuzzy neutrosophic pre σ-nowhere dense set if λ_N is a non-zero fuzzy neutrosophic pre F_σ-set in (X_N, T_N) such that $pint(\lambda_N) = 0_N$.

Example 3.1 Let $X_N = \{a, b, c\}$. The fuzzy neutrosophic sets λ_N , μ_N and γ_N are defined on X_N as follows:

$\lambda_N : X_N \to [0_N, 1_N]$ is defined as,

$\lambda_N = \{(a, (0.7, 0.6, 0.7)), (b, (0.5, 0.6, 0.5)), (c, (0.7, 0.7, 0.6))\}$

$\mu_N : X_N \to [0_N, 1_N]$ is defined as,

$\mu_N = \{(a, (0.5, 0.4, 0.8)), (b, (0.7, 0.6, 0.5)), (c, (0.8, 0.6, 0.6))\}$

$\gamma_N : X_N \to [0_N, 1_N]$ is defined as,

$\gamma_N = \{(a, (0.6, 0.7, 0.5)), (b, (0.5, 0.6, 0.6)), (c, (0.7, 0.6, 0.6))\}$

Then, $T_N = \{0_N , \lambda_N , \mu_N , \gamma_N , \lambda_N \vee \mu_N , \mu_N \vee \gamma_N , \lambda_N \vee \gamma_N , \lambda_N \wedge \mu_N , \mu_N \wedge \gamma_N , \lambda_N \wedge \gamma_N , 1_N \}$ is a fuzzy neutrosophic topology on X_N . The fuzzy neutrosophic set $\beta_N = [(1_N - \lambda_N) \vee (1_N - \mu_N) \vee (1_N - (\lambda_N \vee \mu_N))]$ in (X_N , T_N). Then β_N is a fuzzy neutrosophic pre F_σ-set in (X_N , T_N) and $pint(\beta_N) = 0_N$ and hence β_N is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N).

Example 3.2 Let $X_N = \{a, b\}$.The fuzzy neutrosophic sets α_N and β_N are defined on X_N as follows:

$\alpha_N : X_N \to [0_N, 1_N]$ is defined as $\alpha_N = \{(a, (0.4, 0.3, 0.4)), (b, (0.5, 0.4, 0.3))\}$

$\beta_N : X_N \to [0_N, 1_N]$ is defined as $\beta_N = \{(a, (0.5, 0.3, 0.3)), (b, (0.4, 0.3, 0.5))\}$

Then, $T_N = \{0_N, \alpha_N, \beta_N, \alpha_N \vee \beta_N, \alpha_N \wedge \beta_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. Now, consider

$\eta_N = [(1_N - (\alpha_N \vee \beta_N)) \vee (1_N - (\alpha_N \wedge \beta_N))]$

$\eta_N = 1_N - (\alpha_N \wedge \beta_N)$

Therefore η_N is a fuzzy neutrosopic pre F_σ-set in (X_N , T_N). $pint(\eta_N) \neq 0_N$ .Therefore η_N is not a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N).

Remark 3.1 If λ_N and μ_N are fuzzy neutrosophic pre σ-nowhere dense sets in a fuzzy neutrosophic topological space (X_N , T_N), then $\lambda_N \vee \mu_N$ be a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N). For, consider the following example:

Example 3.3 Let $X_N = \{a, b, c\}$. The fuzzy neutrosophic sets A_N , B_N and C_N are defined X_N as follows:

$A_N : X_N \to [0_N, 1_N]$ is defined as,

$A_N = \{(a, (0.7, 0.6, 0.6)), (b, (0.7, 0.5, 0.6)), (c, (0.5, 0.7, 0.6))\}$

$B_N : X_N \to [0_N, 1_N]$ is defined as,

$B_N = \{(a, (0.5, 0.4, 0.5)), (b, (0.4, 0.7, 0.6)), (c, (0.8, 0.5, 0.6))\}$

$C_N : X_N \to [0_N, 1_N]$ is defined as,

$C_N = \{(a, (0.7, 0.6, 0.6)), (b, (0.6, 0.5, 0.5)), (c, (0.5, 0.4, 0.5))\}$

Then, $T_N = \{0_N , A_N , B_N , C_N , A_N \vee B_N , B_N \vee C_N , A_N \vee C_N , A_N \wedge B_N , B_N \wedge C_N , A_N \wedge C_N , A_N \vee B_N \vee C_N , A_N \wedge B_N \wedge C_N , 1_N\}$ is clearly a fuzzy neutrosophic topology on X_N .

Now, consider, $\alpha_N = [(1_N - A_N) \vee (1_N - B_N) \vee (1_N - C_N)] = [1_N - (A_N \wedge B_N)]$

Therefore α_N is a fuzzy neutrosophic pre F_σ-set in (X_N, T_N).

$$\beta_N = [(1_N - (\beta_N \vee C_N)) \vee (1_N - (A_N \vee C_N)) \vee (1_N - (A_N \wedge B_N))] = [1_N - (A_N \wedge B_N)]$$

$pint(\alpha_N) = 0_N$, which implies that α_N is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N). Then $(1_N - C_N) \vee (1_N - (A_N \vee B_N \vee C_N)) = \gamma_N$ is a fuzzy neutrosophic pre F_σ-set in (X_N, T_N). $pint(\gamma_N) = 0_N$ is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N). $pint(\alpha_N \vee \gamma_N) = 0_N$ is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N).

Proposition 3.1. A fuzzy neutrosophic set λ_N is a fuzzy neutrosophic pre σ- nowehre dense set in a fuzzy neutrosophic toplogical space (X_N, T_N) if and only if $(1_N - \lambda_N)$ is a fuzzy neutrosophic pre dense and fuzzy neutrosophic pre G_δ-set in (X_N, T_N).

Proof. Let λ_N be a fuzzy neutrosophic pre σ-nowhre dense set (X_N, T_N). Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$ where (λ_{N_i})'s are fuzzy neutrosophic pre-closed sets in (X_N, T_N) and $pint(\lambda_N) = 0_N$. Then $1_N - pint(\lambda_N) = 1_N - 0_N = 1_N$ and hence $pcl(1_N - \lambda_N) = 1_N$. Also $(1_N - \lambda_N) = 1_N - \vee_{i=1}^{\infty}(\lambda_{N_i}) = \wedge_{i=1}^{\infty}(1_N - \lambda_{N_i})$, where $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic pre-open sets in (X_N, T_N), implies that $1_N - \lambda_N$ is a fuzzy neutrosophic pre G_δ-set in (X_N, T_N). Hence $(1_N - \lambda_N)$ is a fuzzy neutrosophic pre dense and fuzzy neutrosophic pre G_δ-set in (X_N, T_N).

Conversely,Let λ_N be a fuzzy neutrosophic pre dense and fuzzy neutrosophic pre G_δ-set in (X_N, T_N). Then $\lambda_N = \wedge_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy pre-open sets in (X_N, T_N). Now, $1_N - \lambda_N = 1_N - \wedge_{i=1}^{\infty}(\lambda_{N_i}) = \vee_{i=1}^{\infty}(1_N - \lambda_{N_i})$, where $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic pre-closed sets in (X_N, T_N). Hence $(1_N - \lambda_N)$ is a fuzzy neutrosophic pre F_σ-set in (X_N, T_N) and $pint(1_N - \lambda_N) = 1_N - pcl(\lambda_N) = 1_N - 1_N = 0_N$. [Since λ_N is a fuzzy neutrosophic pre dense in (X_N, T_N)]. Therefore $(1_N - \lambda_N)$ is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N).

Proposition 3.2 If λ_N is a fuzzy neutrosophic pre dense set in a fuzzy neutrosophic topological space (X_N, T_N) such that $\mu_N \leq (1_N - \lambda_N)$, where μ_N is a fuzzy neutrosophic pre F_σ-set in (X_N, T_N), then μ_N is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N).

Proof. Let λ_N be a fuzzy neutrosophic pre dense set in (X_N, T_N) such that $\mu_N \leq (1_N - \lambda_N)$. Now, $\mu_N \leq (1_N - \lambda_N)$, implies that $pint(\mu_N) \leq pint(1_N - \lambda_N)$. Then $pint(\mu_N) \leq 1_N - pcl(\lambda_N) = 1_N - 1_N = 0_N$ and hence $pint(\mu_N) = 0_N$. Therefore, μ_N is a fuzzy neutrosophic pre F_σ-set in (X_N, T_N) such that $pint(\mu_N) = 0_N$ and hence μ_N is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N).

Definition 3.5 Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set λ_N in (X_N, T_N) is called fuzzy neutrosophic pre σ-first category set if $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic pre σ-nowhere dense sets in (X_N, T_N). Any other fuzzy neutrosophic set in (X_N, T_N) is said to be of fuzzy neutrosophic pre σ-second category set in (X_N, T_N).

Definition 3.6 Let λ_N be a fuzzy neutrosophic pre σ-first category set in (X_N, T_N). Then $(1_N - \lambda_N)$ is called a fuzzy neutrosophic pre σ-residual set in (X_N, T_N).

Definition 3.7 A fuzzy neutrosophic topological space (X_N, T_N) is called fuzzy neutrosophic pre σ-first category space is the fuzzy neutrosphic set 1_{X_N} if a fuzzy neutrosophic pre σ-first category set in (X_N, T_N). That is, $1_{X_N} = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic pre σ-nowhere dense sets in (X_N, T_N). Otherwise, (X_N, T_N) will be called a fuzzy neutrosophic pre σ-second category space.

Proposition 3.3 If λ_N is a fuzzy neutrosophic pre σ-first category set in a fuzzy neutrosophic topological space (X_N, T_N), then there is a fuzzy neutrosophic pre F_σ- set δ_N in (X_N, T_N) such that $\lambda_N \leq \delta_N$.

Proof. Let λ_N be a fuzzy neutrosophic pre σ-first category set in (X_N , T_N). Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic pre σ-nowhere dense sets in (X_N , T_N). Now, $[1_N - pcl(\lambda_{N_i})]'s$ arefuzzy neutrosophic pre-open sets in (X_N , T_N). Then $\mu_N = \wedge_{i=1}^{\infty}[1_N - pcl(\lambda_{N_i})]$ is a fuzzy neutrosophic pre $G_\delta -$ set in (X_N , T_N) and $1_N - \mu_N = 1_N - \left[\wedge_{i=1}^{\infty}\left(1_N - pcl(\lambda_{N_i})\right)\right] = \left[\vee_{i=1}^{\infty} pcl(\lambda_{N_i})\right]$. Now, $\lambda_{N_i} \leq pcl(\lambda_{N_i})$, implies that $\vee_{i=1}^{\infty}(\lambda_{N_i}) \leq \left[\vee_{i=1}^{\infty} pcl(\lambda_{N_i})\right]$. Hence $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i}) \leq \left[\vee_{i=1}^{\infty} pcl(\lambda_{N_i})\right] = [1_N - \mu_N]$. That is, $\lambda_N \leq [1_N - \mu_N]$and $[1_N - \mu_N]$ is a fuzzy neutrosophic pre $F_\sigma -$ set in (X_N , T_N). Let$\delta_N = [1_N - \mu_N]$. Hence, if λ_N is a fuzzy neutrosophic pre $\sigma -$ first category set in (X_N , T_N), then there is a fuzzy neutrosophic pre $F_\sigma -$ set in δ_N such that (X_N, T_N).

Proposition 3.4 If λ_N is a fuzzy neutrosophic pre σ-first category set in a fuzzy neutrosophic topological space (X_N , T_N), then there is a fuzzy neutrosophic pre F_σ- set δ_N in (X_N , T_N) such that $\lambda_N \leq \delta_N \leq cl(\lambda_N)$, where δ_N is a fuzzy neutrosophic pre F_σ-set in (X_N, T_N).

Proof. Let λ_N be a fuzzy neutrosophic pre σ-first category set in (X_N , T_N). Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic pre σ-nowhere dense sets in (X_N , T_N). Now, $[1_N - pcl(\lambda_{N_i})]$'s (i = 1 to ∞) are fuzzy neutrosophic pre- open sets in (X_N , T_N). Then, $\mu_N = \wedge_{i=1}^{\infty}(1_N - pcl(\lambda_{N_i}))$ is a fuzzy neutrosophic pre G_δ-set in (X_N , T_N) and $1_N - \mu_N = 1_N - [\wedge_{i=1}^{\infty}(1_N - pcl(\lambda_{N_i}))] = [\vee_{i=1}^{\infty} pcl(\lambda_{N_i})]$. Now, $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i}) \leq \vee_{i=1}^{\infty} pcl(\lambda_{N_i})] \leq \vee_{i=1}^{\infty}(cl(\lambda_{N_i})) \leq cl(\vee_{i=1}^{\infty}(\lambda_{N_i}))$. That is, $\lambda_N \leq [1_N - \mu_N] \leq cl(\lambda_N)$ and $[1_N - \mu_N]$ is a fuzzy neutrosophic pre F_σ-set in (X_N, T_N). Let $\delta_N = [1_N - \mu_N]$. Hence, if λ_N is a fuzzy neutrosophic pre σ-first category set in (X_N , T_N) such that then there is a fuzzy neutrosophic pre F_σ-set δ_N in (X_N , T_N) $\lambda_N \leq \delta_N \leq cl(\lambda_N)$, where δ_N is a fuzzy neutrosophic pre F_σ-set in (X_N, T_N).

Proposition 3.5 If λ_N is a fuzzy neutrosophic pre-closed set in a fuzzy neutrosophic topological space (X_N , T_N) and if $pint(\lambda_N) = 0_N$, then λ_N is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N).

Proof. Let λ_N be a fuzzy neutrosophic pre-closed set in (X_N , T_N). Then we have $pcl(\lambda_N) = \lambda_N$. Now, $pint[pcl(\lambda_N)] = pint(\lambda_N)$ and $pint(\lambda_N) = 0_N$, implies that λ_N is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N).

Proposition 3.6 If λ_N is a fuzzy neutrosophic closed and fuzzy neutrosophic σ-nowehre dense set in a fuzzy neutrosophic topological space (X_N , T_N), then $pint(\lambda_N) = 0_N$ in (X_N , T_N).

Proof. Let λ_N be a fuzzy neutrosophic σ-nowhere dense set in (X_N , T_N). Then λ_N is a fuzzy neutrosophic F_σ-set such that $int(\lambda_N) = 0_N$. We have, $pint(\lambda_N) \leq \lambda_N \wedge intcl(\lambda_N)$. Then, $pint(\lambda_N) \leq \lambda_N \wedge int(\lambda_N)$.[Since λ_N is a fuzzy neutrosophic closed set, $\lambda_N = cl(\lambda_N)$] and hence $pint(\lambda_N) \leq \lambda_N \wedge 0_N$. That is, $pint(\lambda_N) = 0_N$ in (X_N , T_N).

Proposition 3.7 If each fuzzy neutrosophic σ-nowhere dense set λ_N is a fuzzy neutrosophic closed set in a fuzzy neutrosophic topological space (X_N , T_N), then λ_N is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N).

Proof. Let λ_N be a fuzzy neutrosophic σ-nowhere dense set in (X_N , T_N). Then λ_N is a fuzzy neutrosopohic F_σ-set in (X_N , T_N) such that $int(\lambda_N) = 0_N$. We have, $pint(\lambda_N) \leq \lambda_N \wedge intcl(\lambda_N)$. Since λ_N is a fuzzy neutrosophic closed set in (X_N , T_N), $cl(\lambda_N) = \lambda_N$. Then $pint(\lambda_N) \leq \lambda_N \wedge int(\lambda_N)$. That is, $pint(\lambda_N) \leq \lambda_N \wedge 0_N = 0_N$. Hence, $pint(\lambda_N) = 0_N$ and therefore λ_N is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N).

IV FUZZY NEUTROSOPHIC PRE Σ-BAIRE SPACES

Definition 4.1 Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then (X_N, T_N) is called a fuzzy neutrosophic pre σ-Baire Space if $pint(\vee_{i=1}^{\infty}(\lambda_{Ni}) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic pre σ −nowhere dense sets in (X_N, T_N).

Example 4.1 Let $X_N = \{a, b, c\}$. The fuzzy neutrosophic sets A_N, B_N and C_N are defined on X_N as follows:

$A_N : X_N \to [0_N, 1_N]$ is defined as,

$A_N = \{(a, (0.7, 0.6, 0.5)), (b, (0.5, 0.6, 0.8)), (c, (0.7, 0.5, 0.6))\}$

$B_N : X_N \to [0_N, 1_N]$ is defined as,
$B_N = \{(a, (0.6, 0.5, 0.7)), (b, (0.6, 0.6, 0.7)),)c, (0.5, 0.7, 0.5))\}$

$C_N : X_N \to [0_N, 1_N]$ is defined as,

$C_N = \{(a, (0.5, 0.5, 0.7)), (b, (0.7, 0.5, 0.5)), (c, (0.6, 0.5, 0.7))\}$

Then, $T_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, A_N \vee C_N, B_N \vee C_N, A_N \wedge B_N, A_N \wedge C_N, B_N \wedge C_N, A_N \wedge B_N \wedge C_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. Now,

$\alpha_N = [(1_N - B_N) \vee (1_N - C_N) \vee (1_N - (A_N \vee B_N))] = [1_N - (B_N \wedge C_N)]$ is a fuzzy neutrosophic pre F_σ-set in (X_N, T_N).

$pint(\alpha_N) = 0_N$, α_N is a fuzzy neutrosophic pre σ-nowhere dense set in (X_N, T_N). $\beta_N = [(1_N - (A_N \wedge B_N)) \vee (1_N - A_N \wedge C_N) \vee (1_N - (B_N \wedge C_N))]$ is a fuzzy neutrosophic pre F_σ-set in (X_N, T_N). $pint(\beta_N) = 0_N$, β_N is a fuzzy neutrosopic pre σ-nowhere dense set. $pint(\alpha_N \vee \beta_N) = 0_N$, then (X_N, T_N) is a fuzzy neutrosophic pre σ-Baire space.

Proposition 4.1 Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then the following are equivalent:

(1) (X_N, T_N) is a fuzzy neutrosophic pre σ-Baire Space.
(2) $pint(\lambda_N) = 0_N$, for each fuzzy neutrosophic pre σ-first category set in (X_N, T_N).
(3) $pcl(\mu_N) = 1_N$, for each fuzzy neutrosophic pre σ-residual set μ_N in (X_N, T_N).

Proof. (1) ⇒ (2)

Let λ_N be a fuzzy neutrosophic σ-first catergory set in (X_N, T_N). Then $\lambda_N = \vee_{I=1}^{\infty}(\lambda_{N_i})$,
Where $(\lambda_{N_i})'s$ are fuzzy neutrosophic pre σ −nowhere dense sets in (X_N, T_N). Then, $pint(\lambda_N) = pint\left(\vee_{i=1}^{\infty}(\lambda_{N_i})\right)$. Since (X_N, T_N) is a fuzzy neutrosophic σ-Baire space, $pint\left(\vee_{i=1}^{\infty}(\lambda_{N_i})\right) = 0_N$. Hence, $pint(\lambda_N) = 0_N$ for a fuzzy neutrosophic pre σ −first category set λ_N in (X_N, T_N).

(2) ⟹ (3)

Let μ_N be a fuzzy neutrosophic pre σ-residual set μ_N in (X_N, T_N). Then $(1_N - \mu_N)$ is a fuzzy neutrosophic pre σ-first category set in (X_N, T_N). By hypothesis, $pint(1_N - \lambda_N) = 0_N$. Then, $1_N - pcl(\mu_N) = 0_N$. Hence, $pcl(\mu_N) = 1_N$, for a fuzzy neutrosophic pre σ-residual set μ_N in (X_N, T_N).

(3)⟹ (1)

Let λ_N be a fuzzy neutrosophic σ-first category set in (X_N , T_N). Then $\lambda_N = (\vee_{i=1}^{\infty} (\lambda_{N_i}))$, where (λ_{N_i})'s are fuzzy neutrosophic pre σ-nowhere dense sets in (X_N , T_N). Now, λ_N is a fuzzy neutrosophic pre σ-first category set in (X_N , T_N), implies that $(1_N — \lambda_N)$ is a fuzzy neutrosophic pre σ-residual set in (X_N , T_N). By hypothesis, $pcl(1_N — \lambda_N) = 1_N$. Then, $1_N — pint(\lambda_N) = 1_N$. Hence $pint(\lambda_N) = 0_N$. That is, $pint(\vee_{i=}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic pre σ-nowhere dense sets in (X_N , T_N). Hence (X_N , T_N) is a fuzzy neutrosophic pre σ-Baire space.

Proposition 4.2 If the fuzzy neutrosophic topological space (X_N , T_N) is a fuzzy neutrosophic pre σ-Baire space, then (X_N , T_N) is a fuzzy neutrosophic pre σ-second category space.

Proof. Let (X_N , T_N) be a fuzzy neutrosophic pre σ-Baire space. Then, $int(\vee_{i=1}^{\infty} (\lambda_{N_i})) = 0_N$, where (λ_N)'s are fuzzy neutrosophic pre σ-nowhere dense sets in (X_N , T_N). Then $\vee_{i=1}^{\infty}(\lambda_{N_i}) \neq 1_{X_N}$ [Otherwise, $\vee_{i=1}^{\infty} (\lambda_{N_i}) = 1_{X_N}$, implies that $pint(\vee_{i=1}^{\infty} (\lambda_{N_i})) = pint(1_{X_N}) = 1_{X_N}$, which in turn implies that $0_N = 1_N$, a contradiction]. Hence (X_N , T_N) is a fuzzy neutrosophic pre σ-second category space.

Proposition 4.3 Let (X_N , T_N) be a fuzzy neutrosophic topological space.If $\wedge_{i=1}^{\infty} (\lambda_{N_i}) \neq 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic pre dense and fuzzy neutrosophic pre G_δ-sets in (X_N, T_N), then (X_N, T_N) is a fuzzy neutrosophic preσ-second category space.

Proof. Given that $\wedge_{i=1}^{\infty} (\lambda_{N_i}) \neq 0_N$, implies that $1_N — \wedge_{i=1}^{\infty} (\lambda_{N_i}) \neq 1_N — 0_N = 1_N$. Then $\vee_{i=1}^{\infty}(1_N — \lambda_N) \neq 1_N$. Since (λ_N)'s are fuzzy neutrosophic pre dense and fuzzy neutrosophic pre G_δ-set in (X_N , T_N), by proposition 3.1., $(1_N — \lambda_{Ni})$'s are fuzzy neutrosophic pre σ-nowhere dense sets in (X_N , T_N). Hence, $(1_N — \lambda_N) \neq 1_N$, where $(1_N — \lambda_{Ni})$'s are fuzzy neutrosophic pre σ-nowhere dense sets in (X_N , T_N). Hence (X_N , T_N) is not a fuzzy neutrosophic pre σ-first category space. Therefore (X_N, T_N) is a fuzzy neutrosophic pre σ-second category space.

Proposition 4.4 If a fuzzy neutrosophic topological space (X_N , T_N) is a fuzzy neutrosophic pre σ-Baire space, then no non-zero fuzzy neutrosophic pre-open set is a fuzzy neutrosophic σ-first category set in (X_N, T_N).

Proof. Let λ_N be a non-zero fuzzy neutrosophic pre-open set in a fuzzy pre σ- Baire space (X_N , T_N). Suppose that $\lambda_N = \vee_{i=1}^{\infty} (\lambda_{N_i})$, where the fuzzy neutrosophic sets (λ_{Ni})'s are fuzzy neutrosophic pre σ-nowhere de sets in (X_N , T_N). Then $pint(\lambda_N) = pint(\vee_{i=1}^{\infty} (\lambda_{N_i}))$. Since (X_N , T_N) is a fuzzy neutrosophic pre σ-Baire space, $pint(\vee_{i=1}^{\infty} (\lambda_{N_i})) = 0_N$. This implies that, $pint(\lambda_N) = 0_N$. Then we will have $\lambda_N = pint(\lambda_N) = 0_N$, a contradiction. Since λ_N is a non-zero fuzzy neutrosophic set in (X_N , T_N). Hence no non-zero fuzzy neutrosophic pre-open set is a fuzzy neutrosophic pre σ-first category set in (X_N, T_N).

V CONCLUSION

In this study, we have introduced and analyzed the concept of fuzzy neutrosophic pre σ-Baire Spaces, extending classical and fuzzy Baire space theories into the neutrosophic framework. The work lays a foundation for further exploration of fuzzy neutrosophic spaces in advanced topology, particularly in applications.involving decision-making, artificial intelligence and Information systems, where vagueness and indeterminacy play a critical role.

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FUZZY NEUTROSOPHIC REGULAR σ−BAIRE SPACES

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Abstract. In this paper, the concepts of fuzzy neutrosophic regular $\sigma-$nowhere dense sets are introduced and studied. By means of fuzzy neutrosophic regular $\sigma-$nowhere dense sets, the concepts of fuzzy neutrosophic regular $\sigma-$Baire spaces are introduced and several characterizations of fuzzy neutrosophic regular $\sigma-$Baire spaces are studied.

Keywords: Fuzzy neutrosophic dense set, Fuzzy neutrosophic F_σ-set, Fuzzy neutrosophic G_δ-set, Fuzzy neutrosophic Regular $\sigma-$nowhere dense set,Fuzzy neutrosophic Regular $\sigma-$ Baire Space and Fuzzy neutrosophic Regular $\sigma-$first and second category.

1. Introduction

The fuzzy idea was invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [17]. The important concept of fuzzy topological space was offered by C.L. Chang [3]. The idea of fuzzy σ- Baire Spaces was introduced by G. Thangaraj and E. Poongothai [10]. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [16] introduced fuzzy neutrosophic topological spaces. The idea of fuzzy neutrosophic Baire spaces was introduced by E. Poongothai and E. Padmavathi [7].

In this paper, the concepts of fuzzy neutrosophic regular $\sigma-$nowhere dense sets are introduced and studied. By means of fuzzy neutrosophic regular $\sigma-$nowhere dense sets, the concepts of fuzzy neutrosophic regular $\sigma-$Baire spaces are introduced and several characterizations of fuzzy neutrosophic regular $\sigma-$Baire spaces are studied.

2. Preliminaries

Definition 2.1. [2] A fy. neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ x \in X$ where $T, I, F : X \to [0,1]$ and $0 \leq T_A(x) + I_A(x), F_A(x) \leq 3$. With the condition $0 \leq T_{A^*}(x) + I_{A^*}(x), F_{A^*}(x) \leq 2$

Definition 2.2. [2] A fy. neutrosophic set A is a subset of a fy. neutrosophic set B(i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x),\ F_A(x) \geq F_B(x)$

Definition 2.3. [2] Let X be a non-empty set, and
$A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fy. neutrosophic sets. Then

$$\begin{aligned} A \cup B &= \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle \\ A \cap B &= \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle \end{aligned}$$

Definition 2.4. [2] The difference between two fy. neutrosophic sets A and B is defined as $A \backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$

Definition 2.5. [2] A fy. neutrosophic set A over the universe X is said to be null or empty fy. neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N

Definition 2.6. [2] A fy. neutrosophic set A over the universe X is said to be absolute (universe) fy. neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N

Definition 2.7. [2] The complement of a fy. neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x),\ I_{A^C}(x) = 1 - I_A(x),\ F_{A^C}(x) = T_A(x)$. The complement of fy. neutrosophic set A an also be defined as $A^C = 1_N - A$.

Definition 2.8. [2] A fy. neutrosophic topology on a non-empty set X is a τ of fy. neutrosophic sets in X satisfying the following axioms.

(i) $0_N,\ 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1,\ A_2 \in \tau$
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$

In this case the pair (X, τ) is called fy. neutrosophic top. space and any fy. neutrosophic set in τ is known as fy. neutrosophic open set in X

Definition 2.9. [1] The complement of a fy. neutrosophic set A in a fy. neutrosophic top. space (X, τ) is called fy. neutrosophic closed set in X.

Definition 2.10. [1] Let (X, τ) be a fy. neutrosophic top. space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fy. neutrosophic set in X. Then the closure and interior of

A are defined by

$$\begin{aligned} int(A) &= \cup\{G : G \text{ is a } fuzzy \text{ neutrosophic open set in} X \text{ and } G \subseteq A\} \\ cl(A) &= \cap\{G : G \text{ is a } fuzzy \text{ neutrosophic closed set in} X \text{ and } A \subseteq G\} \end{aligned}$$

Definition 2.11. [1] Let (X, τ) be a fuzzy neutrosophic topological space over X. Then (i) $cl(A^C) = (int\ A)^C$, (ii) $int(A^C) = (cl\ A)^C$

Definition 2.12. [7] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic F_σ-set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$

Definition 2.13. [7] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic G_δ-set in (P, τ_N) if $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$

Definition 2.14. [7] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic dense if there exist no $fnCS$ B_N in (P, τ_N) s.t $A_N \subset B_N \subset 1_X$. That is $fn(A_N)^- = 1_N$

Definition 2.15. [7] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic nowh. dense set if there exist no non zero $fnOS$ B_N in (P, τ_N) s.t $B_N \subset fn(A_N)^-$. That is $fn(((A_N)^-)^+) = 0_N$

Definition 2.16. [7] Let (P, τ_N) be a fy. neutrosophic top. space. A fy. neutrosophic set A_N in (P, τ_N) is called fy. neutrosophic one category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Any other fy. neutrosophic set in (P, τ_N) is said to be of fy. neutrosophic two category.

Definition 2.17. [7] A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic one category space if the fy. neutrosophic set 1_X is a fy. neutrosophic one category set in (P, τ_N). That is $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fy. neutrosophic two category space.

Definition 2.18. [4] Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic Set λ_N in (X_N, T_N) is called a fuzzy neutrosophic σ-nowhere dense set if λ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N) such that $int(\lambda_N) = 0_N$.

Definition 2.19. [4] Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set λ_N in (X_N, T_N) is called fuzzy neutrosophic σ- first category if $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$ Where λ_{N_i}'s are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Any other fuzzy neutrosophic set in (X_N, T_N) is said to be fuzzy neutrosophic σ- second category in (X_N, T_N).

Definition 2.20. [4] Let λ_N be a fuzzy neutrosophic σ-first category set in (X_N, T_N). Then $1_N - \lambda_N$ is called a fuzzy neutrosophic σ-residual set in (X_N, T_N).

Definition 2.21. [4] A fuzzy neutrosophic topological space (X_N, T_N) is called fuzzy neutrosophic σ-first category if the fuzzy neutrosophic set 1_{X_N} is a fuzzy neutrosophic set σ-first category set in (X_N, T_N). That is $1_{X_N} = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$ Where λ_{N_i}'s are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Otherwise (X_N, T_N) will be called a fuzzy neutrosophic σ-second category space.

Definition 2.22. [4] Let (X_N, T_N) be a fuzzy neutrosophic topological space then (X_N, T_N) is called a fuzzy neutrosophic σ-Baire space if $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where λ_{N_i}'s are fuzzy neutrosophic σ- nowhere dense sets in (X_N, T_N).

Theorem 2.23. *[5] If λ_N is a fuzzy neutrosophic $\sigma-$nowhere dense set in a fuzzy neutrosophic $P-$space (X_N, T_N), then λ_N is not a fuzzy neutrosophic dense set in (X_N, T_N).*

Theorem 2.24. *[7] Let (P, τ_N) be a fy. neutrosophic top. space. Then the following are equivalent*

(1) *(P, τ_N) is a fy. neutrosophic Baire space.*
(2) *$fn(A_N)^+ = 0_N$, for every fy. neutrosophic one category set A_N in (P, τ_N).*
(3) *$fn(B_N)^+ = 1_N$, for every fy. neutrosophic re. set B_N in (P, τ_N).*

Definition 2.25. [7] Let A_N be a fy. neutrosophic one category set in (P, τ_N). Then $\bar{A_N}$ is called fy. neutrosophic re. set in (P, τ_N).

Definition 2.26. [7] A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic Baire space if $fn(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0_N$, where (A_{N_i})'s are fy. neutrosophic nowh. dense sets in (P, τ_N).

Theorem 2.27. [4] Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then the following are equivalent

(1) (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.
(2) $int(\lambda_N) = 0_N$, for every fuzzy neutrosophic σ- first category set λ_N in (X_N, T_N).
(3) $cl(\mu_N) = 1_N$, for every fuzzy neutrosophic σ-residual set μ_N in (X_N, T_N).

Definition 2.28. [9] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic submaximal space if for each fuzzy neutrosophic set A_N in (X, τ_N) such that $(A_N)^- = 1$ then $A_N \in \tau_N$ in (X, τ_N). That is (X, τ_N) is a fuzzy neutrosophic submaximal space if each fuzzy neutrosophic dense set in (X, τ_N) is a fuzzy neutrosophic open set in (X, τ_N).

Definition 2.29. [9] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic resolvable space if there exist a fuzzy neutrosophic dense set A_N in (X, τ_N) such that $(1 - A_N)^- = 1$. Otherwise, (X, τ_N) is called a fuzzy neutrosophic irresolvable space.

Definition 2.30. [9] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic hyperconnected space if every non-null fuzzy neutrosophic open subset of (X, τ_N) is fuzzy neutrosophic dense in (X, τ_N).

Definition 2.31. [9] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic P-space if each fuzzy neutrosophic G_δ-set in (X, τ_N) is fuzzy neutrosophic open set in (X, τ_N).

Definition 2.32. [9] A fuzzy neutrosophic topological space (X, τ_N) is called a fuzzy neutrosophic almost resolvable space if $\vee_{i=1}^{\infty}(A_{N_i}) = 1$, where (A_{N_i})'s in (X, τ_N) are such that $(A_{N_i})^+ = 0$, otherwise, (X, τ_N) is called a fuzzy neutrosophic almost irresolvable space.

3. FUZZY NEUTROSOPHIC REGULAR $\sigma-$NOWHERE DENSE SET

Definition 3.1. Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set λ_N in (X_N, T_N) is called a fuzzy neutrosophic regular $\sigma-$ nowhere dense set if $\lambda_N = \bigvee_{i=1}^{\infty} cl\ (\lambda_{N_i})$, where int $(\lambda_{N_i}) = 0_N$.

Example 3.2. Let $X_N = \{a, b, c\}$. The fuzzy neutrosophic sets $\lambda_N, \mu_N, \gamma_N$ are defined on X_N as follows:
$\lambda_N : X_N \to [0, 1]$ is defined as $\lambda_N(a) = 0.5; \lambda_N(b) = 0.6; \lambda_N(c) = 0.7$
$\mu_N : X_N \to [0, 1]$ is defined as $\mu_N(a) = 0.6; \mu_N(b) = 0.5; \mu_N(c) = 0.4$
$\gamma_N : X_N \to [0, 1]$ is defined as $\gamma_N(a) = 0.4; \gamma_N(b) = 0.5; \gamma_N(c) = 0.6$
Then, $T_N = \{0_N, \lambda_N, \mu_N, \gamma_N, \lambda_N \vee \mu_N, \mu_N \vee \gamma_N, \lambda_N \wedge \mu_N, \lambda_N \wedge [\mu_N \vee \gamma_N], 1_N\}$ is clearly a fuzzy neutrosophic topology on X_N. Now, consider the following fuzzy neutrosophic sets defined on X_N.
$\alpha_N : X_N \to [0, 1]$ is defined as $\alpha_N(a) = 0.4, \alpha_N(b) = 0.6, \alpha_N(c) = 0.7$
$\beta_N : X_N \to [0, 1]$ is defined as $\beta_N(a) = 0.5, \beta_N(b) = 0.6, \beta_N(c) = 0.3$
$\delta_N : X_N \to [0, 1]$ is defined as $\delta_N(a) = 0.3, \delta_N(b) = 0.5, \delta_N(c) = 0.6$
Now, on computation, $int(\alpha_N) = 0_N, int(\beta_N) = 0_N$, $int(\delta_N) = 0_N$ and $cl(\delta_N) = 1_N - \mu_N$, $cl(\beta_N \wedge \delta_N) = 1_N - (\mu_N \vee \gamma_N)$. Then $cl(\delta_N) \vee cl(\beta_N \wedge \delta_N) = (1_N - \mu_N) \vee [1_N - (\mu_N \vee \delta_N)] = 1_N - \mu_N$. Hence $1_N - \mu_N$ is a fuzzy neutrosophic regular $\sigma-$ nowhere dense set in (X_N, T_N).

Proposition 3.3. *If λ_N is a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N), then $1_N - \lambda_N = \bigwedge_{i=1}^{\infty}(\mu_{N_i})$, where $cl\ (\mu_{N_i}) = 1_N$.*

Proof. Let λ_N be a fuzzy neutrosophic regular $\sigma-$nowhere dense set in (X_N, T_N). Then $\lambda_N = \bigvee_{i=1}^{\infty} cl\ (\lambda_{N_i})$, where int $(\lambda_{N_i}) = 0_N$. Now, int $(\lambda_{N_i}) = 0_N$ implies that $1_N - int\ (\lambda_{N_i}) = 1_N$ and hence we have $cl(1_N - \lambda_{N_i}) = 1_N$. Also, $1_N - \lambda_N = 1_N - \bigvee_{i=1}^{\infty} cl\ (\lambda_{N_i}) = \bigwedge_{i=1}^{\infty}$ int $(1_N - \lambda_{N_i})$, where $cl(1_N - \lambda_{N_i}) = 1_N$. Let $\mu_{N_i} = 1_N - \lambda_{N_i}$. Then $1_N - \lambda_N = \bigwedge_{i=1}^{\infty} int(\mu_{N_i})$, where $cl\ (\mu_{N_i}) = 1_N$.

Proposition 3.4. *If $cl(\lambda_{N_i}) = 1_N$, where λ_{N_i} is a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N), then λ_{N_i} is a fuzzy neutrosophic first category set in (X_N, T_N).*

Proof. Let λ_{N_i} be a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N). Then $\lambda_{N_i} = \bigvee_{i=1}^{\infty} cl\ (\lambda_{N_i})$, where int $(\lambda_{N_i}) = 0_N$. Now, $cl(\lambda_{N_i}) = 1_N$ implies that $cl(\bigvee_{i=1}^{\infty} cl(\lambda_{N_i})) = 1_N$. But $\bigvee_{i=1}^{\infty} cl(cl(\lambda_{N_i})) = 1_N$ implies that $\bigvee_{i=1}^{\infty} intcl(cl(\lambda_{N_i})) = int(1_N)$. This implies that $\bigvee_{i=1}^{\infty} intcl(\lambda_{N_i}) = 1_N$. Then $cl(\lambda_{N_i})$'s are fuzzy neutrosophic nowhere dense sets in (X_N, T_N). Hence $\lambda_N = \bigvee_{i=1}^{\infty}(cl(\lambda_{N_i}))$, where $(cl(\lambda_{N_i}))$'s are fuzzy neutrosophic nowhere dense sets, implies that λ_N is a fuzzy neutrosophic first category set in (X_N, T_N). □

Proposition 3.5. *If $cl(\lambda_N) = 1_N$, where λ_N is a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N), then $1_N - \lambda_N$ is a fuzzy neutrosophic residual set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N) such that $cl(\lambda_N) = 1_N$. Then by proposition-3.4, λ_N is a fuzzy neutrosophic first category set in (X_N, T_N), and hence $1_N - \lambda_N$ is a fuzzy neutrosophic residual set in (X_N, T_N). □

Remark 3.6. :
Let λ_N be a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N). Then we have, $\lambda_N = \bigvee_{i=1}^{\infty} cl(\lambda_{N_i})$, where $int(\lambda_{N_i}) = 0_N$. If (λ_{N_i})'s are fuzzy neutrosophic closed sets in (X_N, T_N), then λ_N is a fuzzy neutrosophic first category set in (X_N, T_N). For, if (λ_{N_i})'s are fuzzy neutrosophic closed sets, then $cl(\lambda_{N_i}) = \lambda_{N_i}$, implies that $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where $intcl(\lambda_{N_i}) = 0_N$ and hence λ_N is a fuzzy neutrosophic first category set in (X_N, T_N). Also, since $int(\lambda_{N_i}) = 0_N, (\lambda_{N_i})$'s are not fuzzy neutrosophic open sets in (X_N, T_N).

Proposition 3.7. *If λ_N is a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N) then λ_N is a fuzzy neutrosophic $F_\sigma-$set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N). Then, we have, $\lambda_N = \bigvee_{i=1}^{\infty} cl(\lambda_{N_i})$, where $int(\lambda_{N_i}) = 0_N$ and hence $\lambda_N = \bigvee_{i=1}^{\infty} cl(\lambda_{N_i})$, implies that λ_N is a fuzzy neutrosophic $F_\sigma-$set in (X_N, T_N). □

Proposition 3.8. *If λ_N is a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N) then $1_N - \lambda_N$ is a fuzzy neutrosophic $G_\delta-$set formed the fuzzy neutrosophic dense sets in (X_N, T_N).*

Proof. The proof follows from proposition 3.7, The following proposition gives the condition for a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space to be a fuzzy neutrosophic $\sigma-$nowhere dense set. □

Proposition 3.9. *If $int(\lambda_N) = 0_N$, for a fuzzy neutrosophic regular $\sigma-$nowhere dense set λ_N in a fuzzy neutrosophic topological space (X_N, T_N), then λ_N is a fuzzy neutrosophic $\sigma-$nowhere dense set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N) such that $int(\lambda_N) = 0_N$. Then by Proposition 3.7, λ_N is a fuzzy neutrosophic $F_\sigma-$ set in (X_N, T_N). Hence λ_N is a fuzzy neutrosophic $F_\sigma-$set such that $int(\lambda_N) = 0_N$. Therefore λ_N is a fuzzy neutrosophic $\sigma-$nowhere dense set in (X_N, T_N). □

Proposition 3.10. *If $int(\lambda_N) = 0_N$, for a fuzzy neutrosophic regular $\sigma-$nowhere dense set λ_N in a fuzzy neutrosophic $P-$space (X_N, T_N), then λ_N is not a fuzzy neutrosophic dense set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N) such that $int(\lambda_N) = 0_N$. Then by Proposition 3.9, λ_N is a fuzzy neutrosophic $\sigma-$nowhere dense set in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic $P-$space, by Theorem 2.23, λ_N is not a fuzzy neutrosophic dense set in (X_N, T_N).

The following proposition ensures the existence of a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space. □

Proposition 3.11. *If λ_N is a fuzzy neutrosophic first category set in a fuzzy neutrosophic topological space (X_N, T_N), then there exists a fuzzy neutrosophic regular $\sigma-$nowhere dense set μ_N in (X_N, T_N) such that $\lambda_N \leq \mu_N$ in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic first category set in (X_N, T_N). Then $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X_N, T_N) and hence $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$ where $int\ cl(\lambda_{N_i}) = 0_N$. Now, $int(\lambda_{N_i}) \leq intcl(\lambda_{N_i})$, implies that $int(\lambda_{N_i}) = 0_N$. Let $\mu_N = \bigvee_{i=1}^{\infty} cl(\lambda_{N_i})$, where $int(\lambda_{N_i}) = 0_N$. Then μ_N is a fuzzy neutrosophic regular $\sigma-$nowhere dense in (X_N, T_N). Now, $\bigvee_{i=1}^{\infty}(\lambda_{N_i}) \leq \bigvee_{i=1}^{\infty} cl(\lambda_{N_i})$, implies that $\lambda_N \leq \mu_N$ in

(X_N, T_N). Therefore, if λ_N is a fuzzy neutrosophic first category set in a fuzzy neutrosophic topological space, then there exist a fuzzy neutrosophic regular $\sigma-$nowhere dense set μ_N in (X_N, T_N) such that $\lambda_N \leq \mu_N$ in (X_N, T_N). □

Proposition 3.12. *If λ_N is a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic submaximal space (X_N, T_N), then λ_N is a fuzzy neutrosophic first category set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic submaximal space (X_N, T_N). Then $\lambda_N = \bigvee_{i=1}^{\infty} cl\ (\lambda_{N_i})$, where $int(\lambda_{N_i}) = 0_N$. Now, $int(\lambda_{N_i}) = 0_N$, implies that $cl\ (1_N - \lambda_{N_i}) = 1_N - int\ (\lambda_{N_i}) = 1_N$. Hence $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic dense sets in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic submaximal space, the fuzzy neutrosophic dense sets $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic open sets in (X_N, T_N) and hence (λ_{N_i})'s are fuzzy neutrosophic closed sets in (X_N, T_N). Then $cl(\lambda_{N_i}) = \lambda_{N_i}$. Now, $intclcl(\lambda_{N_i}) = intcl(\lambda_{N_i}) = int(\lambda_{N_i}) = 0_N$. Then $(cl(\lambda_{N_i}))$'s are fuzzy neutrosophic nowhere dense sets in (X_N, T_N). Hence $\lambda_N = \bigvee_{i=1}^{\infty}(cl(\lambda_{N_i}))$, where $(cl(\lambda_{N_i}))$'s are fuzzy neutrosophic nowhere dense sets, implies that λ_N is a fuzzy neutrosophic first category set in (X_N, T_N). □

Proposition 3.13. *If $int(\lambda_N) = 0_N$, for each fuzzy neutrosophic regular $\sigma-$nowhere dense set λ_N in a fuzzy neutrosophic topological space (X_N, T_N), then (X_N, T_N) is a fuzzy neutrosophic Baire space.*

Proof. Let λ_N be a fuzzy neutrosophic first category set in (X_N, T_N). Then by Proposition 3.11, there exists a fuzzy neutrosophic regular $\sigma-$nowhere dense set μ_N in (X_N, T_N) such that $\lambda_N \leq \mu_N$ in (X_N, T_N). By hypothesis, $int(\mu_N) = 0_N$. Now, $\lambda_N \leq \mu_N$, implies that $int(\lambda_N) \leq int(\mu_N)$. Then we have $int(\lambda_N) \leq 0_N$. That is, $int(\lambda_N) = 0_N$. Hence by Theorem 2.24, (X_N, T_N) is a fuzzy neutrosophic Baire Space. □

Proposition 3.14. *If λ_N is a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic $P-$space (X_N, T_N), then λ_N is a fuzzy neutrosophic closed set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic $P-$space (X_N, T_N). Then by Proposition 3.8, $1_N - \lambda_N$ is a fuzzy neutrosophic $G_\delta-$set in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic $P-$space, then the fuzzy neutrosophic $G_\delta-$set $(1_N - \lambda_N)$ is a fuzzy neutrosophic open set in (X_N, T_N) and hence λ_N is a fuzzy neutrosophic closed set in (X_N, T_N). □

Definition 3.15. A fuzzy neutrosophic set λ_N in a fuzzy neutrosophic topological space (X_N, T_N) is called a fuzzy neutrosophic regular $\sigma-$first category set it $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N).

Definition 3.16. Let λ_N be a fuzzy neutrosophic regular $\sigma-$first category set in (X_N, T_N). Then $1_N - \lambda_N$ is called a fuzzy neutrosophic regular $\sigma-$residual set in (X_N, T_N).

Proposition 3.17. *If a fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic almost resolvable space, then 1_{X_N} is a fuzzy neutrosophic regular $\sigma-$nowhere dense set.*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic almost resolvable space. Then, $\bigvee_{i=1}^{\infty}(\lambda_{N_i}) = 1_N$, where the fuzzy neutrosophic sets (λ_{N_i})'s in (X_N, T_N) are such that $int(\lambda_{N_i}) = 0_N$. Now, $\bigvee_{i=1}^{\infty}(\lambda_{N_i}) \leq \bigvee_{i=1}^{\infty} cl\ (\lambda_{N_i})$, implies that $1_N \leq \bigvee_{i=1}^{\infty} cl\ (\lambda_{N_i})$. That is $\bigvee_{i=1}^{\infty} cl\ (\lambda_{N_i}) = 1_N$, where $int(\lambda_{N_i}) = 0_N$. Hence 1_{X_N} is a fuzzy neutrosophic regular $\sigma-$nowhere dense set. □

Proposition 3.18. *If $1_N - \lambda_N$ is a fuzzy neutrosophic closed and fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N), then $1_N - \lambda_N$ is a fuzzy neutrosophic closed and fuzzy neutrosophic $G_\delta-$set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic open and fuzzy neutrosophic regular $\sigma-$nowhere dense set in (X_N, T_N). Since λ_N is a fuzzy neutrosophic regular $\sigma-$nowhere dense set. By Proposition 3.8., $1_N - \lambda_N$ is a fuzzy neutrosophic $G_\delta-$ set in (X_N, T_N). Hence $1_N - \lambda_N$ is a fuzzy neutrosophic closed and fuzzy neutrosophic $G_\delta-$set in (X_N, T_N). □

Proposition 3.19. *If $1_N - \lambda_N$ is a fuzzy neutrosophic closed and fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N), then $1_N - \lambda_N$ is a fuzzy neutrosophic $F_\sigma-$set in (X_N, T_N).*

Proof. Let $1_N - \lambda_N$ be a fuzzy neutrosophic closed set and fuzzy neutrosophic regular $\sigma-$nowhere dense set in fuzzy neutrosophic topological space (X_N, T_N). Since $1_N - \lambda_N$ is a fuzzy neutrosophic regular $\sigma-$nowhere dense set in (X_N, T_N). By Proposition 3.7, $1_N - \lambda_N$ is a fuzzy neutrosophic $F_\sigma-$ set in (X_N, T_N). □

Proposition 3.20. *If $int(\lambda_N) = 0_N$, for a fuzzy neutrosophic regular $\sigma-$nowhere dense set λ_N in a fuzzy neutrosophic $P-$space (X_N, T_N), then λ_N is a fuzzy neutrosophic nowhere dense set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic open and fuzzy neutrosophic regular $\sigma-$nowhere dense set in a fuzzy neutrosophic $P-$space (X_N, T_N) such that $int(\lambda_N) = 0_N$. Then by proposition 3.14, λ_N is a fuzzy neutrosophic closed set in (X_N, T_N) and hence $cl\ (\lambda_N) = \lambda_N$. Now, $intcl\ (\lambda_N) = int(\lambda_N) = 0_N$ and therefore λ_N is a fuzzy neutrosophic nowhere dense set in (X_N, T_N). □

4. FUZZY NEUTROSOPHIC REGULAR $\sigma-$BAIRE SPACES

Definition 4.1. A fuzzy neutrosophic topological space (X_N, T_N) is called a fuzzy neutrosophic regular $\sigma-$Baire space if $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N).

The following proposition gives the characterization of fuzzy neutrosophic regular $\sigma-$ Baire space interms of fuzzy neutrosophic regular $\sigma-$ first category sets and fuzy neutrosophic regular $\sigma-$ residual sets in a fuzzy neutrosophic topological space.

Proposition 4.2. *Let (X_N, T_N) be a fuzzy neutrosophic topological space then the followings are equivalent.*

(1) *(X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space.*
(2) *$int(\lambda_N) = 0_N$, for every fuzzy neutrosophic $\sigma-$first category set λ_N in (X_N, T_N).*
(3) *$cl\ (\mu_N) = 1_N$ for every fuzzy neutrosophic regular $\sigma-$residual set μ_N in (X_N, T_N).*

Proof. (1) $\implies$ (2):
Let λ_N be a fuzzy neutrosophic regular $\sigma-$first category set in (X_N, T_N). Then $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N) and hence $int(\lambda_N) = int(\bigvee_{i=1}^{\infty}(\lambda_{N_i}))$. Since (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire Space, $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$. Hence $int(\lambda_{N_i}) = 0_N$, for a fuzzy neutrosophic regular $\sigma-$first category set λ_N in (X_N, T_N).
(2) $\implies$ (3):
Let μ_N be a fuzzy neutrosophic regular $\sigma-$residual set in (X_N, T_N). Then $(1_N - \mu_N)$ is a fuzzy neutrosophic regular $\sigma-$first category set in (X_N, T_N). By hypothesis, $int(1_N - \mu_N) = 0_N$. Then $1_N - cl\ (\mu_N) = int(1_N - \mu_N) = 0_N$. Hence $cl(\mu_N) = 1_N$, for a fuzzy neutrosophic regular $\sigma-$residual set μ_N in (X_N, T_N).
(3) $\implies$ (1):
Let λ_N be a fuzzy neutrosophic regular $\sigma-$first category set in (X_N, T_N). Then $\lambda_N = (\bigvee_{i=1}^{\infty}(\lambda_{N_i}))$, where (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). Now, λ_N is a fuzzy neutrosophic regular $\sigma-$first category set in (X_N, T_N), implies that $(1_N - \lambda_N)$ is a fuzzy neutrosophic regular $\sigma-$residual set in (X_N, T_N). By hypothesis,

$cl(1_N - \lambda_N) = 1_N$. Then $1_N - int(\lambda_N) = 1_N$ and hence $int(\lambda_N) = 0_N$. That is $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). Hence (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space. □

Proposition 4.3. *If a fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space, then cl $(\bigwedge_{i=1}^{\infty}(\mu_{N_i})) = 1_N$, Where (μ_{N_i})'s are fuzzy neutrosophic $G_\delta-$sets formed from the fuzzy neutrosophic dense sets in (X_N, T_N).*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic regular $\sigma-$Baire space. Then $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). Now, $1_N - int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 1_N$, implies that $cl\ (\bigwedge_{i=1}^{\infty}(1_N - \lambda_{N_i})) = 1_N$. Since (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). By proposition 3.3, we have $1_N - \lambda_{N_i} = \bigwedge_{j=1}^{\infty} int(\mu_{N_i j})$, where $cl(\mu_{N_i j}) = 1_N$. Let $\delta_{N_i} = 1_N - \lambda_{N_i}$. Then (δ_{N_i})'s are fuzzy neutrosophic $G_\delta-$sets in (X_N, T_N). Hence we have $cl(\bigwedge_{i=1}^{\infty}(\delta_{N_i})) = 1_N$, where (δ_{N_i})'s are fuzzy neutrosophic $G_\delta-$sets formed from the fuzzy neutrosophic dense sets in (X_N, T_N). □

Proposition 4.4. *If a fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space, then (X_N, T_N) is a fuzzy neutrosophic Baire space.*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic regular $\sigma-$Baire space. Then $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where $(\lambda_{N_i})'$s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). Now, $\bigvee_{i=1}^{\infty}[int(\lambda_{N_i})] \leq int[\bigvee_{i=1}^{\infty}(\lambda_{N_i})]$, implies that $\bigvee_{i=1}^{\infty}[int(\lambda_{N_i})] \leq 0_N$. That is $\bigvee_{i=1}^{\infty}[int(\lambda_{N_i})] = 0_N$. This implies that $int(\lambda_{N_i}) = 0_N$, for each i. Thus for the fuzzy neutrosophic regular $\sigma-$nowhere dense sets (λ_{N_i})'s in (X_N, T_N). We have $int(\lambda_{N_i}) = 0_N$. Therefore by proposition 3.13, (X_N, T_N) is a fuzzy neutrosophic Baire space. □

Proposition 4.5. *If (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space, then $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic first category sets in (X_N, T_N).*

Proof. Let (λ_{N_i})'s be a fuzzy neutrosophic first category sets in (X_N, T_N). Then by proposition 3.11, there exists fuzzy neutrosophic regular $\sigma-$nowhere dense sets μ_{N_i} in (X_N, T_N) such that $\lambda_{N_i} \leq \mu_{N_i}$ in (X_N, T_N). Then we have $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) \leq int(\bigvee_{i=1}^{\infty}(\mu_{N_i}))$. Since (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space, $int(\bigvee_{i=1}^{\infty}(\mu_{N_i})) = 0_N$. Then, $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) \leq 0_N$. That is $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$. Thus $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic first category sets in (X_N, T_N). □

Proposition 4.6. *If (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space, then (X_N, T_N) is a fuzzy neutrosophic Baire space.*

Proof. Let λ_N be a fuzzy neutrosophic first category set in (X_N, T_N). Then $\lambda_N \leq \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where $(\lambda_{N_i})'$s are fuzzy neutrosophic first category sets in (X_N, T_N) in which $\lambda_{N_1} = \lambda_N$. Now, $\lambda_N \leq \bigvee_{i=1}^{\infty}(\lambda_{N_i})$ implies that $int(\lambda_N) \leq int(\bigvee_{i=1}^{\infty}(\lambda_{N_i}))$. Since (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space. By proposition 4.5, $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$. Then $int(\lambda_N) \leq 0_N$. That is, $int(\lambda_N) = 0_N$. Hence we have $int(\lambda_N) = 0_N$, for a fuzzy neutrosophic first category set in (X_N, T_N) and therefore, by Theorem 2.24, (X_N, T_N) is a fuzzy neutrosophic Baire space. □

Proposition 4.7. *If (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space, then (X_N, T_N) is a fuzzy neutrosophic $\sigma-$Baire space.*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic regular $\sigma-$Baire space. Then,$int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). Now, $\bigvee_{i=1}^{\infty}[int(\lambda_{N_i})] \leq [\bigvee_{i=1}^{\infty}(\lambda_{N_i})]$, implies that $\bigvee_{i=1}^{\infty}[int(\lambda_{N_i})] \leq 0_N$. That is $\bigvee_{i=1}^{\infty}[int(\lambda_{N_i})] = 0_N$. This implies that $int(\lambda_{N_i}) = 0_N$, for each i. By proposition 3.9, (λ_{N_i})'s are fuzzy neutrosophic $\sigma-$ nowhere dense sets in (X_N, T_N). Hence we have, $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy $\sigma-$ nowhere dense sets in (X_N, T_N). Therefore (X_N, T_N) is a fuzzy neutrosophic $\sigma-$Baire space. □

Definition 4.8. A fuzzy neutrosophic topological space (X_N, T_N) is called a fuzzy neutrosophic regular $\sigma-$first category space if the fuzzy set 1_{X_N} is a fuzzy neutrosophic regular $\sigma-$first category set in (X_N, T_N). That is, $1_{X_N} = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). Otherwise, (X_N, T_N) will be called a fuzzy neutrosophic regular $\sigma-$second category space.

Proposition 4.9. *If a fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$first category space, then $\bigwedge_{i=1}^{\infty}(\mu_{N_i}) = 0_N$, where (μ_{N_i})'s are fuzzy neutrosophic $G_\delta-$ sets formed from the fuzzy neutrosophic dense sets in (X_N, T_N).*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic regular $\sigma-$first category space. Then, $1_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). Now, $1_N - \bigvee_{i=1}^{\infty}(\lambda_{N_i}) = 0_N$. Then $\bigwedge_{i=1}^{\infty}(1_N - \lambda_{N_i}) = 0_N$. since (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). By proposition 3.8, $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic $G_\delta-$sets formed from the fuzzy neutrosophic dense sets in (X_N, T_N). Let $\mu_{N_i} = (1_N - \lambda_{N_i})$. Then $\bigwedge_{i=1}^{\infty}(\mu_{N_i}) = 0_N$, where (μ_{N_i})'s are fuzzy neutrosophic $G_\delta-$ sets formed from the fuzzy neutrosophic dense sets in (X_N, T_N). □

Proposition 4.10. *If a fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space, then (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$second category space.*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic regular $\sigma-$Baire space. Then,$int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). Now, we claim that, $\bigvee_{i=1}^{\infty}(\lambda_{N_i}) \neq 1_{X_N}$. Suppose that $\bigvee_{i=1}^{\infty}(\lambda_{N_i}) = 1_{X_N}$. Then, this will implies that $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = int(1_{X_N}) = 1_{X_N} \neq 0_N$, this is a contradiction. Hence we must have $\bigvee_{i=1}^{\infty}(\lambda_{N_i}) \neq 1_{X_N}$ and (X_N, T_N) is not a fuzzy neutrosophic regular $\sigma-$first category space. Therefore (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$second category space. □

Proposition 4.11. *If the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$first category space, then (X_N, T_N) is not a fuzzy neutrosophic regular $\sigma-$Baire space.*

Proof. Let the fuzzy neutrosophic topological space (X_N, T_N) be a fuzzy neutrosophic regular $\sigma-$first category space. Then, $\bigvee_{i=1}^{\infty}(\lambda_{N_i}) = 1_N$, where the fuzzy sets $(\lambda_{N_i})'$s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). Now $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = int(1_N) = 1_N \neq 0_N$. Hence (X_N, T_N) is not a fuzzy neutrosophic regular $\sigma-$Baire space. □

Proposition 4.12. *If the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space, then no non-zero fuzzy neutrosophic open set is a fuzzy neutrosophic regular $\sigma-$first category set in (X_N, T_N).*

Proof. Let λ_N be a non-zero fuzzy neutrosophic open set in a fuzzy neutrosophic regular $\sigma-$ Baire space (X_N, T_N) such that $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where the fuzzy neutrosophic sets $(\lambda_{N_i})'$s are fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N). Then $int(\lambda_N) = int(\bigvee_{i=1}^{\infty}(\lambda_{N_i}))$. Since (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space, $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$. This implies that $int(\lambda_N) = 0_N$. Then we will have $\lambda_N = 0_N$, This is a contradiction. [Since $\lambda_N \in T_N, int(\lambda_N) = \lambda_N$ in (X_N, T_N)]. Hence $\lambda_N \neq \bigvee_{i=1}^{\infty}(\lambda_{N_i})$ and therefore no non-zero fuzzy neutrosophic open set is a fuzzy neutrosophic regular $\sigma-$first category set in (X_N, T_N). □

Proposition 4.13. *If the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$Baire space, then (X_N, T_N) is a fuzzy neutrosophic almost irresolvable space.*

Proof. Let (X_N, T_N) be a fuzzy neutrosophic regular $\sigma-$Baire space. By proposition 4.11, (X_N, T_N) is a fuzzy neutrosophic regular $\sigma-$second category space and hence (X_N, T_N) is not a fuzzy neutrosophic regular σ-first category space. Then, $\bigvee_{i=1}^{\infty}(\lambda_{N_i}) \neq 1_N$, where (λ_{N_i})'s are

fuzzy neutrosophic regular $\sigma-$nowhere dense sets in (X_N, T_N).

Now, $\bigvee_{i=1}^{\infty}[int(\lambda_{N_i})] \leq int[\bigvee_{i=1}^{\infty}(\lambda_{N_i})]$, implies that $\bigvee_{i=1}^{\infty}[int(\lambda_{N_i})] \leq 0_N$. That is, $\bigvee_{i=1}^{\infty}[int(\lambda_{N_i})] = 0_N$. This implies that, $int(\lambda_{N_i}) = 0_N$, for each i. Hence $\bigvee_{i=1}^{\infty}(\lambda_{N_i}) \neq 1_N$, where $int(\lambda_{N_i}) = 0_N$ and therefore (X_N, T_N) is a fuzzy neutrosophic almost irresolvable space.

□

Remark 4.14. :

The relation between fuzzy neutrosophic regular $\sigma-$Baire space, fuzzy neutrosophic $\sigma-$Baire space and fuzzy neutrosophic almost irresolvable space, can be summarized as follows.

Fuzzy neutrosophic regular $\sigma-$Baire space.

↓

Fuzzy neutrosophic $\sigma-$Baire space.

↓

Fuzzy neutrosophic almost irresolvable space.

5. Conclusion

The study of fuzzy neutrosophic regular σ-Baire Spaces contributes significantly to the advancement of fuzzy neutrosophic topology by extending the classical notion of Baire spaces within the framework of uncertainty and indeterminacy.These spaces provide a rotust structure where the interplay of fuzziness,neutrosophic logic and regularity conditions can be analyzed systematically. The key conclusion is that under appropriate conditions, the fuzzy neutrosophic regular σ-Baire space maintains essential topological properties such as density of fuzzy neutrosophic open sets and the preservation of σ-Baire conditions under regularity. This class of spaces has potential applications in areas where precise classification is impossible, such as decision-making, computational intelligence and Mathematical Modelling. Furthermore, the results offer a groundwork for futher generalizations and investigations into fuzzy neutrosophic connectedness, compactness and continuity within more generalized space.

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FUZZY NEUTROSOPHİC SETTİNG ON P-SPACES, WEAK P-SPACES AND ALMOST P-SPACES

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ABSTRACT

In this paper we introduce the concepts of fuzzy neutrosophic P-space, weak fuzzy neutrosophic P-space and Fuzzy neutrosophic almost P-space. Also we discuss several characterizations of fuzzy neutrosophic P-space, weak Fuzzy neutrosophic P-space and Fuzzy neutrosophic almost P-space. Several examples are given to illustrate the concepts introduced in this paper.

Keywords: Fuzzy neutrosophic regular-open(closed) sets, Fuzzy neutrosophic Pspace, weak fuzzy neutrosophic P-space, fuzzy neutrosophic almost P-space and fuzzy neutrosophic Baire space.

1. INTRODUCTION

The fuzzy idea was invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [21]. The important concept of fuzzy topological space was offered by C.L. Chang [3]. The idea of fuzzy σ- Baire Spaces was introduced by G. Thangaraj and E. Poongothai [13]. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [20] introduced fuzzy neutrosophic topological spaces. The idea of fuzzy neutrosophic Baire spaces was introduced by E. Poongothai and E. Padmavathi [10].In this paper we introduce the concepts of fuzzy neutrosophic P-space, weak fuzzy neutrosophic P-space and Fuzzy neutrosophic almost P-space. Also we discuss several characterizations of fuzzy neutrosophic P-space, weak Fuzzy neutrosophic P-space and Fuzzy neutrosophic almost P-space. Several examples are given to illustrate the concepts introduced in this paper.

2. PRELIMINARIES

Definition 2.1. [5] A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $x \in X$ where $T, I, F : X \to [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. With the condition $0 \leq T_A*(x) + I_A*(x) + F_A*(x) \leq 2$.

Definition 2.2. [5] A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \leq F_B(x)$.

Definition 2.3. [5] Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$$

$$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$$

Definition 2.4. [5] The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$

Definition 2.5. [5] A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0, I_A(x) = 0, F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N.

Definition 2.6. [5] A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N.

Definition 2.7. [5] The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_AC(x), I_AC(x), F_AC(x)\rangle$ where $T_AC(x) = F_A(x), I_AC(x) = 1 - I_A(x), F_AC(x) = (T_A(x)$.

The complement of a fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8. [20] A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X

(i) $0_N, 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$
(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$. Satisfying the following axioms. In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9. [26] The complement A^C of a fuzzy neutrosophic set in A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10. [26] Let (X, τ_N) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$int(A) = \cup\{G : G \text{ is a fuzzy neutrosophic open set in } X \text{ and } G \subseteq A\}$$

$$cl(A) = \cap\ \{G : G \text{ is a fuzzy neutrosophic closed set in } X \text{ and } G \subseteq G\}.$$

Definition 2.11 [1] Let (X, τ) be a fuzzy neutrosophic topological space over X. Then the following properties hold, $(i)\ cl(A^C) = (int\ A)^C$, $(ii)\ int(A^C) = (cl\ A)^C$.

Definition 2.12 [10] A fy. neutrosophic set A_N in a fy. neutrosophic top. Space (P, τ_N) is called a fy. neutrosophic F_σ-set if $A_N = \vee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$.

Definition 2.13 [10] A fy. neutrosophic set A_N in a fy. neutrosophic top. Space (P, τ_N) is called a fy. neutrosophic G_δ-set in (P, τ_N) if $A_N = \wedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$.

Definition 2.14 [10] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic dense if there exist no $fnCS\ B_N$ in (P, τ_N) s.t $A_N \subset B_N \subset 1_X$. That is $fn(A_N)^- = 1_N$.

Definition 2.15 [10] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic nowh. dense set if there exist no non zero $fnOS\ B_N$ in (P, τ_N) s.t $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$

Definition 2.16 [10] Let (P, τ_N) be a fy. neutrosophic top. space. A fy. neutrosophic set A_N in (P, τ_N) is called fy. neutrosophic one category set if $A_N = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Any other fy. neutrosophic set in (P, τ_N) is said to be of fy. neutrosophic two category.

Definition 2.17 [10] A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic one category space if the fy. neutrosophic set 1_X is a fy. Neutrosophic one category set in (P, τ_N). That is $1_X = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i} 's are fy. Neutrosophic nowh. dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fy. neutrosophic two category space.

Definition 2.18 [10] Let A_N be a fy. neutrosophic one category set in (P, τ_N). Then $\overline{A_N}$ is called fy. neutrosophic re. set in (P, τ_N).

Definition 2.19 [10] A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic Baire space if $fn(\vee_{i=1}^{\infty}(A_{N_i}))^{+} = 0_N$, where $(A_{N_i})'s$ are fy. neutrosophic nowh. dense sets in (P, τ_N).

Theorem 2.1 [10] Let (P, τ_N) be a fy. neutrosophic top. space. Then the following are equivalent

(1) (P, τ_N) is a fy. neutrosophic Baire space.

(2) $fn(A_N)^{+} = 0_N$, for every fy. neutrosophic one category set A_N in (P, τ_N).

(3) $fn(B_N)^{-} = 1_N$, for every fy. neutrosophic re. set B_N in (P, τ_N).

Definition 2.20 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic Set λ_N in (X_N, T_N) is called a fuzzy neutrosophic σ-nowhere dense set if λ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N) such that $int(\lambda_N) = 0_N$.

Definition 2.21 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set λ_N in (X_N, T_N) is called fuzzy neutrosophic σ- first category set if $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where $(\lambda_{N_i})'s$ are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Any other fuzzy neutrosophic set in (X_N, T_N) is said to be fuzzy neutrosophic σ- second category sets in (X_N, T_N).

Definition 2.22 [5] Let λ_N be a fuzzy neutrosophic σ-first category set in (X_N, T_N). Then $1_N - \lambda_N$ is called a fuzzy neutrosophic σ-residual set in (X_N, T_N).

Definition 2.23 [5] A fuzzy neutrosophic topological space (X_N, T_N) is called fuzzy neutrosophic σ-first category space if the fuzzy neutrosophic set 1_{X_N} is a fuzzy neutrosophic σ-first category set in (X_N, T_N). That is $1_{X_N} = \vee_{i=1}^{\infty}(\lambda_{N_i})$ where $(\lambda_{N_i})'s$ are fuzzy neutrosophic σ- nowhere dense sets in (X_N, T_N). Otherwise (X_N, T_N) will be called a fuzzy neutrosophic σ- second category space.

Definition 2.24 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then (X_N, T_N) is called a fuzzy neutrosophic σ-Baire space if $\text{int}(\vee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$,where $(\lambda_{N_i})'s$ are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N).

Theorem 2.2 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then the following are equivalent

(1) (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.

(2) $int(\lambda_N) = 0_N$, for every fuzzy neutrosophic σ- first category set λ_N in (X_N, T_N).

(3) $cl(\mu_N) = 1_N$, for every fuzzy neutrosophic σ-residual set μ_N in (X_N, T_N).

Definition 2.25 [4] FNS λ_N in FNTS (X, τ) is called Fuzzy neutrosophic regular open set (Briefly, FNR-open) if $\lambda_N = FNInt(FNcl(\lambda_N))$.

Definition 2.26 [4] FNS λ_N in FNTS (X, τ) is called Fuzzy neutrosophic regular closed set (Briefly, FNR-closed) if $\lambda_N = FNcl(FNInt(\lambda_N))$.

3. FUZZY NEUTROSOPHIC P-SPACES

Definition 3.1 [12] A fuzzy neutrosophic topological space (X_N, T_N) is called a fuzzy neutrosophic P-Space if countable intersection of fuzzy neutrosophic open sets in (X_N, T_N) is fuzzy neutrosophic open. That is, every non-zero fuzzy neutrosophic G_δ—set in (X_N, T_N) is fuzzy neutrosophic open in (X_N, T_N).

Example 3.1 Let $X_N = \{x_1, x_2\}$. The fuzzy neutrosophic sets λ_N and μ_N are defined on X_N as follows:

$\lambda_N : X_N \to [0_N, 1_N]$ is defined as $\lambda_N(x_1) = (1, 0.2, 0.1), \lambda_N(x_2) = (0.6, 0.3, 0.1)$.

$\mu_N : X_N \to [0_N, 1_N]$ is defined as $\mu_N(x_1) = (0.8, 0.1, 0.1), \mu_N(x_2) = (1, 0, 0)$.

Then, $T_N = \{0, \lambda_N, \mu_N, X_N\}$ is a fuzzy neutrosophic topology on X_N.

Now, the intersection of fuzzy neutrosophic sets are λ_N, μ_N is $\lambda_N \wedge \mu_N =$ $\{(x_1, (0.8, 0.2, 0.1)), (x_2, (0.6, 0.3, 0.1))\}$. This implies that $(\lambda_N \wedge \mu_N) \in T_N$, then every fuzzy neutrosophic G_δ—set is fuzzy neutrosophic open. Hence (X_N, T_N) is a fuzzy neutrosophic P-space.

Example 3.2 Let $X_N = \{x_1, x_2\}$. The fuzzy neutrosophic sets α_N and β_N are defined on X_N as follows:

$\alpha_N : X_N \to [0_N, 1_N]$ is defined as, $\alpha_N(x_1) = (1, 0.2, 0.1), \alpha_N(x_2) = (0.5, 0.3, 0.2)$.

$\beta_N : X_N \to [0_N, 1_N]$ is defined as $\beta_N(x_1) = (0.6, 0.1, 0.3), \beta_N(x_2) = (0.7, 0.2, 0.2)$.

Then $T_N = \{0_N, \alpha_N, \beta_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. Now, the fuzzy neutrosophic sets are α_N, β_N is $\eta_N = \alpha_N \wedge \beta_N$ is a fuzzy neutrosophic G_δ—set. But $\eta_N \notin T_N$ in (X_N, T_N). Hence the fuzzy neutrosophic topological space (X_N, T_N) is not a fuzzy neutrosophic P-space.

Proposition 3.1 If λ_N is a non-zero fuzzy neutrosophic F_σ—set in a fuzzy neutrosophic P-Space (X_N, T_N), then λ_N is a fuzzy neutrosophic closed set in (X_N, T_N).

Proof. Given that λ_N is a non-zero fuzzy neutrosophic F_σ—set in (X_N, T_N), $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where the fuzzy neutrosophic sets $(\lambda_{N_i})'s$ are fuzzy neutrosophic closed in (X_N, T_N). Then $1_N - \lambda_N = 1_N - (\vee_{i=1}^{\infty}(\lambda_{N_i})) = \wedge_{i=1}^{\infty}(1_N - \lambda_{N_i})$. Now, $(\lambda_{N_i})'s$ are fuzzy neutrosophic closed in (X_N, T_N), implies that $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic open in (X_N, T_N). Hence we have $1_N - \lambda_N = \wedge_{i=1}^{\infty}(1_N - \lambda_{N_i})$, where $1_N - \lambda_{N_i} \in T_N$. Then $1_N - \lambda_N$ is a fuzzy neutrosophic G_δ—set in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic P-space, $1_N - \lambda_N$ is fuzzy neutrosophic open in (X_N, T_N). Therefore λ_N is a fuzzy neutrosophic closed set in (X_N, T_N).

Proposition 3.2 If the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic P-space, then $cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty} cl(\lambda_{N_i})$, where $\lambda_{N_i}'s$ are non-zero fuzzy neutrosophic closed sets in (X_N, T_N).

Proof. Let $\lambda_{N_i}'s$ be a non-zero fuzzy neutrosophic closed sets in a fuzzy neutrosophic P-space (X_N, T_N).Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$ is a non-zero fuzzyneutrosophic F_σ—set in (X_N, T_N). By Proposition 3.1., λ_N is a fuzzy neutrosophic closed set in (X_N, T_N). Hence $cl(\lambda_N) = \lambda_N$, which implies that $cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty}(\lambda_{N_i}) = \vee_{i=1}^{\infty}(cl(\lambda_{N_i}))$. [since $\lambda_{N_i}'s$ are fuzzy neutrosophic

closed, $cl(\lambda_{N_i}) = \lambda_{N_i}$]. Therefore $cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty} cl(\lambda_{N_i})$, where λ_{N_i}'s are fuzzy neutrosophic closed sets in (X_N, T_N).

Proposition 3.3 If $(\lambda_{N_i})'s$ are fuzzy neutrosophic regular-closed sets in a fuzzy neutrosophic P-space (X_N, T_N), then $cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty}(\lambda_{N_i})$.

Proof. Let $(\lambda_{N_i})'s$ be fuzzy neutrosophic regular-closed sets in a fuzzy neutrosophic P-space (X_N , T_N). Then $(\lambda_{N_i})'s$ are fuzzy neutrosophic closed sets in (X_N, T_N), which implies that (1_N — λ_{Ni})'s are fuzzy neutrosophic open sets in (X_N, T_N). Let $\mu_N = \wedge_{i=1}^{\infty}[1_N - (\lambda_{N_i})]$. Then μ_N is a non-zero fuzzy neutrosophic G_δ—set in (X_N, T_N). Since the fuzzy neutrosophic topological space (X_N , T_N) is a fuzzy neutrosophic P-space, $\text{int}(\mu_N) = \mu_N$, which implies that $\text{int}(\wedge_{i=1}^{\infty}[1_N - (\lambda_{N_i})]) = \wedge_{i=1}^{\infty}[1_N - (\lambda_{N_i})]$. Then $1_N - cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = 1_N - \vee_{i=1}^{\infty}(\lambda_{N_i})$. Hence we have $cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty}(\lambda_{N_i})$.

Proposition 3.4 If the fuzzy neutrosophic topological space (X_N , T_N) is a fuzzy neutrosophic P-space and if λ_N is a fuzzy neutrosophic first category set in (X_N , T_N), then λ_N is a fuzzy neutrosophic dense set in (X_N, T_N).

Proof. Assume that the contrary. Suppose that λ_N is a fuzzy neutrosophic first category set in (X_N, T_N) such that $cl(\lambda_N) = 1_N$. Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where $(\lambda_{N_i})'s$ are fuzzy neutrosophic nowhere dense sets in (X_N, T_N). Now, 1_N — $cl(\lambda_{Ni})$ is a fuzzy neutrosophic open set in (X_N, T_N). Let $\mu_N = \wedge_{i=1}^{\infty}[1_N - (\lambda_{N_i})]$. Then μ_N is a non-zero fuzzy neutrosophic G_δ— set in (X_N, T_N). Now, we have
$\wedge_{i=1}^{\infty}[1_N - cl(\lambda_{N_i})] = 1_N - \vee_{i=1}^{\infty}(cl(\lambda_{N_i})) \le 1_N - \vee_{i=1}^{\infty}(\lambda_{N_i}) = 1_N - \lambda_N$.
Hence $\mu_N \le 1_N — \lambda_N$. Then $int(\mu_N) \le int(1_N — \lambda_N) = 1_N — cl(\lambda_N) = 1_N — 1_N = 0_N$. That is, $int(\mu_N) = 0_N$. Since (X_N, T_N) is a fuzzy neutrosophic P-space, $\mu_N = int(\mu_N)$ which implies that $\mu_N = 0_N$, a contradiction to μ_N being a non-zero fuzzy neutrosophic G_δ—set in (X_N, T_N). Hence $cl(\lambda_N) \neq 1_N$.

Proposition 3.5 If the fuzzy neutrosophic topolgical space (X_N, T_N) is a fuzzy neutrosophic P-space and if λ_N is afuzzy neutrosophic first category set in (X_N, T_N), then λ_N is not a fuzzy neutrosophic nowhere dense set in (X_N, T_N).

Proof: Let λ_N be a fuzzy neutrosophic first category set in fuzzy neutrosophic P-space (X_N, T_N). Then, we have $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{Ni})$, where $(\lambda_{N_i})'s$ are fuzzy neutrosophic nowhere dense sets in (X_N, T_N). Now, $\text{int}\, cl(\lambda_N) = \text{int}\, cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) \ge \text{int}[\vee_{i=1}^{\infty} cl(\lambda_{N_i})]$ and $[\vee_{i=1}^{\infty} cl(\lambda_{N_i})]$ is a fuzzy neutrosophic F_σ - set in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic P-space, by proposition 3.1, $[\vee_{i=1}^{\infty} cl(\lambda_{N_i})]$ is a non-zero fuzzy neutrosophic regular-closed set in (X_N, T_N). Also interior of a fuzzy neutrosophic closed is a fuzzy neutrosophic regular-open set, $\text{int}[\vee_{i=1}^{\infty} cl(\lambda_{N_i})]$ is a non-zero fuzzy neutrosophic regular-open set in (X_N, T_N). Hence we have $0_N \neq \text{int}[\vee_{i=1}^{\infty} cl(\lambda_{N_i})] \le \text{int}\, cl(\lambda_N)$, implies that $\text{int}\, cl(\lambda_N) \neq 0_N$. Therefore λ_N is not a fuzzy neutrosophic nowhere dense set in (X_N, T_N).

Proposition 3.6 If λ_N is a fuzzy neutrosophic first category set in a fuzzy neutrosophic P-space (X_N, T_N) such that $\mu_N \leq 1_N - \lambda_N$, where μ_N is a non-zro fuzzy neutrosophic dense G_δ-set in (X_N, T_N), then λ_N is a fuzzy neutrosophic nowhere dense set in (X_N, T_N).

Proof: Let λ_N be a fuzzy neutrosophic first category set in (X_N, T_N).Then, $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{Ni})$, where $(\lambda_{N_i})'s$ are fuzzy neutrosophic nowhere dense sets in (X_N, T_N). Now, $1_N - cl(\lambda_{N_i})$ is a fuzzy neutrosophic open set in (X_N, T_N). Let $\mu_N = \wedge_{i=1}^{\infty}[1_N - cl(\lambda_{N_i})]$. Then μ_N is a non-zero fuzzy neutrosophic dense G_δ-set in (X_N, T_N). Now, we have $\wedge_{i=1}^{\infty}[1_N - cl(\lambda_{N_i})] = 1_N - \vee_{i=1}^{\infty}(cl(\lambda_{N_i})) \leq 1_N - \vee_{i=1}^{\infty}(\lambda_{N_i}) = 1_N - \lambda_N$. Hence $\mu_N \leq (1_N - \mu_N)$. Then, we have $\lambda_N \leq 1_N - cl\,\mathrm{int}(\mu_N)$. Now, $\mathrm{int}\,cl(\lambda_N) \leq \mathrm{int}\,cl(1_N - \mu_N)$, which implies that $\mathrm{int}\,cl(\lambda_N) \leq 1_N - cl\,\mathrm{int}(\mu_N)$. Since (X_N, T_N) is a fuzzy neutrosophic P-space, the fuzzy neutrosophic G_δ-set μ_N is a fuzzy neutrosophic open in (X_N, T_N) and $\mathrm{int}(\mu_N) = \mu_N$. Therefore $\mathrm{int}\,cl(\lambda_N) \leq 1_N - cl(\mu_N) = 1_N - 1_N = 0_N$. (since μ_N is a fuzzy neutrosophic dense). Then $\mathrm{int}\,cl(\lambda_N) = 0_N$ and hence λ_N is a fuzzy neutrosophic nowhere dense set in (X_N, T_N).

Proposition 3.7 If λ_N is a fuzzy neutrosophic first category set in a fuzzy neutrosophic P-space (X_N, T_N) such that $\mu_N \leq 1_N - \lambda_N$, where μ_N is a non-zero fuzzy neutrosophic dense G_δ-set in (X_N, T_N), then (X_N, T_N) is a fuzzy neutrosophic Baire Space.

Proof. Let λ_N be a fuzzy neutrosophic first category set in (X_N, T_N). As in the proof of the Proposition 3.6., we have $\mathrm{int}\,cl(\lambda_N) = 0_N$. Then $\mathrm{int}(\lambda_N) \leq \mathrm{int}\,cl(\lambda_N)$, implies that $\mathrm{int}(\lambda_N) = 0_N$ and hence by Theorem 2.1, (X_N, T_N) is a fuzzy neutrosophic Baire Space.

Proposition 3.8. If the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic P-space and if λ_N is a fuzzy neutrosophic dense and fuzzy first category set in (X_N, T_N), then there is no non-zero fuzzy neutrosophic G_δ-set μ_N in (X_N, T_N) such that $\mu_N \leq 1_N - \lambda_N$.

Proof. Let λ_N be a fuzzy neutrosophic first category set in (X_N, T_N). As in the proof of the Proposition 3.6, we have a neutrosophic G_δ-set μ_N in (X_N, T_N) such that $\mu_N \leq 1_N - \lambda_N$. Then $\mathrm{int}(\mu_N) \leq (1_N - \lambda_N)$, implies that $\mathrm{int}(\mu_N) \leq 1_N - cl(\lambda_N) = 1_N - 1_N = 0_N$. [Since λ_N is a fuzzy neutrosophic dense, $cl(\lambda_N) = 1_N$]. That is, $\mathrm{int}(\mu_N) = 0_N$. Since (X_N, T_N) is a fuzzy neutrosophic P-space, $\mathrm{int}(\mu_N) = \mu_N$ and hence we have $\mu_N = 0_N$. Hence, if λ_N is a fuzzy neutrosophic dense and fuzzy neutrosophic first category set in (X_N, T_N), then there is no non-zero fuzzy neutrosophic G_δ-set μ_N in (X_N, T_N) such that $\mu_N \leq 1_N - \lambda_N$.

4 WEAK FUZZY NEUTROSOPHIC P-SPACES

Definition 4.1 A fuzzy neutrosophic topological space (X_N, T_N) is called a weak fuzzy neutrosophic P-space if the countable intersection of fuzzy neutrosophic regular-open sets in (X_N, T_N) is a fuzzy neutrosophic regular-open set in (X_N, T_N). That is, $\wedge_{i=1}^{\infty}(\lambda_{N_i})$ is fuzzy neutrosophic regular-open in (X_N, T_N), where $(\lambda_{N_i})'s$ are fuzzy neutrosophic regular-open sets in (X_N, T_N).

Example 4.1 Let $X_N = \{x_1, x_2\}$. The fuzzy neutrosophic sets, λ_N, μ_N are defined on X_N as follows:

$\lambda_N : X_N \to [0_N, 1_N]$ is defined as $\lambda_N(x_1) = (0.6, 0.2, 0.2), \lambda_N(x_2) = (0.3, 0.5, 0.2)$.

$\mu_N : X \to [0_N, 1_N]$ is defined as $\mu_N(x_1) = (0.5, 0.3, 0.2), \mu_N(x_2) = (0.2, 0.2, 0.6)$.

Then, $T_N = \{0_N, \lambda_N, \mu_N, \lambda_N \vee \mu_N, \lambda_N \wedge \mu_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N.

Now, λ_N, μ_N, $\lambda_N \vee \mu_N$ and $\lambda_N \wedge \mu_N$ are fuzzy neutrosophic regular-open sets in (X_N, T_N) and

$(\lambda_N) \wedge (\mu_N) \wedge (\lambda_N \vee \mu_N) \wedge (\lambda_N \wedge \mu_N) = \lambda_N \wedge \mu_N$ is a fuzzy neutrosophic regular-open set in (X_N, T_N). Hence (X_N, T_N) is a weak fuzzy neutrosophic P-space.

Proposition 4.1. A fuzzy neutrosophic topological space (X_N, T_N) is a weak fuzzy neutrosophic P-space if and only if $\vee_{i=1}^{\infty}(\mu_{N_i})$, where $(\mu_{N_i})'s$ are fuzzy neutrosophic regular-closed sets in (X_N, T_N) is fuzzy neutrosophic regular-closed in (X_N, T_N).

Proof: Let (X_N, T_N) be a weak fuzzy neutrosophic P-space. Then $\operatorname{int} cl(\wedge_{i=1}^{\infty}(\lambda_{N_i})) = \wedge_{i=1}^{\infty}(\lambda_{N_i})$, where $(\lambda_{N_i})'s$ are fuzzy neutrosophic regular-open sets in (X_N, T_N). Now $1_N - [\operatorname{int} cl(\wedge_{i=1}^{\infty}(\lambda_{N_i})] = 1_N - \wedge_{i=1}^{\infty}(\lambda_{N_i})$, implies that $cl \operatorname{int}[(\vee_{i=1}^{\infty}(1_N - \lambda_{N_i}))] = \vee_{i=1}^{\infty}(1_N - \lambda_{N_i})$. Let $\mu_{N_i} = (1_N - \lambda_{N_i})$ Since λ_{N_i} is a fuzzy neutrosophic regular-open set in (X_N, T_N), μ_{N_i} is a fuzzy neutrosophic regular-closed set in (X_N, T_N). Then we have $cl \operatorname{int}(\vee_{i=1}^{\infty}(\mu_{N_i})) = \vee_{i=1}^{\infty}(\mu_{N_i})$. Hence $\vee_{i=1}^{\infty}(\mu_{N_i})$ is a fuzzy neutrosophic regular-closed in (X_N, T_N). Conversely, suppose that $cl \operatorname{int}(\vee_{i=1}^{\infty}(\mu_{N_i})) = \vee_{i=1}^{\infty}(\mu_{N_i})$, where μ_{N_i}'s are fuzzy neutrosophic regular-closed sets in (X_N, T_N). Then $1_N - cl \operatorname{int}(\vee_{i=1}^{\infty}(\mu_{N_i})) = 1_N - \vee_{i=1}^{\infty}(\mu_{N_i})$, which implies that $\operatorname{int} cl(\wedge_{i=1}^{\infty}(1_N - \mu_{N_i})) = \wedge_{i=1}^{\infty}(1_N - \mu_{N_i})$, where $(1_N - \mu_{N_i})'s$ are fuzzy neutrosophic regular-open sets in (X_N, T_N). Therefore (X_N, T_N) is a weak fuzzy neutrosophic P-space.

Proposition 4.2 If a fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic P-space, then (X_N, T_N) is a weak fuzzy neutrosophic P-space.

Proof. Let $(\lambda_{N_i})'s$ be fuzzy neutrosophic regular-closed sets in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic P-space by proposition 3.3, we have $cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty}(\lambda_{N_i})$. Now, $cl \operatorname{int}(\vee_{i=1}^{\infty}(\lambda_{N_i})) \leq cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty}(\lambda_{N_i})$. That is $cl \operatorname{int}(\vee_{i=1}^{\infty}(\lambda_{N_i})) \leq \vee_{i=1}^{\infty}(\lambda_{N_i})$ ……..(1).

Since $(\lambda_{N_i})'s$ are fuzzy neutrosophic regular-closed sets in (X_N, T_N), $cl \operatorname{int}(\lambda_{N_i}) = \lambda_{N_i}$ .Then $\vee_{i=1}^{\infty} cl \operatorname{int}(\lambda_{N_i}) = \vee_{i=1}^{\infty}(\lambda_{N_i})$, which implies that $\vee_{i=1}^{\infty}(\lambda_{N_i}) \leq cl \operatorname{int}(\vee_{i=1}^{\infty}(\lambda_{N_i}))$ ……..(2).

From (1) and (2), we have, $cl \operatorname{int}(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty}(\lambda_{N_i})$. Hence by proposition 4.1, (X_N, T_N) is a weak fuzzy neutrosophic P-space.

Remark 4.1 A weak fuzzy neutrosophic P-space need not be a fuzzy neutrosophic P-space. For, consider the following example.

Example 4.2 Let $X_N = \{a, b\}$. The fuzzy neutrosophic sets λ_N and μ_N are defined on X_N as follows:

$\lambda_N : X_N \to [0_N, 1_N]$ is defined as $\lambda_N = \{(a, (0.5, 0.3, 0.4)), (b, (0.4, 0.6, 0.4))\}$

$\mu_N : X_N \to [0_N, 1_N]$ is defined as $\mu_N = \{(a, (0.5, 0.3, 0.3)), (b, (0.4, 0.5, 0.4))\}$

Then, $T_N = \{0_N, \lambda_N, \mu_N, \lambda_N \vee \mu_N, \lambda_N \wedge \mu_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. Now, the fuzzy neutrosophic sets μ_N and $\lambda_N \vee \mu_N$ are fuzzy neutrosophic regular- open sets in (X_N, T_N) and $(\mu_N) \wedge (\lambda_N \vee \mu_N) = \mu_N$ is a fuzzy neutrosophic regular- open set in (X_N, T_N) and hence (X_N, T_N) is a weak fuzzy neutrosophic P-space. But (X_N, T_N) is not a fuzzy neutrosophic P-space, since the fuzzy neutrosophic G_δ—set $(\lambda_N \wedge \mu_N) \wedge (\lambda_N \vee \mu_N)$ is not fuzzy neutrosophic open in (X_N, T_N).

5 FUZZY NEUTROSOPHIC ALMOST P-SPACES

Definition 5.1 A fuzzy neutrosophic topological space (X_N, T_N) is called a fuzzy neutrosophic almost P-space if for eevery non-zero fuzzy G_δ - set λ_N in (X_N, T_N) , $\text{int}(\lambda_N) \neq 0_N$ in (X_N, T_N)

Example 5.1 Let $X_N = \{a, b\}$. The fuzzy neutrosophic sets α_N, β_N and γ_N are defined on X_N as follows:

$\alpha_N : X_N \to [0_N, 1_N]$ is defined as $\alpha_N = \{(a, (0.5, 0.6, 0.4)), (b, (0.6, 0.4, 0.5))\}$

$\beta_N : X_N \to [0_N, 1_N]$ is defined as $\beta_N = \{(a, (0.4, 0.7, 0.5)), (b, (0.5, 0.6, 0.5))\}$

$\gamma_N : X_N \to [0_N, 1_N]$ is defined as $\gamma_N = \{(a, (0.5, 0.8, 0.6)), (b, (0.8, 0.7, 0.5))\}$

Then, $T_N = \{0_N, \alpha_N, \beta_N, \gamma_N, \alpha_N \vee \beta_N, \beta_N \vee \gamma_N, \alpha_N \vee \gamma_N, \alpha_N \wedge \beta_N, \beta_N \wedge \gamma_N, \alpha_N \wedge \gamma_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. Now, for the fuzzy neutrosophic G_δ—sets $(\alpha_N \wedge \beta_N)$, $(\beta_N \wedge x\, \gamma_N)$ and $(\alpha_N \wedge \gamma_N)$ in (X_N, T_N), $\text{int}[(\alpha_N \wedge \beta_N) \wedge (\beta_N \wedge \gamma_N) \wedge (\alpha_N \wedge \gamma_N)] = (\beta_N \wedge \gamma_N) \neq 0_N$. Hence (X_N, T_N) is a fuzzy neutrosophic almost P-space.

Remark 5.1 (1) Clearly every fuzzy neutrosophic P-space is a fuzzy neutrosophic almost fuzzy neutrosophic P-space, since for every non-zero fuzzy neutrosophic G_δ set δ_N in (X_N, T_N), we have $\text{int}(\delta_N) = \delta_N \neq 0_N$. But the converse need not be true. For in example 4.2, for every non-zero fuzzy neutrosophic G_δ set δ_N in (X_N, T_N), we have $\text{int}(\delta_N) \neq 0_N$ in (X_N, T_N). Hence (X_N, T_N) is a fuzzy eutrosophic almost P-space, but (X_N, T_N) is not a fuzzy neutrosophic P-space, since the fuzzy neutrosophic G_δ -set $\{[\lambda_N \vee \mu_N] \wedge [\lambda_N \vee \gamma_N] \wedge [\mu_N \vee \gamma_N]\}$ is not fuzzy neutrosophic open in (X_N, T_N).

(2) A fuzzy neutrosophic almost P-space need not be a weak fuzzy neutrosophic P-space. For, consider the following example.

Example 5.2 Let $X_N = \{a, b\}$. The fuzzy neutrosophic sets A_N, B_N and C_N are defined on X_N as follows:

$A_N : X_N \to [0_N, 1_N]$ is defined as $A_N = \{(a, (0.8, 0.6, 0.7)), (b, (0.7, 0.6, 0.6))\}$

$B_N : X_N \to [0_N, 1_N]$ is defined as $B_N = \{(a, (0.9, 0.8, 0.6)), (b, (0.6, 0.5, 0.6))\}$

$C_N : X_N \to [0_N, 1_N]$ is defined as $C_N = \{(a, (0.7, 0.5, 0.9)), (b, (0.9, 0.6, 0.8))\}$

Then, $T_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, B_N \vee C_N, C_N \vee A_N, A_N \wedge B_N, B_N \wedge C_N, A \wedge C_N, A_N \vee B_N \vee C_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. In every non zero fuzzy neutrosophic G_δ-sets $(B_N \vee C_N) \wedge (B_N \wedge C_N)$, we have $\text{int}[(B_N \vee C_N) \wedge (B_N \wedge C_N)] \neq 0_N$ in (X_N, T_N). Hence (X_N, T_N) is a fuzzy neutrosophic almost P-space, but (X_N, T_N) is not a weak fuzzy neutrosophic P-space.

(3) A weak fuzzy neutrosophic P-space need not be a fuzzy neutrosophic almost P-space. For, consider the for consider the following examples:

Example 5.3 Let $X_N = \{a, b\}$. The fuzzy neutrosophic sets λ_N, μ_N and γ_N are defined on X_N as follows :

$\lambda_N : X_N \to [0_N, 1_N]$ is defined as $\lambda_N = \{(a, (0, 0.4, 0.5)), (b, (0.4, 0.5, 0.6))\}$

$\mu_N : X_N \to [0_N, 1_N]$ is defined as $\mu_N = \{(a, (0.6, 0, 0.4)), (b, (0.4, 0.6, 0.6))\}$

$\gamma_N : X_N \to [0_N, 1_N]$ is defined as $\gamma_N = \{(a, (0.5, 0.6, 0)), (b, (0.6, 0, 0.5))\}$

Then, $T_N = \{0_N, \lambda_N, \mu_N, \gamma_N, \lambda_N \vee \mu_N, \mu_N \vee \gamma_N, \lambda_N \vee \gamma_N, \lambda_N \wedge \mu_N, \mu_N \wedge \gamma_N, \lambda_N \wedge \gamma_N, \lambda_N \vee \mu_N \vee \gamma_N, \lambda_N \wedge \mu_N \wedge \gamma_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. Now, $(\lambda_N \wedge \mu_N), (\lambda_N \wedge \gamma_N), (\lambda_N \wedge \mu_N \wedge \gamma_N), (\lambda_N \vee \gamma_N)$ are fuzzy neutrosophic regular-open sets in (X_N, T_N) and $[(\lambda_N \wedge \mu_N) \wedge (\lambda_N \wedge \gamma_N) \wedge (\lambda_N \wedge \mu_N \wedge \gamma_N) \wedge (\lambda_N \vee \gamma_N)] = (\lambda_N \wedge \gamma_N)$ is a fuzzy neutrosophic regular-open set in (X_N, T_N) and hence (X_N, T_N) is a weak fuzzy neutrosophic P-space. But (X_N, T_N) is not a fuzzy neutrosophic almost P- space, since for the non-zero fuzzy neutrosophic G_δ—set $\{(\lambda_N \vee \mu_N) \wedge (\mu_N \vee \gamma_N) \wedge (\lambda_N \vee \gamma_N) \wedge (\lambda_N \wedge \mu_N)\}$, we have $\text{int}\{[(\lambda_N \vee \mu_N) \wedge (\mu_N \vee \gamma_N) \wedge (\lambda_N \vee \gamma_N) \wedge (\lambda_N \wedge \mu_N)]\} = 0_N$.

Proposition 5.1 If a fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic almost P-space if and only if the fuzzy neutrosophic F_σ -set λ_N such that $cl(\lambda_N) = 1_N$ in (X_N, T_N) is 1_{X_N}.

Proof. Let $\lambda_N (\neq 1_N)$ be a fuzzy neutrosophic F_σ -set such that $cl(\lambda_N) = 1_N$ in (X_N, T_N). Then, $1_N - \lambda_N$ is a non-zero fuzzy neutrosophic G_δ -set in (X_N, T_N). Now, $1_N - cl(\lambda_N) = 0_N$, implies that $\text{int}(1_N - \lambda_N) = 0_N$, which is a contradiction to (X_N, T_N) being a fuzzy neutrosophic almost P-space in which $\text{int}(\delta_N) \neq 0_N$ for any non-zero fuzzy neutrosophic G_δ -set δ_N in (X_N, T_N). Hence our assumption that $cl(\lambda_N) = 1_N$ does not hold. Therefore there is no non-zero fuzzy neutrosophic F_σ -set (other than 1_{X_N}) in (X_N, T_N) such that $cl(\lambda_N) = 1_N$.

Conversely, let us assume that the only fuzzy neutrosophic F_σ -set λ_N in (X_N, T_N) such that $cl(\lambda_N) = 1_N$ is 1_{X_N}. Let λ_N be a fuzzy neutrosophic G_δ -set in (X_N, T_N). Then $\lambda_N = \wedge_{i=1}^{\infty}(\lambda_{N_i})$,where $(\lambda_{N_i})'s$ are fuzzy neutrosophic open sets in (X_N, T_N).Now, $1_N - \lambda_N = \vee_{i=1}^{\infty}(1_N - \lambda_{N_i})$. Then $1_N - \lambda_N$ is a fuzzy neutrosophic F_σ -set in (X_N, T_N). By hypothesis, $cl(1_N - \lambda_N) \neq 1_N$. Then, $1_N - \text{int}(\lambda_N) \neq 1_N$, this implies that $\text{int}(\lambda_N) \neq 0_N$. Hence for the non-zero fuzzy neutrosophic G_δ -set λ_N in (X_N, T_N), we have $\text{int}(\lambda_N) \neq 0_N$ and therefore (X_N, T_N) is a fuzzy neutrosophic almost P-space.

Remark 5.2: If a fuzzy neutrosophic topological space (X_N, T_N) has non-zero fuzzy neutrosophic nowhere dense fuzzy neutrosophic G_δ -sets, then (X_N, T_N) is not a fuzzy neutrosophic almost P-space. For consider the following proposition.

Proposition 5.2. If λ_N is a non-zero fuzzy neutrosophic nowhere dense fuzzy neutrosopic G_δ -set in a fuzzy neutrosophic topological space (X_N, T_N) , then (X_N, T_N) is not a fuzzy neutrosophic almost P-space.

Proof. Let λ_N be a non-zero fuzzy neutrosophic nowhere dense fuzzy neutrosophic G_δ -set λ_N in (X_N, T_N). Then $\text{int}(\lambda_N) \leq \text{int}\, cl(\lambda_N)$ and $\text{int}\, cl(\lambda_N) = 0_N$, implies that $\text{int}(\lambda_N) = 0_N$.Hence for the non-zero fuzzy

neutrosophic G_δ -set λ_N in (X_N, T_N) , $\text{int}(\lambda_N) = 0_N$ in (X_N, T_N) Theorefore (X_N, T_N) is not a fuzzy neutrosophic almost P-space.

6 CONCLUSION

Inconclusion Fuzzy neutrosophic P-space $\rightarrow$ Fuzzy neutrosophic weak P-space $\Leftrightarrow$ Fuzzy neutrosophic almokst P-space. The systematic development and analysis of these fuzzy neutrosophic spaces enhance our ability to model and study complex systems under uncertainty, supporting advancements in applied Mathematics, artificial intelligence and decision support systems.

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A Short Note On Fuzzy Neutrosophic μ-Strongly Category Sets and Fuzzy Neutrosophic μ-Strongly Baire Spaces

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Abstract

In this paper, we introduced some types of fuzzy neutrosophic sets such as μ-nowhere dense set, μ-first and second category, μ-residual, μ-strongly nowhere dense, μ-strongly first and second category, μ-strongly residual and their characterizations and examples are explained. Also we discuss about fuzzy neutrosophic μ-strongly Baire Space and its properties are to be described.

Keywords: Fuzzy neutrosophic dense, fuzzy neutrosophic μ-nowhere dense set, fuzzy neutrosophic μ-category sets, fuzzy neutrosophic μ-strongly nowhere dense, fuzzy neutrosophic μ-strongly category sets, fuzzy neutrosophic μ-strongly residual and fuzzy neutrosophic μ-strongly Baire space.

1 Introduction

The fuzzy idea was invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [22]. The important concept of fuzzy topological space was offered by C.L. Chang [3]. The idea of fuzzy σ- Baire Spaces was introduced by G. Thangaraj and E. Poongothai [14]. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [21] introduced fuzzy neutrosophic topological spaces. The idea of fuzzy neutrosophic Baire spaces was introduced by E. Poongothai and E. Padmavathi [11].

In this paper, we introduced some types of fuzzy neutrosophic sets such as μ-nowhere dense set, μ-first and second category, μ-residual, μ-strongly nowhere dense, μ-strongly first and second category, μ-strongly residual and their characterizations and examples are explained. Also we discuss about fuzzy neutrosophic μ-strongly Baire Space and its properties are to be described.

2 Preliminaries

Definition 2.1 [2] A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ x \in X$ where $T, I, F : X \to [0,1]$ and $0 \leqslant T_A(x) + I_A(x) + F_A(x) \leqslant 3$.

Definition 2.2 [2] A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B(i.e.,) $A \subseteq B$ for all x if $T_A(x) \leqslant T_B(x),\ I_A(x) \leqslant I_B(x),\ F_A(x) \geqslant F_B(x)$.

Definition 2.3 [2] Let X be a non-empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle,\ B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x))\rangle$$
$$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x))\rangle$$

Definition 2.4 [2] The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$.

Definition 2.5 [2] A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N.

Definition 2.6 [2] A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N.

Definition 2.7 [2] The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as $A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$ where $T_{A^C}(x) = F_A(x),\ I_{A^C}(x) = 1 - I_A(x),\ F_{A^C}(x) = T_A(x)$. The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A$.

Definition 2.8 [1] A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X satisfying the following axioms.

(i) $0_N,\ 1_N \in \tau$

(ii) $A_1 \cap A_2 \in \tau$ for any $A_1,\ A_2 \in \tau$

(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$

In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [1] The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10 [1] Let (X, τ) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$\begin{aligned} int(A) &= \cup\{G : G \textit{ is a fuzzy neutrosophic open set in} X \textit{ and } G \subseteq A\} \\ cl(A) &= \cap\{G : G \textit{ is a fuzzy neutrosophic closed set in} X \textit{ and } A \subseteq G\} \end{aligned}$$

Definition 2.11 [1] Let (X, τ) be a fuzzy neutrosophic topological space over X. Then the following properties hold, (i) $cl(A^C) = (int\ A)^C$, (ii) $int(A^C) = (cl\ A)^C$.

Definition 2.12 [11] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic F_σ-set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$

Definition 2.13 [11] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic G_δ-set in (P, τ_N) if $A_N = \bigwedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$

Definition 2.14 [11] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic dense if there exist no $fnCS\ B_N$ in (P, τ_N) s.t $A_N \subset B_N \subset 1_X$. That is $fn(A_N)^- = 1_N$

Definition 2.15 [11] A fy. neutrosophic set A_N in a fy. neutrosophic top. space (P, τ_N) is called a fy. neutrosophic nowh. dense set if there exist no non zero $fnOS\ B_N$in (P, τ_N) s.t $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$

Definition 2.16 [11] Let (P, τ_N) be a fy. neutrosophic top. space. A fy. neutrosophic set A_N in (P, τ_N) is called fy. neutrosophic one category set if $A_N = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Any other fy. neutrosophic set in (P, τ_N) is said to be of fy. neutrosophic two category.

Definition 2.17 [11] A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic one category space if the fy. neutrosophic set 1_X is a fy. neutrosophic one category set in (P, τ_N). That is $1_X = \bigvee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fy. neutrosophic nowh. dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fy. neutrosophic two category space.

Definition 2.18 [11] Let A_N be a fy. neutrosophic one category set in (P, τ_N). Then $\bar{A_N}$ is called fy. neutrosophic re. set in (P, τ_N).

Definition 2.19 [11] A fy. neutrosophic top. space (P, τ_N) is called fy. neutrosophic Baire space if $fn(\bigvee_{i=1}^{\infty}(A_{N_i}))^+ = 0_N$, where (A_{N_i})'s are fy. neutrosophic nowh. dense sets in (P, τ_N).

Theorem 2.1 [11] Let (P, τ_N) be a fy. neutrosophic top. space. Then the following are equivalent

(1) (P, τ_N) is a fy. neutrosophic Baire space.

(2) $fn(A_N)^+ = 0_N$, for every fy. neutrosophic one category set A_N in (P, τ_N).

(3) $fn(B_N)^+ = 1_N$, for every fy. neutrosophic re. set B_N in (P, τ_N).

Definition 2.20 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic Set λ_N in (X_N, T_N) is called a fuzzy neutrosophic σ-nowhere dense set if λ_N is a fuzzy neutrosophic F_σ-set in (X_N, T_N) such that $int(\lambda_N) = 0_N$.

Definition 2.21 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set λ_N in (X_N, T_N) is called fuzzy neutrosophic σ- first category set if $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic σ-nowhere dense sets in (X_N, T_N). Any other fuzzy neutrosophic set in (X_N, T_N) is said to be fuzzy neutrosophic σ- second category sets in (X_N, T_N).

Definition 2.22 [5] Let λ_N be a fuzzy neutrosophic σ-first category set in (X_N, T_N). Then $1_N - \lambda_N$ is called a fuzzy neutrosophic σ-residual set in (X_N, T_N).

Definition 2.23 [5] A fuzzy neutrosophic topological space (X_N, T_N) is called fuzzy neutrosophic σ-first category space if the fuzzy neutrosophic set 1_{X_N} is a fuzzy neutrosophic σ-first category set in (X_N, T_N). That is $1_{X_N} = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic σ- nowhere dense sets in (X_N, T_N). Otherwise (X_N, T_N) will be called a fuzzy neutrosophic σ- second category space.

Definition 2.24 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then (X_N, T_N) is called a fuzzy neutrosophic σ-Baire space if $int(\bigvee_{i=1}^{\infty}(\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic σ- nowhere dense sets in (X_N, T_N).

Theorem 2.2 [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then the following are equivalent

(1) (X_N, T_N) is a fuzzy neutrosophic σ-Baire space.

(2) $int(\lambda_N) = 0_N$, for every fuzzy neutrosophic σ- first category set λ_N in (X_N, T_N).

(3) $cl(\mu_N) = 1_N$, for every fuzzy neutrosophic σ-residual set μ_N in (X_N, T_N).

3 Fuzzy Neutrosophic μ-Strongly Nowhere Dense Sets

Definition 3.1 *Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic λ_N is called fuzzy neutrosopic μ-nowhere dense set in (X_N, T_N) if $[\lambda_N \wedge (1_N - \lambda_N)]$ is a fuzzy neutrosophic nowhere dense set in (X_N, T_N).*

Definition 3.2 *Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set λ_N is called fuzzy neutrosophic μ-first category in (X_N, T_N) if $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic μ-nowhre dense sets in (X_N, T_N). Otherwise are called as fuzzy neutrosophic μ-second category set in (X_N, T_N).*

Definition 3.3 *The complement of fuzzy neutrosopic μ-first category sets are called fuzzy neutrosophic μ-residual set in (X_N, T_N).*

Definition 3.4 *let (X_N, T_N) be a fuzzy neturosophic topological space. A fuzzy neutrosophic set λ_N is called fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N) if $[\lambda_N \wedge (1_N - \lambda_N)]$ is a fuzzy neutrosophic μ-nowhere dense set in (X_N, T_N). That is, λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N) if $\mu - \{Intcl[\lambda_N \wedge (1_N - \lambda_N)]\} = 0_N$ in $(X_N.T_N)$.*

Example 3.1 *Let $X_N = \{a, b, c\}$ the fuzzy neutrosophic sets A_N, B_N and C_N are defined on X_N as follows:*

$A_N : X_N \to [0_N, 1_N]$ is defined as,

$A_N = \{\langle a, (0.6, 0.7, 0.6)\rangle, \langle b, (0.5, 0.6, 0.7)\rangle, \langle c, (0.7, 0.6, 0.6)\rangle\}$

$B_N : X_N \to [0_N, 1_N]$ is defined as,

$B_N = \{\langle a, (0.5, 0.6, 0.7)\rangle, \langle b, (0.8, 0.6, 0.6)\rangle, \langle c, (0.6, 0.5, 0.5)\rangle\}$

$C_N : X_N \to [0_N, 1_N]$ is defined as,

$C_N = \{\langle a, (0.7, 0.6, 0.8)\rangle, \langle b, (0.8, 0.8, 0.6)\rangle, \langle c, (0.5, 0.5, 0.5)\rangle\}$

Then, $T_N = \{0_N, A_N, B_N, C_N, A_N \vee B_N, B_N \vee C_N, A_N \wedge B_N, B_N \wedge C_N, A_N \wedge C_N A_N \wedge B_N \wedge C_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. We computation see that μ-$Intcl[(A_N \wedge B_N \wedge C_N) \wedge (1_N - (A_N \wedge B_N \wedge C_N))] = 0_N$. Hence, $(A_N \wedge B_N \wedge C_N)$ is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N).

Proposition 3.1 *If λ_N is a fuzzy neutrosophic μ-nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N), then λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic μ-nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N), then $\mu - Intcl(\lambda_N) = 0_N$ in (X_N, T_N). Since $[\lambda_N \wedge (1_N - \lambda_N)] \leqslant \lambda_N$ in (X_N, T_N). We obtain that $\mu - Intcl[\lambda_N \wedge (1_N - \lambda_N)] \leqslant \mu - Intcl(\lambda_N)$

and hence $\mu - \{Intcl[\lambda_N \wedge (1_N - \lambda_N)]\} \leqslant 0_N$. That is $\mu - \{Intcl[\lambda_N \wedge (1_N - \lambda_N)]\} = 0_N$. Hence λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N).

Proposition 3.2 *If $(1_N - \lambda_N)$ is a fuzzy neutrosophic μ-nowhere dense set in a fuzzy neutrosophic topological space in (X_N, T_N), then λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N).*

Proof. Suppose that $(1_N - \lambda_N)$ is a fuzzy neutrosophic μ-nowhere dense set in a fuzzy neutrosophic topological space (X_N, T_N), then $\mu - [Intcl(1_N - \lambda_N)] = 0_N$ in (X_N, T_N). Since $[\lambda_N \wedge (1_N - \lambda_N)] \leqslant (1_N - \lambda_N), \mu - \{Intcl[\lambda_N \wedge (1_N - \lambda_N)]\} \leqslant \mu - [Intcl(1_N - \lambda_N)]$ and hence $\mu - \{Intcl[\lambda_N \wedge (1_N - \lambda_N)]\} \leqslant 0_N$. implies that $\mu - \{Intcl[\lambda_N \wedge (1_N - \lambda_N)]\} = 0_N$. Hence λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N).

Proposition 3.3 *If $\mu - [Clint(1_N - \lambda_N)] = 1_N$, for a fuzzy neutrosophic set λ_N defined on (X_N, T_N), then λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N).*

Proof. Suppose that $\mu - [Clint(1_N - \lambda_N)] = 1_N$ in (X_N, T_N). Then $1_N - \{\mu - [Clint(1_N - \lambda_N)]\} = 0_N$ which implies that $1_N - 1_N - \{\mu - [Intcl(1_N - 1_N - \lambda_N)]\} = 0_N$. We obtain $\mu - [Intcl(\lambda_N)] = 0_N$. Thus λ_N is a fuzzy neutrosophic μ-nowhere dense set in (X_N, T_N). By proposition 3.2, λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N).

Proposition 3.4 *If λ_N is a fuzzy neutrosophic μ-strongly nowhere dense in (X_N, T_N), then $(1_N - \lambda_N)$ is also a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N) which implies that $\mu - \{Intcl[\lambda_N \wedge (1_N - \lambda_N)]\} = 0_N$ in (X_N, T_N). Now, $\mu - \{[Intcl[(1_N - \lambda_N) \wedge (1_N - 1_N - \lambda_N)]]\} = \mu - \{[Intcl[(1_N - \lambda_N) \wedge \lambda_N]]\} = 0_N$. This implies that $(1_N - \lambda_N)$ is also a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N).

Proposition 3.5 *If λ_N is a fuzzy neutrosophic μ-nowhere dense set in fuzzy neutrosophic topological space (X_N, T_N) then $(1_N - \lambda_N)$ is a fuzzy neutrosophic μ-strongly nowehre dense set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic μ-nowhere dense set in (X_N, T_N). Now, by proposition 3.2, we get λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N) and by proposition 3.4, we obtain that $(1_N - \lambda_N)$ is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N).

Proposition 3.6 *If λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N) then $\mu - cl[\lambda_N \wedge (1_N - \lambda_N)] = 1_N$.*

Proof. Let λ_N be a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N). Then $\mu - cl[\lambda_N \wedge (1_N - \lambda_N)] = 0_N$ implies that $\mu - cl[1_N - Int[\lambda_N \wedge (1_N - \lambda_N)]] = 1_N$ which implies that $\mu - cl[Int(1_N - \lambda_N) \wedge (1_N - 1_N - \lambda_N)] = 1_N$. But $\mu - [Clint[(1_N - \lambda_N) \wedge \lambda_N] \leqslant \mu - [cl[(1_N - \lambda_N) \vee \lambda_N]]$, this implies that $1_N \leqslant \mu - cl[(1_N - \lambda_N) \vee \lambda_N]$. Hence we get $\mu - cl[\lambda_N \vee (1_N - \lambda_N)] = 1_N$.

4 Fuzzy Neutrosophic μ-strongly first(second) category sets

Definition 4.1 *A fuzzy neturosophic set λ_N is said to be fuzzy neutrosophic μ-strongly first category set in (X_N, T_N) if $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy*

neutrosophic μ-strongly nowhere dense sets in (X_N, T_N). Otherwise are called as fuzzy neutrosophic μ-strongly second category set in (X_N, T_N).

Definition 4.2 *The complement of fuzzy neutrosophic μ-strongly first category sets are called fuzzy neutrosophic μ-strongly residual set in (X_N, T_N).*

Example 4.1 *Let (X_N, T_N) be a fuzzy neutrosophic topological space, where $X_N = \{a, b\}$ and defined the fuzzy neutrosophic sets are A_N, B_N, C_N, D_N, E_N as following*

$A_N : X_N \to [0_N, 1_N]$ is defined as $A_N = \{\langle a, (0.6, 0.4, 0.8)\rangle, \langle b, (0.8, 0.6, 0.9)\rangle\}$

$B_N : X_N \to [0_N, 1_N]$ is defined as $B_N = \{\langle a, (0.6, 0.3, 0.8)\rangle, \langle b, (0.9, 0.2, 0.7)\rangle\}$

$C_N : X_N \to [0_N, 1_N]$ is defined as $C_N = \{\langle a, (0.5, 0.4, 0.9)\rangle, \langle b, (0.7, 0.8, 0.9)\rangle\}$

$D_N : X_N \to [0_N, 1_N]$ is defined as $D_N = \{\langle a, (0.4, 0.6, 0.9)\rangle, \langle b, (0.6, 0.8, 0.9)\rangle\}$

$E_N : X_N \to [0_N, 1_N]$ is defined as $E_N = \{\langle a, (0.3, 0.7, 0.9)\rangle, \langle b, (0.5, 0.9, 0.9)\rangle\}$

Then, $T_N = \{0_N, A_N, B_N, C_N, D_N, E_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. We computation see that, fuzzy neutrosophic μ-strongly first category set is B_N and fuzzy neutrosophic μ-strongly second category sets are $\{0_N, A_N, C_N, D_N, E_N, 1_N\}$. Also, the fuzzy neutrosophic μ-strongly residual set is $1_N - B_N$.

Proposition 4.1 *If λ_N is a fuzzy neutrosophic μ-first category set in (X_N, T_N),then λ_N is a fuzzy neutrosophic μ-strongly first category set in (X_N, T_N).*

Proof. Let λ_N be a fuzzy neutrosophic μ-first category set in a fuzzy neutrosophic topological space (X_N, T_N). Then $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic μ-nowhere dense sets in (X_N, T_N). The fact "Every fuzzy neutrosophic μ-nowhere dense set is fuzzy neutrosophic μ-strongly nowhere dense set" in (X_N, T_N). We deduce that (λ_{N_i})'s are fuzzy neutrosophic μ-strongly nowhere dense sets and hence $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic μ-strongly

nowhere dense sets in (X_N, T_N). Hence λ_N is a fuzzy neutrosophic $\mu-$strongly first category set in (X_N, T_N).

Remark 4.1 *A fuzzy neutrosophic μ-strongly first category sets need not be a fuzzy neutrosophic μ-first category set in (X_N, T_N). For, consider the following example.*

Example 4.2 *Let (X_N, T_N) be a fuzzy neutrosophic topological space. $X_N = \{a\}$ and define the fuzzy neutrosophic sets are A_N, B_N, C_N, as follows:*

$A_N : X_N \to [0_N, 1_N]$ is defined as $A_N = \{\langle a, (0.6, 0.4, 0.6)\rangle\}$

$B_N : X_N \to [0_N, 1_N]$ is defined as $B_N = \{\langle a, (0.3, 0.4, 0.6)\rangle\}$

$C_N : X_N \to [0_N, 1_N]$ is defined as $C_N = \{\langle a, (0.9, 0.7, 0.6)\rangle\}$

Then, $T_N = \{0_N, A_N, B_N, C_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. We computation see that, A_N and B_N fuzzy neutrosophic μ-strongly nowhere dense sets in (X_N, T_N). Also $(A_N \vee B_N) = A_N$ this implies that, A_N is a fuzzy neutrosophic μ-strongly first category set in (X_N, T_N). Now, $(1_N - B_N)$ and $(1_N - C_N)$ are fuzzy neutrosophic μ-nowhere dense sets in (X_N, T_N). Then $[(1_N - B_N) \vee (1_N - C_N)] = D_N$ is not a fuzzy neutrosophic μ-first category set in (X_N, T_N).

Proposition 4.2 *If $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic closed sets with $\mu - Int(\lambda_N) = 0_N$, then λ_N is a fuzzy neutrosophic μ-strongly first category set in (X_N, T_N)*

Proof. Suppose $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic closed sets with $\mu - Int(\lambda_N) = 0_N$ in (X_N, T_N). If λ_N is a fuzzy neutrosophic closed in fuzzy neutrosophic topological space (X_N, T_N) with $\mu - Int(\lambda_{N_i}) = 0_N$, then λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set. Then use this concept $\lambda_N$is a

fuzzy neutrosophic closed in fuzzy neutrosophic topological space (X_N, T_N) with $\mu - Int(\lambda_{N_i}) = 0_N$. Thus, λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set and then we have $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic μ-strongly nowhere dense sets in (X_N, T_N). Therefore λ_N is a fuzzy neutrosophic μ-strongly first category set in (X_N, T_N).

5 Fuzzy Neutrosophic μ-strongly Baire Spaces

Definition 5.1 *A fuzzy neutrosophic topological space (X_N, T_N) is called fuzzy neutrosophic μ-strongly Baire Space if $\mu - cl[\bigvee_{i=1}^{\infty}(\lambda_{N_i})] = 1_N$, where (λ_{N_i})'s are fuzzy neutrosophic μ-strongly nowhere dense sets in (X_N, T_N).*

Example 5.1 *Let (X_N, T_N) be a fuzzy neutrosophic topological space. Where $X_N = \{a\}$ and define the fuzzy neutrosophic sets are A_N, B_N and C_N as follows:*

$A_N : X_N \to [0_N, 1_N]$ is defined as $A_N = \{\langle a, (0.7, 0.8, 0.9)\rangle\}$

$B_N : X_N \to [0_N, 1_N]$ is defined as $B_N = \{\langle a, (0.3, 0.4, 0.6)\rangle\}$

$C_N : X_N \to [0_N, 1_N]$ is defined as $C_N = \{\langle a, (0.9, 0.7, 0.6)\rangle\}$

Then, $T_N = \{0_N, A_N, B_N, C_N, 1_N\}$ is a fuzzy neutrosophic topology on X_N. We computation see that, fuzzy neutrosophic μ-strongly nowhere dense sets are A_N and B_N. Also, $\mu - cl[A_N \vee B_N] = 1_N$. Hence (X_N, T_N) is a fuzzy neutrosophic μ-strongly Baire Space.

Proposition 5.1 *Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then the following are equivalent*

(i) (X_N, T_N) is a fuzzy neutrosophic μ-strongly Baire space.

(ii) $\mu - [cl(\lambda_N)] = 1_N$, for every fuzzy neutrosopic μ-strongly first category set in (X_N, T_N).

(iii) $\mu - [Int(\gamma_N)] = 0_N$, *for every fuzzy neutrosophic* μ*-strongly residual set in* (X_N, T_N).

Proof.

(i) $\Longrightarrow$ (ii)

Let λ_N be a fuzzy neutrosophic μ-strongly first category set in (X_N, T_N). Then $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic μ-strongly nowhere dense sets in (X_N, T_N). Since (X_N, T_N) is a fuzzy neutrosophic μ-strongly Baire space implies that $\mu - cl[\bigvee_{i=1}^{\infty}(\lambda_{N_i})] = 1_N$. Hence $\mu - [cl(\lambda_N)] = 1_N$.

(ii) $\Longrightarrow$ (iii)

Let γ_N be a fuzzy neutrosophic μ-strongly residual set in (X_N, T_N). Then $1_N - \gamma_N$ is a fuzzy neutrosophic μ-strongly first category set in (X_N, T_N). We obtain that $\mu - cl[(1_N - \gamma_N)] = 1_N$ implies that $1_N - [\mu - Int(\gamma_N)] = 1_N$. Hence $\mu - Int(\gamma_N) = 0_N$.

(iii) $\Longrightarrow$ (i)

Let λ_N be a fuzzy neutrosophic μ-strongly first category set in (X_N, T_N). Then $\lambda_N = \bigvee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic μ-strongly nowehre dense sets in (X_N, T_N). We have λ_N is a fuzzy neutrosophic μ-strongly first categorty set then $(1_N - \lambda_N)$ is a fuzzy neutrosophic $\mu-$strongly residual set in (X_N, T_N). We obtain that $\mu - Int(1_N - \lambda_N) = 0_N$ which gives that $1_N - [\mu - cl(\lambda_{N_i})] = 0_N$. Therefore we get $\mu - cl(\lambda_{N_i}) = 1_N$ and hence $\mu - cl[\bigvee_{i=1}^{\infty}(\lambda_{N_i})] = 1_N$, where (λ_{N_i})'s are fuzzy neutrosophic μ-strongly nowhere dense sets in (X_N, T_N). Hence (X_N, T_N) is a fuzzy neutrosophic μ-strongly Baire space.

Proposition 5.2 *If* λ_N *be a fuzzy neutrosophic open and dense set in a fuzzy neutrosophic topological space* (X_N, T_N), *then* (X_N, T_N) *is a fuzzy neutrosophic* μ*-strongly Baire space.*

Proof. If λ_N be a fuzzy neutrosophic open and dense set then λ_N is a fuzzy neutrosophic μ-strongly nowhere dense set in (X_N, T_N). In this fact use, we obtain that (λ_{N_i})'s are fuzzy neutrosophic μ-strongly nowhere dense sets in (X_N, T_N). Let $\gamma_N = \bigvee_{i=1}^{\infty}(\gamma_{N_i})$, where (γ_{N_i})'s are fuzzy neutrosophic μ-strongly first category sets in (X_N, T_N). Now, $\mu - cl[(\gamma_N)] = \mu - cl[\bigvee_{i=1}^{\infty}(\gamma_{N_i})] \geqslant \bigvee_{i=1}^{\infty}[\mu - cl[(\gamma_{N_i})]] = 1_N$. Hence, (X_N, T_N) is a fuzzy neutrosophic μ-strongly Baire space.

6 Conclusion

In this study, we introduced and explored the concept of fuzzy neutrosophic μ-strongly various sets and fuzzy neutrosophic μ-strongly Baire space. We have examined the foundational properties of fuzzy neutrosophic μ-strongly various sets and fuzzy neutrosophic μ-strongly Baire space. Future work may include the study of other functions, compactness and connectedness in these sets and spaces as well as potential Generalizations and real-world applications.

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FUZZY NEUTROSOPHIC SUPRA DENSE G_δ-SPACES

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ABSTRACT

In this paper a new class of fuzzy neutrosophic supra topological space, namely fuzzy neutrosophic supra σ-nowhere dense set and fuzzy neutrosophic supra Dense G_δ-spaces, are introduced and studied. Several characterizations of fuzzy neutrosophic supra σ-nowhere dense set and fuzzy neutrosophic supra Dense G_δ-spaces, are established. The condition under which fuzzy neutrosophic supra σ-nowhere dense set become fuzzy neutrosophic supra nowhere dense set, fuzzy neutrosophic supra residual set, fuzzy neutrosophic supra first category and fuzzy neutrosophic supra G_δ-spaces, are obtained.

Keywords: Fuzzy neutrosophic supra F_σ −set, Fuzzy neutrosophic supra G_δ − set, Fuzzy neutrosophic supra σ-nowhere dense set, Fuzzy neutrosophic supra σ-first and σ-second category, Fuzzy neutrosophic supra σ-residual set, Fuzzy neutrosophic supra Dense G_δ-space.

1. INTRODUCTION

In recent years, fuzzy and neutrosophic set theories have emerged as powerful tools for dealing with uncertainty in various areas of Mathematics and Computer science. Zadeh [17] was first introduced in 1965 the concept of fuzzy sets and fuzzy set operations in his classical paper. The concept of fuzzy topological spaces, introduced by Chang in 1968 [6], has been extensively studied and applied in many fields, including artificial intelligence, decision making and image processing.

Baire spaces, introduced by Haworth and McCoy in 1977 [7], are an important concept in topology. In 1983, Mashhour et al. [8] extended the notion of supra topological spaces. Abd El-Monsef and Ramadan in 1987 [1] investigated the concept of fuzzy supra topological spaces, which has since been further studied by researchers such as Ahmed, Chandra Chetia, and others [2].

Neutrosophic sets have been introduced to the literature by smarandache to handle incomplete, indeterminate, and inconsistent information. The concept of fuzzy supra σ-Baire space are introduced and studied by Poongothai and Thangaraj [9] investigated fuzzy settings on supra σ-Baire spaces.

The purpose of this paper the concept of fuzzy neutrosophic supra σ-nowhere dense set, fuzzy neutrosophic supra σ-first category, fuzzy neutrosophic supra σ-residual and fuzzy neutrosophic supra Dense G_δ -spaces, are introduced and studied. Several characterizations of fuzzy neutrosophic supra σ-nowhere dense set, fuzzy neutrosophic supra σ-first category fuzzy neutrosophic supra σ-residual and fuzzy neutrosophic supra Dense G_δ-spaces are established. The conditions under which fuzzy neutrosophic supra σ-nowhere dense set become fuzzy neutrosophic supra nowhere dense set, fuzzy neutrosophic supra residual, fuzzy neutrosophic supra first category and fuzzy neutrosophic supra Dense G_δ-spaces are obtained. The existence of fuzzy neutrosophic supra nowhere dense set in fuzzy neutrosophic supra Dense G_δ-spaces are established by means of fuzzy neutrosophic supra σ-nowhere dense set. Several examples are given in this concept.

2. PRELIMINARIES

Definition 2.1 [4]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ $x \in X$ where $T, I, F: X \rightarrow [0,1]$ and $0 \leq \{T_A(x) + I_A(x) + F_A(x)\} \leq 3$.

Definition 2.2 [4]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$

Definition 2.3 [4]:

Let X be a non-empty set, and $A = T_A(x), I_A(x), F_A(x)$, $B = T_B(x), I_B(x), F_B(x)$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, \max\big(T_A(x), T_B(x)\big), \max\big(I_A(x), I_B(x)\big), \min\big(F_A(x), F_B(x)\big)\rangle$$

$$A \cap B = \langle x, \min\big(T_A(x), T_B(x)\big), \min\big(I_A(x), I_B(x)\big), \max\big(F_A(x), F_B(x)\big)\rangle$$

Definition 2.4 [4]:

The difference between two fuzzy neutrosophic sets A and B is defined as $\mathrm{A\backslash B(x)} = \langle \mathrm{x}, \min\big(\mathrm{T_A(x)}, \mathrm{F_B(x)}\big), \min\big(\mathrm{I_A(x)}, \mathrm{1\text{-}I_B(x)}\big), \max\big(\mathrm{F_A(x)}, \mathrm{T_B(x)}\big)\rangle$

Definition 2.5 [4]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T(x) = 0, I(x) = 0, F_A(x) = 1$ for all $x \in X$ It is denoted by 0_{N} .

Definition 2.6 [4]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T(x) = 1, I(x) = 1, F_A(x) = 0$ for all $x \in X$ It is denoted by 0_N.

Definition 2.7 [4]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as

$A^C = \langle T_{A^C}(x), I_{A^C}(x), F_{A^C}(x) \rangle$

where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fuzzy neutrosophic set A can also be defined as $\mathrm{A^C} = 1_\mathrm{N}$-A

Definition 2.8 [5]:

A fuzzy neutrosophic topology on a non-empty set X is a τ_N of fuzzy neutrosophic sets in X Satisfying the following axioms.

$(i) 0_N, 1_N \in \tau$

$(ii) A_1 \cap A_2 \in \tau \ for\ any\ A_1, A_2 \in \tau$

$(iii) \cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i : i \in J\} \in \tau$

In this case the pair(X,τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [3]:

A fuzzy neutrosophic supra topology (FNST) a non-empty set X is a τ^μ of fuzzy neutrosophic supra subsets in X satisfying the following axioms.

$(a) 0_N, 1_N \in \tau^\mu$

$(b) \cup G_i \in \tau^\mu,\ for\ all\ \{G_i : i \in J\} \subseteq \tau^\mu$

In this case the pair(X,τ^μ) is called a fuzzy neutrosophic supra topological space(FNSTS) and any fuzzy neutrosophic supra set in τ^μ is known as fuzzy neutrosophic supra open set(FNSOS) in X. The element of τ^μ are called open fuzzy neutrosophic supra sets. The complement of FNSOS in the FNSTS (X,τ^μ) is called fuzzy neutrosophic supra closed set (FNSCS).

Definition 2.10 [3]:

Let (X,τ^μ) be FNSTS and P=(y,T_P,I_P,F_P) be a FNSS in X then the fuzzy neutrosophic supra interior (FNSI) and fuzzy neutrosophic supra closure (FNSC) of P are defined by FNScl(P)= $\cap \{Q : Q\ is\ a\ FNSCS\ in\ X\ and\ P \subseteq Q\}$.

FNS int(P) = $\cup \{R : R\ is\ a\ FNSOS\ in\ X\ and\ R \subseteq P\}$

Now that Cl(P) is a FNSCS & int(P) is a FNSOS in X, Further

1. P is FNSCS in X iff cl(P) =P

2. P is a FNSOS in X iff int(P)=P.

Definition 2.11 [3]:

Let (X,τ^{μ}) be a FNSTS over X. then the following properties hold.

1. FNScl(CO(P))=CO(FNS int(P))

2. FNS int(CO(P))=CO(FNScl(P)).

Definition 2.12 [5]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X,τ) is called fuzzy neutrosophic closed set in X.

Definition 2.13 [13]:

A fuzzy neutrosophic set X_N^* in fuzzy neutrosophic supra topological space (X_N*,T_N*) is called a fuzzy neutrosophic supra nowhere dense set if there exists no non-zero fuzzy supra open set Y_N^* in (X_N*,T_N*) such that $Y_N^*<fn(cl^*l(X_N^*))$=0, in (X_N*,T_N*). That is $fn(\text{int}^*cl^*(X_N^*))=0$ in (X_N*,T_N*).

Definition 2.14 [13]:

A fuzzy neutrosophic supra set X_N^* in a FNSTS (X_N*,T_N*) is called as neutrosophic supra first category set if $X_N^*=\bigvee_{i=1}^{\infty}(X_{N_i}^*)$'s are neutrosophic supra fuzzy nowhere dense set in (X_N*,T_N*). Any other fuzzy set in (X_N*,T_N*) is said to be neutrosophic supra fuzzy second category.

Definition 2.15 [13]:

A fuzzy neutrosophic supra topological space (X_N*,T_N*) is called a fuzzy neutrosophic supra first category if $\bigvee_{i=1}^{\infty}(X_{N_i}^*)$ =1, where$(X_{N_i}^*)$'s are fuzzy neutrosophic supra nowhere dense set in (X_N*,T_N*). A topological space which is not of fuzzy neutrosophic supra first category, is said to be fuzzy neutrosophic second category.

Definition 2.16 [13]:

Let X_N^* a fuzzy neutrosophic supra first category set in (X_N*,T_N*), Then $1-X_N^*$ is called a fuzzy neutrosophic supra residual set in (X_N*,T_N*).

Definition 2.17 [13]:

Let (X_N*,T_N*) be a fuzzy neutrosophic supra topological spaces. Then (X_N*,T_N*) is called fuzzy neutrosophic Baire space if$fn(int^*(\bigvee_{i=1}^{\infty}(X_{N_i}^*)))$ =0, where $\left(X_{N_i}^*\right)$'s are fuzzy neutrosophic supra nowhere dense sets in (X_N*,T_N*).

Definition 2.18 [14]

Let (X_N*,T_N*) be a fuzzy neutrosophic supra topological space. A fuzzy set $A_N{}^*$ in (X_N*,T_N*) is called a fuzzy neutrosophic supra residual set if $A_N{}^*=\bigwedge_{i=1}^{\infty}A_{N_i}{}^*$, where the fuzzy sets $\left(A_{N_i}{}^*\right)$,s are such that $fn\ cl^*\left[fn\ int^*\left(A_{N_i}{}^*\right)\right]=1_N$ in (X_N*,T_N*).

3. FUZZY NEUTROSOPHIC SUPRA σ-NOWHERE DENSE SET

Definition 3.1

A fuzzy neutrosophic supra set ${A_N}^*$ in a fuzzy neutrosophic supra topological space (X_N*,T_N*) is called a fuzzy neutrosophic supra σ-nowhere dense set if ${A_N}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*) such that $int({A_N}^*) = 0$

Example 3.2

Let ${X_N}^* = \{a, b, c\}$. The fuzzy neutrosophic supra sets ${A_N}^*, {B_N}^*$ and ${C_N}^*$ on ${X_N}^*$ as follows:

$${A_N}^* = \left(X, \left(\frac{a}{0.3}, \frac{b}{0.7}, \frac{c}{0.4}\right), \left(\frac{a}{0.7}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.7}, \frac{c}{0.3}\right)\right),$$

$${B_N}^* = \left(X, \left(\frac{a}{0.5}, \frac{b}{0.7}, \frac{c}{0.8}\right), \left(\frac{a}{0.4}, \frac{b}{0.8}, \frac{c}{0.5}\right), \left(\frac{a}{0.8}, \frac{b}{0.5}, \frac{c}{0.5}\right)\right),$$

$${C_N}^* = \left(X, \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.6}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.2}, \frac{b}{0.6}, \frac{c}{0.6}\right)\right),$$

Then $T^* = \{0, {A_N}^*, {B_N}^*, {C_N}^*, {A_N}^* \vee {B_N}^*, {B_N}^* \vee {C_N}^*, {A_N}^* \vee {C_N}^*, {A_N}^* \wedge {B_N}^*, {B_N}^* \wedge {C_N}^*, {A_N}^* \wedge {C_N}^*, 1\}$. On computation, we see that ${E_N}^* = (1 - {B_N}^*) \vee (1 - {A_N}^* \vee {B_N}^*) \vee (1 - {A_N}^* \vee {C_N}^*)$ this implies that ${E_N}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*). Then $int^*({E_N}^*) = 0$ and hence ${E_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Also, we see that $int^*({F_N}^*) = (1 - {A_N}^*) \vee (1 - {B_N}^*) \vee (1 - {C_N}^*) = {B_N}^* \wedge {C_N}^* \neq 0$ and hence $int^*({F_N}^*) \neq 0$ this implies that ${F_N}^*$ is a fuzzy neutrosophic supra F_σ -set in (X_N*,T_N*) but ${F_N}^*$ is not a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proposition 3.3

In a fuzzy neutrosophic supra topological space (X_N*,T_N*), a fuzzy neutrosophic supra set ${A_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*) if and only if $1 - {A_N}^*$ is a fuzzy neutrosophic supra dense and fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*).

Proof.

Let ${A_N}^*$ be a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*). Then ${A_N}^* = \bigvee_{i=1}^{\infty} {A_{N_i}}^*$, where $1 - {A_{N_i}}^* \in {T_N}^*$, for $i \in I$ and $int^*({A_N}^*) = 0$. Then $cl^*(1 - {A_N}^*) = 1 - int^*({A_N}^*) = 1 - 0 = 1$. This implies that $1 - {A_N}^*$ is a fuzzy neutrosophic supra dense set in (X_N*,T_N*). Also $1 - {A_N}^* = 1 - \bigvee_{i=1}^{\infty}({A_{N_i}}^*) = \bigwedge_{i=1}^{\infty} 1 - ({A_{N_i}}^*)$, where $1 - {A_{N_i}}^* \in {T_N}^*$, for $i \in I$. This implies that $1 - {A_N}^*$ is a fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*). Therefore $1 - {A_N}^*$ is a fuzzy neutrosophic supra dense and fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*).

Conversely, let ${A_N}^*$ be a fuzzy neutrosophic supra dense and fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*). Then ${A_N}^* = \bigwedge_{i=1}^{\infty}({A_{N_i}}^*)$, where ${A_{N_i}}^* \in {T_N}^*$, for $i \in I$ and $cl^*({A_N}^*) = 1$., Now $1 - {A_N}^* = 1 - \bigwedge_{i=1}^{\infty} {A_{N_i}}^* = \bigvee_{i=1}^{\infty}(1 - {A_{N_i}}^*)$, where $(1 - {A_{N_i}}^*)$'s are fuzzy neutrosophic supra closed set in (X_N*,T_N*). This

implies that $1 - {A_N}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*). Now $int^*(1 - {A_N}^*) = 1 - cl^*({A_N}^*) = 1 - 1 = 0$. Hence, $1 - {A_N}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*) and $int^*(1 - {A_N}^*) = 0$. Therefore $1 - {A_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proposition 3.4

If ${A_N}^*$ is a fuzzy neutrosophic supra dense in a fuzzy neutrosophic supra topological space (X_N*,T_N*) such that ${B_N}^* \leq (1 - {A_N}^*)$, where ${B_N}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*), then ${B_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proof.

Let ${A_N}^*$ is a fuzzy neutrosophic supra dense in (X_N*,T_N*) such that ${B_N}^* \leq (1 - {A_N}^*)$. Then $cl^*({A_N}^*) = 1$ and ${B_N}^* \leq (1 - {A_N}^*)$. Now ${B_N}^* \leq (1 - {A_N}^*)$, implies that $int^*({B_N}^*) \leq int^*(1 - {A_N}^*) = cl^*({A_N}^*) = 1 - 1 = 0$ and hence $int^*({B_N}^*) \leq 0$. That is $int^*({B_N}^*) = 0$. Since ${B_N}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*) and $int^*({B_N}^*) = 0$, ${B_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proposition 3.5

If ${A_N}^*$ is a fuzzy neutrosophic supra F_σ-set and fuzzy neutrosophic supra nowhere dense in a fuzzy neutrosophic supra topological space (X_N*,T_N*), then ${A_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proof.

It is clear that ${A_N}^* \leq cl^*({A_N}^*)$ for any fuzzy neutrosophic supra set in (X_N*,T_N*). Then $int^*({A_N}^*) \leq int^*cl^*({A_N}^*)$. Since ${A_N}^*$ is a fuzzy neutrosophic supra nowhere dense set in (X_N*,T_N*), $int^*cl^*({A_N}^*) = 0$ and hence $int^*({A_N}^*) = 0$, Now ${A_N}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*) and $int^*({A_N}^*) = 0$, implies that ${A_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proposition 3.6

If ${A_N}^* \leq {B_N}^*$, where ${A_N}^*$ is a fuzzy neutrosophic supra F_σ-set and ${B_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra topological space (X_N*,T_N*), then ${A_N}^*$ is also a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proof.

Now ${A_N}^* \leq {B_N}^*$ implies that $int^*({A_N}^*) \leq int^*({B_N}^*)$. Since ${B_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*), ${B_N}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*) such that $int^*({B_N}^*) = 0$. Then $int^*({A_N}^*) \leq 0$. That is., $int^*({A_N}^*) = 0$. Hence ${A_N}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*) such that $int^*({A_N}^*) \leq 0$. Therefore ${A_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*).

Proposition 3.7

If ${A_N}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra topological space (X_N*,T_N*), then $1 - {A_N}^* = \wedge_{i-1}^{\infty}({B_{N_i}}^*)$, where $cl^*({B_{N_i}}^*) = 1$.

Proof.

Let $A_N{}^*$ be a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*), Then $A_N{}^*$ is a fuzzy neutrosophic F_σ-set in (X_N*,T_N*) such that $int^*(A_N{}^*) = 0$. Hence $A_N{}^* = \bigvee_{i=1}^{\infty}(A_{N_i}{}^*)$, where $1 - A_{N_i}{}^* \in T_N{}^*$ for $i \in I$ and $int^*(A_N{}^*) = 0$. Now $int^*(A_N{}^*) = int^*\left(\bigvee_{i=1}^{\infty}(A_{N_i}{}^*)\right) = 0$, Then $0 = int^*\left(\bigvee_{i=1}^{\infty}(A_{N_i}{}^*)\right)$. But, $\bigvee_{i=1}^{\infty}\left(int^*(A_{N_i}{}^*)\right) \leq int^*\left(\bigvee_{i=1}^{\infty}(A_{N_i}{}^*)\right)$. This implies that $\bigvee_{i=1}^{\infty}\left(int^*(A_{N_i}{}^*)\right) \leq 0$, that is., $\bigvee_{i=1}^{\infty}\left(int^*(A_{N_i}{}^*)\right) = 0$ and hence $int^*(A_{N_i}{}^*) = 0$. Then $int^*(A_{N_i}{}^*) = 1 - int^*(A_{N_i}{}^*) = 1 - 0 = 1$. Now $1 - A_N{}^* = 1 - \bigvee_{i=1}^{\infty}(A_{N_i}{}^*) = \bigwedge_{i-1}^{\infty}(1 - A_{N_i})$, where $cl^*(1 - A_{N_i}{}^*) = 1$. Let $B_{N_i}{}^* = 1 - A_{N_i}{}^*$. Therefore, $1 - A_N{}^* = \bigwedge_{i-1}^{\infty}(B_{N_i}{}^*)$, where $cl^*(B_{N_i}{}^*) = 1$.

4. FUZZY NEUTROSOPHIC SUPRA σ- FIRST CATEGORY AND FUZZY NEUTROSOPHIC SUPRA σ-SECOND CATEGORY SET

Definition 4.1:

A fuzzy neutrosophic supra set $A_N{}^*$ in a fuzzy neutrosophic supra topological space (X_N*,T_N*) is called a fuzzy neutrosophic supra σ-first category set if $A_N{}^* = \bigvee_{i=1}^{\infty}(A_{N_i}{}^*)$, where $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra σ-nowhere dense sets in (X_N*,T_N*). Any other fuzzy neutrosophic supra set in (X_N*,T_N*) is said to be a fuzzy neutrosophic supra σ-second category set in (X_N*,T_N*).

Definition 4.2:

Let $A_N{}^*$ be a fuzzy neutrosophic supra σ-first category set in a fuzzy neutrosophic supra topological space (X_N*,T_N*). Then $1 - A_N{}^*$ is called a fuzzy neutrosophic supra σ-residual set in (X_N*,T_N*).

Proposition 4.3:

If $A_N{}^*$ is a fuzzy neutrosophic supra σ-first category set in a fuzzy neutrosophic supra topological space (X_N*,T_N*), then there is a fuzzy neutrosophic supra F_σ-set $C_N{}^*$ in (X_N*,T_N*) such that $A_N{}^* \leq C_N{}^*$.

Proof.

Let $A_N{}^*$ be a fuzzy neutrosophic supra σ-first category set in (X_N*,T_N*). Then $A_N{}^* = \bigvee_{i=1}^{\infty}(A_{N_i}{}^*)$, where $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*). Now $\left(1 - cl^*(A_{N_i}{}^*)\right)$'s are fuzzy neutrosophic supra open sets in (X_N*,T_N*). Then, $B_N{}^* = \bigwedge_{i-1}^{\infty}\left(1 - cl^*(A_{N_i}{}^*)\right)$ is a fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*) and $1 - B_N{}^* = 1 - \left[\bigwedge_{i-1}^{\infty}\left(1 - cl^*(A_{N_i}{}^*)\right)\right] = \bigvee_{i=1}^{\infty}\left(cl^*(A_{N_i}{}^*)\right)$ but $\bigvee_{i=1}^{\infty}(A_{N_i}{}^*) \leq \bigvee_{i=1}^{\infty}\left(cl^*(A_{N_i}{}^*)\right)$, implies that $A_N{}^* \leq 1 - B_N{}^*$. Since $B_N{}^*$ is a fuzzy neutrosophic supra G_δ-set, $1 - B_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*). Let $C_N{}^* \leq 1 - B_N{}^*$. Hence, if $A_N{}^*$ is a fuzzy neutrosophic supra σ-first category set in (X_N*,T_N*), then there is a fuzzy neutrosophic supra F_σ-set $C_N{}^*$ in (X_N*,T_N*) such that $A_N{}^* \leq C_N{}^*$.

Proposition 4.4:

If A_N^* is a fuzzy neutrosophic supra σ-first category set in a fuzzy neutrosophic supra topological space (X_N^*,T_N^*), then there is a fuzzy neutrosophic supra G_δ-set B_N^* in (X_N^*,T_N^*) such that $1 - A_N^* \geq B_N^*$.

Proof.

Let A_N^* be a fuzzy neutrosophic supra σ-first category set in (X_N^*,T_N^*). Then by Proposition 4.3, there is a fuzzy neutrosophic supra F_σ-set C_N^* in (X_N^*,T_N^*) such that $A_N^* \leq C_N^*$. Then $1 - C_N^* \leq 1 - A_N^*$. Since C_N^* is a fuzzy neutrosophic supra F_σ-set $1 - C_N^*$ is a fuzzy neutrosophic supra G_δ-set in (X_N^*,T_N^*). Let $B_N^* \leq 1 - C_N^*$. Hence, if A_N^* is a fuzzy neutrosophic supra σ-first category set in (X_N^*,T_N^*), then there is a fuzzy neutrosophic supra G_δ-set B_N^* in (X_N^*,T_N^*) such that $1 - A_N^* \geq B_N^*$.

Proposition 4.5:

If A_N^* is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra topological space (X_N^*,T_N^*), then A_N^* is a fuzzy neutrosophic supra first category set in (X_N^*,T_N^*).

Proof.

Let A_N^* is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N^*,T_N^*), then A_N^* is a fuzzy neutrosophic supra F_σ-set in (X_N^*,T_N^*) such that $int^*(A_N^*) = 0$. Since A_N^* is a fuzzy neutrosophic supra F_σ-set, $A_N^* = \bigvee_{i=1}^{\infty}(A_{N_i}^*)$, where $(A_{N_i}^*)$'s are fuzzy neutrosophic supra closed sets in (X_N^*,T_N^*). Now $int^*(A_N^*) = 0$ implies that $int^*\left(\bigvee_{i=1}^{\infty}(A_{N_i}^*)\right) = 0$. But $\bigvee_{i=1}^{\infty} int^*(A_{N_i}^*) \leq int^*\left(\bigvee_{i=1}^{\infty}(A_{N_i}^*)\right)$. This implies that $\bigvee_{i=1}^{\infty} int^*(A_{N_i}^*) \leq 0$. That is $\bigvee_{i=1}^{\infty} int^*(A_{N_i}^*) = 0$ and hence $int^*(A_{N_i}^*) = 0$ for each i. Since $(A_{N_i}^*)$'s are fuzzy neutrosophic supra closed sets in (X_N^*,T_N^*), $cl^*(A_{N_i}^*) = A_{N_i}^*$. Then $int^* cl^*(A_{N_i}^*) = int^*(A_{N_i}^*) = 0$. Hence $(A_{N_i}^*)$'s are fuzzy neutrosophic supra nowhere dense set in (X_N^*,T_N^*). This implies that $A_N^* = \bigvee_{i=1}^{\infty}(A_{N_i}^*)$, where $(A_{N_i}^*)$'s are fuzzy neutrosophic supra nowhere dense set in (X_N^*,T_N^*). Therefore A_N^* is a fuzzy neutrosophic supra first category set in (X_N^*,T_N^*).

Proposition 4.6:

If A_N^* is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra topological space (X_N^*,T_N^*), then $1 - A_N^*$ is a fuzzy neutrosophic supra residual set in (X_N^*,T_N^*).

Proof.

Let A_N^* is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N^*,T_N^*), then by Proposition 4.5, A_N^* is a fuzzy neutrosophic supra first category set in (X_N^*,T_N^*). Therefore $1 - A_N^*$ is a fuzzy neutrosophic supra residual set in (X_N^*,T_N^*).

Proposition 4.7:

If A_N^* is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra topological space (X_N^*,T_N^*), then there is a fuzzy neutrosophic supra F_σ-set C_N^* in (X_N^*,T_N^*) such that $A_N^* \leq C_N^*$.

Proof.

Let $A_N{}^*$ be a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*), then by Proposition 4.5, $A_N{}^*$ is a fuzzy neutrosophic supra first category set in (X_N*,T_N*). Then there is a fuzzy neutrosophic supra F_σ-set $C_N{}^*$ in (X_N*,T_N*) such that $A_N{}^* \leq C_N{}^*$.

5. FUZZY NEUTROSOPHIC SUPRA DENSE G_δ-SPACES

Definition 5.1:

A fuzzy neutrosophic supra topological space (X_N*,T_N*) is called a fuzzy neutrosophic supra Dense G_δ-space if each fuzzy neutrosophic supra dense set in (X_N*,T_N*) is a fuzzy neutrosophic G_δ-set in (X_N*,T_N*).

Example 5.2:

Let $X_N{}^* = \{a, b, c\}$. The fuzzy neutrosophic supra sets $A_N{}^*$, $B_N{}^*$ and $C_N{}^*$ are defined on $X_N{}^*$ as follows:

$$A_N{}^* = \{\langle a, 0.8, 0.6, 0.7\rangle,\ \langle b, 0.7, 0.6, 0.8\rangle,\ \langle c, 0.6, 0.5, 0.5\rangle\}$$

$$B_N{}^* = \{\langle a, 0.6, 0.7, 0.6\rangle, \langle b, 0.6, 0.5, 0.7\rangle, \langle c, 0.5, 0.6, 0.7\rangle\}$$

$$C_N{}^* = \{\langle a, 0.6, 0.8, 0.6\rangle, \langle b, 0.8, 0.8, 0.8\rangle, \langle c, 0.7, 0.7, 0.6\rangle\}$$

Now, $T_N{}^* = \{0, A_N{}^*, B_N{}^*, C_N{}^*, A_N{}^* \vee B_N{}^*, B_N{}^* \vee C_N{}^*, A_N{}^* \vee C_N{}^*, A_N{}^* \wedge B_N{}^*, B_N{}^* \wedge C_N{}^*, A_N{}^* \wedge C_N{}^*, A_N{}^* \vee (B_N{}^* \wedge C_N{}^*), B_N{}^* \vee (C_N{}^* \wedge A_N{}^*), A_N{}^* \vee (B_N{}^* \wedge C_N{}^*), A_N{}^* \vee B_N{}^* \vee C_N{}^*, 1\}$, is a fuzzy neutrosophic supra topology on $X_N{}^*$, On computation, we see that the fuzzy neutrosophic supra dense set in (X_N*,T_N*) is the fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*). Now $E_N{}^* = [A_N{}^* \vee B_N{}^*] \wedge [B_N{}^* \vee C_N{}^*] \wedge [C_N{}^* \vee A_N{}^*] \wedge [A_N{}^* \vee (B_N{}^* \wedge C_N{}^*)] \wedge [B_N{}^* \vee (C_N{}^* \wedge A_N{}^*)] \wedge [A_N{}^* \vee (B_N{}^* \wedge C_N{}^*)]$, and hence the fuzzy neutrosophic supra dense set $E_N{}^*$ in (X_N*,T_N*) is a fuzzy neutrosophic supra dense G_δ- set in (X_N*,T_N*). Hence (X_N*,T_N*) is a fuzzy neutrosophic supra Dense G_δ-space.

Proposition 5.3:

If $A_N{}^*$ is a fuzzy neutrosophic supra dense set in a fuzzy neutrosophic supra Dense G_δ-space in (X_N*,T_N*), then $A_N{}^* = \wedge_{i=1}^{\infty}(B_{N_i}{}^*)$, where $(B_{N_i}{}^*)$'s are fuzzy neutrosophic supra dense and fuzzy neutrosophic supra open sets in (X_N*,T_N*).

Proof.

Let $A_N{}^*$ be a fuzzy neutrosophic supra dense set in (X_N*,T_N*). Then $cl(A_N{}^*) = 1$. Since (X_N*,T_N*) is a fuzzy neutrosophic supra G_δ-space. $A_N{}^*$ is fuzzy neutrosophic supra G_δ-set in (X_N*,T_N*). Then $A_N{}^* = \wedge_{i=1}^{\infty}(A_{N_i}{}^*)$, where $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra open sets in (X_N*,T_N*). Now $cl(A_N{}^*) = cl[\wedge_{i=1}^{\infty}(A_{N_i}{}^*)]$ in (X_N*,T_N*). But $cl[\wedge_{i=1}^{\infty}(A_{N_i}{}^*)] = [\wedge_{i=1}^{\infty} cl(A_{N_i}{}^*)]$, implies that $1 = [\wedge_{i=1}^{\infty} cl(A_{N_i}{}^*)]$. That is,$\wedge_{i=1}^{\infty} cl(A_{N_i}{}^*) = 1$ and then $cl(A_{N_i}{}^*) = 1$, in (X_N*,T_N*). Hence, $A_N{}^* = \wedge_{i=1}^{\infty}(A_{N_i}{}^*)$, where $(A_{N_i}{}^*)$'s are fuzzy neutrosophic supra dense and fuzzy neutrosophic supra open sets in (X_N*,T_N*).

Proposition 5.4:

If A_N^* is a fuzzy neutrosophic supra dense sets in a fuzzy neutrosophic supra Dense G_δ-space (X_N^*,T_N^*), then A_N^* is a fuzzy neutrosophic supra residual set in (X_N^*,T_N^*).

Proof.

Let A_N^* is a fuzzy neutrosophic supra dense sets in (X_N^*,T_N^*). Since (X_N^*,T_N^*) is a fuzzy neutrosophic supra Dense G_δ-space Proposition 5.3, then $A_N^* = \wedge_{i=1}^{\infty}(A_{N_i}^*)$, where $(A_{N_i}^*)$ are fuzzy neutrosophic dense and fuzzy neutrosophic supra open sets in (X_N^*,T_N^*). Then $1 - A_N^* = 1 - \wedge_{i=1}^{\infty}(A_{N_i}^*) = \vee_{i-1}^{\infty}(1 - A_{N_i}^*)$. Now $int\ cl(1 - A_{N_i}^*) = 1 - cl\ int(A_{N_i}^*) = 1 - cl(A_{N_i}^*) = 1 - 1 = 0$ and thus $(1 - A_{N_i}^*)$'s fuzzy neutrosophic supra nowhere dense sets in (X_N^*,T_N^*). Hence $1 - A_N^* = \vee_{i-1}^{\infty}(1 - A_{N_i}^*)$, where$(1 - A_{N_i}^*)$'s are fuzzy neutrosophic supra nowhere dense sets in (X_N^*,T_N^*), implies that $1 - A_N^*$ is a fuzzy neutrosophic supra first category set in (X_N^*,T_N^*). Thus A_N^* is a fuzzy neutrosophic supra residual set in (X_N^*,T_N^*).

Proposition 5.5:

If A_N^* is a fuzzy neutrosophic supra nowhere dense sets in a fuzzy neutrosophic supra Dense G_δ-space (X_N^*,T_N^*), then A_N^* is a fuzzy neutrosophic supra F_σ-set in (X_N^*,T_N^*).

Proof.

Let A_N^* is a fuzzy neutrosophic supra nowhere dense sets in (X_N^*,T_N^*). Then, $int\ cl(A_N^*) = 0$, in (X_N^*,T_N^*). But $int(A_N^*) \leq int\ cl(A_N^*)$ implies that $int^*(A_N^*) = 0$ in (X_N^*,T_N^*). Now $cl(1 - A_N^*) = 1 - int(A_N^*) = 1 - 0 = 1$ and hence $1 - A_N^*$ is a fuzzy neutrosophic dense set in (X_N^*,T_N^*). Since (X_N^*,T_N^*) is a fuzzy neutrosophic supra Dense G_δ-space, the fuzzy neutrosophic dense set $1 - A_N^*$ is a fuzzy neutrosophic supra G_δ-set in (X_N^*,T_N^*). Hence A_N^* is a fuzzy neutrosophic supra F_σ-set in (X_N^*,T_N^*).

Proposition 5.6:

If A_N^* is a fuzzy neutrosophic supra nowhere dense sets in a fuzzy neutrosophic supra Dense G_δ-space (X_N^*,T_N^*), then A_N^* is a fuzzy neutrosophic supra σ –nowhere dense set in (X_N^*,T_N^*).

Proof.

Let (X_N^*,T_N^*) be a fuzzy neutrosophic supra Dense G_δ-space and A_N^* be a fuzzy neutrosophic supra nowhere dense set in (X_N^*,T_N^*). The, by Proposition 5.5, A_N^* is a fuzzy neutrosophic supra F_σ-set in (X_N^*,T_N^*). Now, $int(A_N^*) \leq int\ cl(A_N^*)$ and $int\ cl(A_N^*) = 0$, implies that $int\ (A_N^*) = 0$ in (X_N^*,T_N^*). Thus, Hence A_N^* is a fuzzy neutrosophic supra F_σ-set such that $int\ (A_N^*) = 0$ in (X_N^*,T_N^*). Therefore A_N^* is a fuzzy neutrosophic supra σ –nowhere dense set in (X_N^*,T_N^*).

Proposition 5.7:

If A_N^* is a fuzzy neutrosophic supra σ-nowhere dense set in a fuzzy neutrosophic supra Dense G_δ-space (X_N^*,T_N^*), then there exists a fuzzy neutrosophic supra F_σ-set E_N^* in (X_N^*,T_N^*) such that $A_N^* \leq E_N^*$.

Proof.

Let $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*). Then by Proposition 4.3, $A_N{}^* \leq 1 - C_N{}^*$. Let $E_N{}^* = 1 - C_N{}^*$. Then $E_N{}^*$ is a fuzzy neutrosophic supra F_σ-set in (X_N*,T_N*). Thus, if $A_N{}^*$ is a fuzzy neutrosophic supra σ-nowhere dense set in (X_N*,T_N*), then there exists a fuzzy neutrosophic supra F_σ-set $E_N{}^*$ in (X_N*,T_N*) such that $A_N{}^* \leq E_N{}^*$.

Proposition 5.8:

If $int^*\left(\bigvee_{i=1}^{\infty}\left(A_{N_i}{}^*\right)\right) = 0$, where $\left(A_{N_i}{}^*\right)$,s are fuzzy neutrosophic supra nowhere dense set in a fuzzy neutrosophic supra Dense G_δ -space in (X_N*,T_N*), then (X_N*,T_N*) is a fuzzy neutrosophic supra σ-nowhere dense set.

Proof.

Suppose that $int^*\left(\bigvee_{i=1}^{\infty}\left(A_{N_i}{}^*\right)\right) = 0$, where $\left(A_{N_i}{}^*\right)$,s are fuzzy neutrosophic supra nowhere dense set in a fuzzy neutrosophic supra Dense G_δ-space (X_N*,T_N*). Since $\left(A_{N_i}{}^*\right)$'s are fuzzy neutrosophic supra nowhere dense set in the fuzzy neutrosophic supra Dense G_δ-space (X_N*,T_N*), by Proposition 5.6, $\left(A_{N_i}{}^*\right)$'s are fuzzy neutrosophic supra nowhere dense set in (X_N*,T_N*).

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ON FUZZY NEUTROSOPHIC SUPRA SEMI-BAIRE SPACES

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ABSTRACT

In this paper the concept of fuzzy neutrosophic supra semi-open (closed) sets, fuzzy neutrosophic supra semi-nowhere dense sets are introduced and studied. By means of fuzzy neutrosophic supra semi-nowhere dense set, the concepts of fuzzy neutrosophic supra semi-Baire space are introduced and characterization of fuzzy neutrosophic supra semi - Baire space are studied.

Keywords: Fuzzy neutrosophic supra semi- open set, Fuzzy neutrosophic supra semi -closed set, Fuzzy neutrosophic supra semi-dense set, Fuzzy neutrosophic supra semi-nowhere dense set, Fuzzy neutrosophic supra semi-first and second category, Fuzzy neutrosophic supra semi-Baire spaces.

1. INTRODUCTION

In 1984, Mashhour et al [5] introduced the concept of fuzzy supra topological space spaces. In 1987, Abd E-Monsef et al [1] introduced fuzzy supra topological space and defined basic notion of fuzzy supra topological Space.

Arockiarani I, Sumathi, and Martina Jency [2] introduced the idea of fuzzy neutrosophic supra sets with the concept of fuzzy neutrosophic topological spaces. In their study, Poongothai and Thangaraj [6] investigated fuzzy settings on supra σ-Baire spaces, while Poongothai and Priya [7] studied fuzzy supra semi Baire spaces.

Building upon Smarandache.F in 2005[11]Neutrosophic supra set. A Generalization of the Intuitionistic fuzzy sets. Salama and Alblowi [10] investigated neutrosophic sets and neutrosophic topological spaces. Poongothai.E and Padmavathi.E [8] introduced the concept of fuzzy neutrosophic semi-Baire spaces.

Through our investigation of neutrosophic supra semi-Baire spaces on fuzzy neutrosophic supra topological spaces, we aim to contribute to the development of the theory of fuzzy neutrosophic supra semi-Baire spaces and provide a foundation for further research in this area.

2. PRELIMINARIES

Definition 2.1 [2]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$ With the condition $0 \leq T_{A^\bullet}(x) + I_{A^\bullet}(x) + F_{A^\bullet}(x) \leq 2.$

Definition 2.2 [2]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x),\ I_A(x) \leq I_B(x), F_A(x) \geq F_B(x).$

Definition 2.3 [2]:

Let X be a non-emptyset, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$$

$$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$$

Definition 2.4 [2]:

The difference between two fuzzy neutrosophic sets A and B is defined as $A \backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.5 [2]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0,\ I_A(x) = 0,\ F_A(x) = 1\ for\ all\ x\ \in X.$ It is denoted by 0_N.

Definition 2.6 [2]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1,\ I_A(x) = 1,\ F_A(x) = 0\ for\ all\ x\ \in X.$ It is denoted by 1_N.

Definition 2.7 [2]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as

$A^C = \langle x, T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$

Where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N - A.$

Definition 2.8 [2]:

A fuzzy neutrosophic topology on a non-empty set X is a τ_N of fuzzy neutrosophic sets in X

$(i) 0_N, 1_N \in \tau$

$(ii) A_1 \cap A_2 \in \tau\ \ for\ any\ A_1, A_2 \in \tau$

$(iii)\ \cup A_i \in \tau\ for\ any\ arbitrary\ family\ \{A_i : i \in J\}\ \in \tau$

Satisfying the following axioms.

In this case the pair(X,τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9 [2]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10 [8].

A fuzzy neutrosophic set A_N in fuzzy neutrosophic topological space (X,τ_N) is called fuzzy neutrosophic semi-(briefly as *fn.S*) nowhere dense if there exists no non-zero fuzzy neutrosophic semi-

open set $B_N \subsetneq fn.S(A_N)$. That is., $fn.S.((A_N)^-)^+ = 0$.

Definition 2.11 [8].

Let A_N be a fuzzy neutrosophic semi-one category set in (X,τ_N). Then $1 - A_N$ is called fuzzy neutrosophic semi-residual set in (X,τ_N).

Definition 2.12 [8].

A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (X,τ_N) is called a fuzzy neutrosophic semi-dense if there exist no $fnCS$ B_N in (X, τ_N) such that $A_N < B_N < 1$.That is, $fn.S(A_N)^- = 1$.

Definition 2.13 [8].

Let (X,τ_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set A_N in (X,τ_N) is called fuzzy neutrosophic semi-one category set if $A_N = \bigvee_{i=1}^{\infty}(A_{N_i}{}^*)$, where A_{N_i}'s, are fuzzy neutrosophic semi-nowhere dense sets in (X,τ_N). Any other fuzzy neutrosophic set in (X,τ_N) is said to be of fuzzy neutrosophic semi-two category.

Definition 2.14 [8].

Let A_N be a fuzzy neutrosophic semi-one category set in (X,τ_N). Then $1 - A_N$ is called fuzzy neutrosophic semi-residual set in (X,τ_N).

Definition 2.15 [8].

A fuzzy neutrosophic topological space (X,τ_N) is called fuzzy neutrosophic semi-one category space if the fuzzy neutrosophic set 1 is a fuzzy neutrosophic semi-one category set in (X,τ_N). That is $1 = \bigvee_{i=1}^{\infty}(A_{N_i}{}^*)$, where A_{N_i}'s, are fuzzy neutrosophic semi-nowhere dense sets in (X,τ_N). Otherwise (X,τ_N) will be called a fuzzy neutrosophic semi-two category space.

Definition 2.16 [8].

A fuzzy neutrosophic topological space (X,τ_N) is called fuzzy neutrosophic semi-Baire space if $fn.S(\bigvee_{i=1}^{\infty}(A_{N_i}{}^*))^+$,where A_{N_i}'s, are fuzzy neutrosophic semi-nowhere dense sets in (X,τ_N).

Definition 2.17 [9]

Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X,τ_N). Then A_N is said to be fuzzyneutrosophic semi-open set in (X,τ_N) if there exists a fuzzy neutrosophic open set such that $fnO \leq A_N \leq (fnO)$.

Definition 2.18 [9]

Let A_N be a fuzzy neutrosophic set of a fuzzy neutrosophic topological space (X,τ_N). Then A_N is said to be fuzzy neutrosophic semi-closed set in (X,τ_N) if there exists a fuzzy neutrosophic closed (fnC) set such that $(fnC)^+ \leq A_N \leq (fnC)$.

Definition 2.19 [9]

Let (X,τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X,τ_N), the fuzzy neutrosophic semi-interior of A_N is the union of all fuzzy neutrosophic semi-open sets of (X,τ_N) contained in A_N.That is, $(A_N)^+ = \bigvee\{G_N : G_N$ is a fuzzy neutrosophic semi - open set in (X, τ_N) and $G_N \leq A_N\}$

Definition 2.20[9]

Let (X,τ_N) be a fuzzy neutrosophic topological space. Then for a fuzzy neutrosophic set A_N of (X,τ_N), the

fuzzy neutrosophic semi-closure of A_N [fnS (A_N) for short] is the intersection of all fuzzy neutrosophic semi-closed sets of (X,τ_N) contained in A_N. That is,$(A_N)^{-}=\wedge\{K_N:K_N$ $is\ a\ fnSC\ set\ in$ (X,τ_N) and $K_N \geq A_N\}$.

3. FUZZY NEUTROSOPHIC SUPRA SEMI−OPEN SETS, SEMI- CLOSED SETS, SEMI-INTERIOR SETS AND SEMI-CLOSURE SETS

3.1 Fuzzy neutrosophic supra semi−open sets (closed sets) in fuzzy neutrosophic supra topological spaces.

In this section, we introduce the concept of fuzzy neutrosophic supra semi-open set and also discussed theircharacterizations.

Definition 3.1::

Let A_{N^*} be a fuzzy neutrosophic supra set of a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}). Then A_{N^*} is said to be fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}) if there exists a fuzzy neutrosophic supra open set such that $fnsO \leq A_{N^*} \leq fnsc(fnsO)$.

The following proposition is the characterization of fuzzy neutrosophic supra semi-open set in fuzzy neutrosophic supra topological space.

Proposition 3.1:

A fuzzy neutrosophic supra set A_{N^*} in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) is fuzzy neutrosophic supra semi-open set if only if $A_{N^*} \leq cl^*(int^*(A_{N^*}))$

Proof:

Sufficiency: Let $A_{N^*} \leq cl^*(int^*(A_{N^*}))$ Then for $fnsO = int^*(A_{N^*})$, we have $fnsO \leq A_{N^*} \leq fnsc(fnsO)$.

Necessity: Let A_{N^*} be a fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}). Then $fnsO \leq A_{N^*} \leq fns\,c(fnsO)$ for some fuzzy neutrosophic supra semi-open set A_{N^*} But $fnsO = int^*(A_{N^*})$ and thus $fnsc\,(fnsO) \leq cl^*(int^*(A_{N^*}))$ Hence $A_{N^*} \leq fnsc(fnsO) \leq cl^*(int^*(A_{N^*}))$

Proposition 3.2:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space. Then the union of two fuzzy neutrosophic supra semi-open sets is a fuzzy neutrosophic supra semi-open set in the fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}).

Proof:

Let A_{N^*} and B_{N^*} are fuzzy neutrosophic supra semi-open sets in (X_{N^*},T_{N^*}). Then $A_{N^*} \leq cl^*(int^*(A_{N^*}))$ and $B_{N^*} \leq cl^*(int^*(B_{N^*}))$. Therefore $A_{N^*} \vee B_{N^*} \leq cl^*(int^*(A_{N^*})) \vee cl^*(int^*(B_{N^*})) = cl^*(nt^*(A_{N^*})) \vee cl^*(int^*(B_{N^*})) \leq cl^*(int^*(A_{N^*} \vee B_{N^*}))$. By propositin3.1. Hence $A_{N^*} \vee B_{N^*}$ is fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}).

Proposition 3.3:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space. If $\{A_{N_i}{}^*\}_{i\in\triangle}$ is a collection of fuzzy neutrosophic supra semi-open sets in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}). Then $\vee_{i\in\triangle}$ $A_{N_i}{}^*$ is fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}).

Proof:

For each $i \in \triangle$, we have a fuzzy neutrosophic supra semi-open set $fnsO_i$ such that $fnsO_i \leq A_{N_i}{}^* \leq fns$

cl* ($fnsO_i$).Then $V_{i\in\triangle}\, fnO_i \leq V_{i\in\triangle}\, A_{N_i}^{*} \leq V_{i\in\triangle}\, fns$ cl* ($fnsO_i$) $\leq fns$ cl* ($V_{i\in\triangle}\, fns$) . Hence let $fnsO = V_{i\in\triangle}\, fns$.

Remark 3.1

The intersection of any two fuzzy neutrosophic supra semi-open sets need not be a fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}) asshown by the following example.

Example 3.1:

Let $X_{N^*} = \{a, b, c\}$ and

$A_{N^*} = \{\langle a, 0.5,0.3,0.4\rangle, \langle b, 0.5,0.2,0.6\rangle, \langle c, 0.4,0.1,0.4\rangle\}$

$B_{N^*} = \{\langle a, 0.6,0.1,0.5\rangle, \langle b, 0.5,0.8,0.1\rangle, \langle c, 0.3,0.2,0.2\rangle\}$

$C_{N^*} = \{\langle a, 0.2,0.4,0.8\rangle, \langle b, 0.4,0.2,0.4\rangle, \langle c, 0.4,0.2,0.3\rangle\}$

Then $T_{N^*} = \{ 0_{N^*}, A_{N^*}, B_{N^*}, C_{N^*}, 1_{N^*}\}$ is fuzzy neutrosophic supra topological space on (X_{N^*},T_{N^*}). Now, we define the two fuzzy neutrosophic suprasemi−open sets as follows:

$E_{N^*} = \{\langle a, 0.5,0.4,0.8\rangle, \langle b, 0.5,0.3,0.5\rangle, \langle c, 0.6,0.2,0.4\rangle\}$ and

$G_{N^*} = \{\langle a, 0.5,0.3,0.5\rangle, \langle b, 0.5,0.3,0.7\rangle, \langle c, 0.4,0.2,0.4\rangle\}$. Here int*($E_{N^*}$) = C_{N^*} and cl*(int*(E_{N^*})) = 1 and int* (F_{N^*}) = A_{N^*} and cl*(int*((F_{N^*})) = But $E_{N^*} \wedge F_{N^*}$ = $\{\langle a, 0.5,0.3,0.5\rangle, \langle b, 0.5,0.3,0.5\rangle, \langle c, 0.4,0.2,0.4\rangle\}$ is not a fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}).

Proposition 3.4:

Let A_{N^*} be a fuzzy neutrosophic supra semi-open in the fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) and suppose $A_{N^*} \leq B_{N^*} \leq$cl*($A_{N^*}$) . Then B_{N^*} is a fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}).

Proof:

There exists a fuzzy neutrosophic supra open set $fnsO$ such that $fnsO \leq A_{N^*} \leq$ cl*($fnsO$). Then $A_{N^*} \leq B_{N^*}$ But cl*(A_{N^*}) $\leq$ cl*(($fnsO$)) and thus $B_{N^*} \leq$ cl*($fn(fnsO)$). Hence $fnsO \leq B_{N^*} \leq$ cl*($fnsO$) and B_{N^*} is a fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}).

Proposition 3.5:

Every fuzzy neutrosophic supra open set in the fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) is fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}).

Proof:

Let A_{N^*} be fuzzy neutrosophic supra open set in fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}). Then $A_{N^*} \leq$ int*(A_{N^*}) Also int*(A_{N^*}) $\leq$cl*(int*(A_{N^*})) This implies that $A_{N^*} \leq$ cl*(int*(A_{N^*})) Hence by proposition3.1, is a fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}).

Remark 3.2:

In a fuzzy neutrosophic supra topological space, every fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}) is need not to be fuzzyneutrosophic supra open set in (X_{N^*},T_{N^*}) as shown by the example.

Example 3.2:

Let $X_{N^*} = \{a, b, c\}$ and with $T_{N^*} = \{ 0_{N^*}, A_{N^*}, B_{N^*}, C_{N^*}, D_{N^*}, E_{N^*}, F_{N^*}, 1_{N^*}\}$. Some of the fuzzy neutrosophic supra semi-open sets are

$A_{N^*} = \{\langle a, 0.5,0.4,0.2\rangle, \langle b, 0.3,0.1,0.2\rangle, \langle c, 0.8,0.6,0.9\rangle\}$

$B_{N^*} = \{\langle a, 0.4,0.2,0.5\rangle, \langle b, 0.2,0.1,0.1\rangle, \langle c, 0.8,0.5,0.6\rangle\}$

$C_{N^*} = \{\langle a, 0.1,0.6,0.5\rangle, \langle b, 0.3,0.1,0.4\rangle, \langle c, 0.5,0.9,0.8\rangle\}$

$D_{N^*} = \{\langle a, 0.4,0.5,0.3\rangle, \langle b, 0.2,0.1,0.6\rangle, \langle c, 0.5,0.8,0.7\rangle\}$

$E_{N^*} = \{\langle a, 0.6,0.5,0.1\rangle, \langle b, 0.1,0.6,0.4\rangle, \langle c, 0.8,0.5,0.9\rangle\}$

$F_{N^*} = \{\langle a, 0.5,0.4,0.3\rangle, \langle b, 0.2,0.3,0.1\rangle, \langle c, 0.8,0.7,0.5\rangle\}$

Here A_{N^*}, B_{N^*}, C_{N^*}, D_{N^*}, E_{N^*}, F_{N^*} are fuzzy neutrosophic supra semi-open sets but are not fuzzy neutrosophic supra open sets in (X_{N^*},T_{N^*}).

3.2 Fuzzy Neutrosophic supra Semi−Closed Sets In Fuzzy Neutrosophic supra Topological Spaces.

In this section, the concept of fuzzy neutrosophic supra semi-closed set is introduced and also discussed their properties.

Definition 3.2:

Let A_{N^*} be a fuzzy neutrosophic supra set of a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}). Then A_{N^*} is said to be fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}) if there exists a fuzzy neutrosophic supra closed $(fnsC)$ set such that $f(\text{int}^*(fnsC)) \leq A_{N^*} \leq (fnsC)$.

Proposition 3.6:

A fuzzy neutrosophic supra set A_{N^*} in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) is fuzzy neutrosophic supra semi-closed set if only if $\text{int}^*(\text{cl}^*((A_{N^*}))\leq A_{N^*}$.

Proof:

Sufficiency: $\text{int}^*(\text{cl}^*((A_{N^*})) \leq A_{N^*}$. Then for $fnsC = \text{cl}^*(A_{N^*})$, we have $\text{int}^*(fnsC) \leq A_{N^*} \leq fnsC$.

Necessity: Let A_{N^*} be a fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}). Then $\text{int}^*(fnsC) \leq A_{N^*} \leq fnsC$ for some fuzzy neutrosophic supra semi-closed set $fnsC$. But $fns(\text{cl}^*(A_{N^*}) \leq fnsC$ and thus $\text{int}^*(\text{cl}^*((A_{N^*})) \leq fns(\text{int}^*(fnC))$ Hence $\text{int}^*(\text{cl}^*((A_{N^*})) \leq fns(\text{nt}^*(fnC)) \leq A_{N^*}$.

Proposition 3.7:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space and A_{N^*} be a fuzzy neutrosophic supra set of (X_{N^*},T_{N^*}). Then A_{N^*} is a fuzzy neutrosophic supra semi-closed set if and only if complement of (A_{N^*}) is fuzzy neutrosophic supra semi−open set in (X_{N^*},T_{N^*}).

Proof:

Let A_{N^*} be a fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}). Then by proposition 3.6, $\text{int}^*(\text{cl}^*((A_{N^*})) \leq A_{N^*}$. Taking complement on both sides, $(A_{N^*}) \leq \text{int}^*(\text{cl}^*((A_{N^*})) = \text{cl}^*(\text{cl}^*((A_{N^*}))$ .Then $(A_{N^*}) \leq \text{cl}^*(\text{int}^*((A_{N^*}))$. Then (A_{N^*}) is fuzzy neutrosophic supra semi-open. Conversely let (A_{N^*}) is fuzzy neutrosophic supra semi-open set $\text{int}^*(\text{cl}^*(A_{N^*}))$. Then A_{N^*} is a fuzzy neutrosophic supra semi-closed set.

Proposition 3.8:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space. Then intersection of two fuzzy neutrosophic supra semi-closed sets is a fuzzy neutrosophic supra semi-closed set in fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}).

Proof:

Let A_{N^*} and B_{N^*} are fuzzy neutrosophic supra semi-closed sets in (X_{N^*},T_{N^*}). Then $\text{int}^*(\text{cl}^*((A_{N^*}))\leq A_{N^*}$ and $\text{int}^*(\text{cl}^*(A_{N^*})) \leq B_{N^*}$. Therefore $B_{N^*} \geq \text{int}^*(\text{cl}^*(A_{N^*})) \wedge \text{int}^*(\text{cl}^*(B_{N^*})) = \text{cl}^*(A_{N^*}) \vee \text{int}^*(\text{cl}^*(B_{N^*})) \geq \text{int}^*(\text{cl}^*(A_{N^*} \vee B_{N^*}))$. Hence $A_{N^*} \wedge B_{N^*}$ is fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}).

Proposition 3.9:

Let $\{A_{N_i}^{*}\}_{\in\triangle}$ be a collection of fuzzy neutrosophic supra semi-closed sets in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}). Then $\wedge_{i\in\triangle} A_{N_i}^{*}$ is fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}).

Proof:

For each $i \in\triangle$, we have a fuzzy neutrosophic supra semi-closed set $fnsC_i$ such that $\text{int}^*(fnsC_i) \leq A_{N^*} \leq fnsC_i$. Then $\text{int}^*(\wedge_{i\in\triangle} fnsC_i) \leq \wedge_{i\in\triangle} f(\text{int}^*(fnsC_i)) \leq \wedge_{i\in\triangle} A_{N^*} \leq \wedge_{i\in\triangle} fnsC_i$. Hence let $fnsC = \wedge_{i\in\triangle} fnsC_i$.

Remark 3.3

The union of any two fuzzy neutrosophic supra semi-closed sets need not be a fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}) as shown by the following example.

Example 3.3:

Let $X_{N^*} = \{a, b, c\}$ and

$A_{N^*} = \{\langle a, 0.1,0.5,0.5\rangle\}$,

$B_{N^*} = \{\langle a, 0,0.9,0.2\rangle\}$,

$C_{N^*} = \{\langle a, 0.9,0.2,0.1\rangle\}$.

Then $T_{N^*} = \{0_{N^*}, A_{N^*}, B_{N^*}, C_{N^*}, A_{N^*} \vee B_{N^*}, B_{N^*} \vee C_{N^*}, A_{N^*} \vee C_{N^*}, A_{N^*} \wedge B_{N^*}, B_{N^*} \wedge C_{N^*}, A_{N^*} \wedge C_{N^*}, A_{N^*} \vee B_{N^*} \vee C_{N^*}, 1_{N^*}\}$ is a fuzzy neutrosophic supra topological space on (X_{N^*},T_{N^*}).

Here, $\text{cl}^*(\text{in}^*(1 - B_{N^*})) = 1 = \text{int}^*(1 - B_{N^*}) = 0$ and $\text{cl}^*(\text{int}^*(1 - C_{N^*})) = 1 = \text{int}^*(1 - C_{N^*}) = 0$ are fuzzy neutrosophic supra semi-closed sets in (X_{N^*},T_{N^*}).But $(1 - B_{N^*}) \vee \text{int}^*(1 - C_{N^*}) = ((1 - B_{N^*}) \vee \text{int}^*(\text{cl}^*(C_{N^*}))) = \text{int}^*(1 - B_{N^*} \vee C_{N^*}) = A_{N^*} \neq 0$ is not a fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}).

Proposition 3.10:

Let A_{N^*} be fuzzy neutrosophic supra semi-closed set in the fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) and suppose $\text{int}^*(A_{N^*}) \leq B_{N^*} \leq A_{N^*}$. Then B_{N^*} is fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}).

Proof:

There exists a fuzzy neutrosophic supra semi-closed set $(fnsC)$ such that $\text{int}^*(fnsC) \leq A_{N^*} \leq fnsC$. Then $B_{N^*} \leq fnsC$. But $\text{int}^*(fnsC) \leq \text{int}^*(A_{N^*})$and thus fn $(\text{int}^*(fnsC) \leq B_{N^*}$. Hence $\text{int}^*(fnsC) \leq B_{N^*} \leq fnsC$ and B_{N^*} is fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}).

Proposition 3.11:

Every fuzzy neutrosophic supra closed set in the fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) is fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}).

Proof:

Let A_{N^*} be fuzzy neutrosophic supra closed set in fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}). Then $\text{cl}^*(A_{N^*}) = A_{N^*}$. Also $\text{int}^*(\text{cl}^*((A_{N^*})) = \text{cl}^*((A_{N^*}))$. This implies that $\text{int}^*(\text{cl}^*((A_{N^*})) \leq A_{N^*}$. Then A_{N^*} is a fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}).

Remark 3.4:

In a fuzzy neutrosophic supra topological space, every fuzzy neutrosophic supra semi-closed set in

(X_{N^*},T_{N^*}) is need not to be fuzzyneutrosophic supra closed set in (X_{N^*},T_{N^*}). For the following example.

Example 3.4:

Let $X_{N^*}= \{a, b, c\}$ and with $T_{N^*} = \{ 0_{N^*}, A_{N^*}, B_{N^*}, C_{N^*}, D_{N^*}, E_{N^*}, 1_{N^*}\}$. Some of the fuzzy neutrosophic supra semi-open sets are

$A_{N^*} = \{\langle a, 0.3,0.6,0.5\rangle, \langle b, 0.9,0.7,0.1\rangle, \langle c, 0.4,0.6,0.1\rangle\}$

$B_{N^*}= \{\langle a, 0.7,0.4,0\rangle, \langle b, 0.9,0.6,0.1\rangle, \langle c, 0.8,0.5,0.5\rangle\}$

$C_{N^*} = \{\langle a, 0.5,0.4,0.3\rangle, \langle b, 0.1,0.3,0.9\rangle, \langle c, 0.1,0.4,0.4\rangle\}$

$D_{N^*} = \{\langle a, 0,0.6,0.7\rangle, \langle b, 0.1,0.4,0.9\rangle, \langle c, 0.5,0.5,0.8\rangle\}$

$E_{N^*} = \{\langle a, 0.9,0.4,0.2\rangle, \langle b, 0.9,0.2,0\rangle, \langle c, 0.1,0.2,0.3\rangle\}$

Now, $\text{int}^*(\text{cl}^*(1 - C_{N^*})) = \text{int}^*(1 - C_{N^*}) = 0$, $\text{int}^*(\text{cl}^* (1 - D_{N^*})) = \text{int}^* (1 - D_{N^*}) = 0$, $\text{int}^*(\text{cl}^*((1 - E_{N^*})) = \text{int}^*(1 - E_{N^*}) = 0$.

Here the fuzzy neutrosophic supra semi−closed sets are $1 - C_{N^*}$, $1 - D_{N^*}$, $1 - E_{N^*}$ but are not a fuzzy neutrosophic supra closed sets in (X_{N^*},T_{N^*}).

3.3 Fuzzy Neutrosophic supra Semi-Interior In Fuzzy Neutrosophic supra Topological Spaces.

In this section, we introduce the fuzzy neutrosophic supra semi-interior and their properties in fuzzy neutrosophic supra topologicalspace.

Definition 3.3:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space. Then for a fuzzy neutrosophic supra set A_{N^*} of (X_{N^*},T_{N^*}), the fuzzy neutrosophic supra semi-interior of A_{N^*} is the union of all fuzzy neutrosophic supra semi-open sets of (X_{N^*},T_{N^*}) contained in A_{N^*}.That is, $\text{int}^*(A_{N^*}) = \vee\{ G_{N^*}: G_{N^*}$ is a fuzzy neutrosophic supra semi - open set in (X_{N^*},T_{N^*}) and $G_{N^*} \leq A_{N^*}\}$

Proposition 3.12:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space. Then for any fuzzy neutrosophic supra sets A_{N^*} and B_{N^*} of a fuzzy neutrosophic supratopological space (X_{N^*},T_{N^*}) we have

(i) $\text{int}^*(A_{N^*}) \leq A_{N^*}$

(ii) A_{N^*} is $fnSO$ set in $(X_{N^*},T_{N^*}) \Leftrightarrow fn\,(\text{int}^*(A_{N^*})) = A_{N^*}$

(iii) $(\text{int}^*(\text{int}^*(A_{N^*})))= fnS\,(\text{int}^*(A_{N^*}))$

(iv) If $A_{N^*} \leq B_{N^*}$ then $\text{int}^*(A_{N^*}) \leq fnS\,(\text{int}^*(B_{N^*})$.

Proof:

(i) follows from definition 3.3

Let A_{N^*} be fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}).Then $A_{N^*} \leq fn(\text{int}^*(A_{N^*}))$. Conversely assume that $A_{N^*} = \text{int}^*(A_{N^*})$. By using definition 3.3, A_{N^*} is a fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}). Thus (ii) proved. By using (ii), $(\text{int}^*(\text{int}^*(A_{N^*}))) = fnS(\text{int}^*(A_{N^*}))$. This proved (iii).Since $A_{N^*} \leq B_{N^*}$ by using (i), $\text{int}^* (A_{N^*}) \leq A_{N^*} \leq B_{N^*}$. Thus $\text{int}^*(A_{N^*}) \leq A_{N^*}$. By (iii), $(\text{int}^*(\text{int}^*(A_{N^*}))) = fnS(\text{int}^* (B_{N^*}))$.Thus $\text{int}^*(A_{N^*}) \leq (\text{int}^*(B_{N^*}))$. This proves (iv).

Proposition 3.13:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space. Then for any fuzzy neutrosophic supra sets A_{N^*} and B_{N^*} of a fuzzyneutrosophic supra topological space (X_{N^*},T_{N^*}) we have

(i) $\text{int}^*(A_{N^*} \wedge B_{N^*}) = (\text{int}^*(A_{N^*})) \wedge f(\text{int}^*(B_{N^*}))$.

(ii) $\text{int}^*(A_{N^*} \vee B_{N^*}) \geq fn(\text{int}^*(A_{N^*})) \vee fnS(\text{int}^*(B_{N^*}))$.

Proof:

Since $A_{N^*} \wedge B_{N^*} \leq A_{N^*}$ and $A_{N^*} \wedge B_{N^*} \leq B_{N^*}$. By using proposition 3.12 (iv), $(\text{int}^*(A_{N^*} \wedge B_{N^*})) \leq fnS(\text{int}^*(A_{N^*}))$ and $fnS(\text{int}^*(A_{N^*} \wedge B_{N^*})) \leq fnS(\text{int}^*(B_{N^*}))$. This implies that $\text{int}^*(A_{N^*} \wedge B_{N^*}) = (\text{int}^*(A_{N^*})) \wedge fnS(\text{int}^*(B_{N^*})) \longrightarrow (1)$. By using proposition 3.12 (i), $\text{int}^*(A_{N^*}) \leq A_{N^*}$ and $\text{int}^*(B_{N^*}) \leq B_{N^*}$. This implies that $\text{int}^*(A_{N^*}) \wedge (\text{int}^*(B_{N^*})) \leq A_{N^*} \wedge B_{N^*}$. Now applying proposition 3.12 (iv), $(\text{int}^*(A_{N^*}) \wedge fn(\text{int}^*(B_{N^*}))) \leq fnS(\text{int}^*(A_{N^*} \wedge B_{N^*})) \longrightarrow (2)$. By (1), $(\text{int}^*(fn(\text{int}^*(A_{N^*})))) \wedge fnS(\text{int}^*(fnS(\text{int}^*(B_{N^*})))) \leq fnS(\text{int}^*(A_{N^*} \wedge B_{N^*}))$. By proposition 3.12(iii), $\text{int}^*(A_{N^*}) \wedge (\text{int}^*(B_{N^*})) \leq fnS(\text{int}^*(A_{N^*} \wedge B_{N^*}))$. From (1) and (2), $\text{int}^*(A_{N^*} \wedge B_{N^*}) = f(\text{int}^*(A_{N^*})) \wedge fnS(\text{int}^*(B_{N^*}))$. This implies (i). Since $A_{N^*} \leq A_{N^*} \vee B_{N^*}$ and $B_{N^*} \leq A_{N^*} \vee B_{N^*}$, by using proposition 3.12 (iv), $(\text{int}^*(A_{N^*})) \leq fnS(\text{int}^*(A_{N^*} \vee B_{N^*}))$ and $fnS(\text{int}^*(B_{N^*})) \leq fnS(\text{int}^*(A_{N^*} \vee B_{N^*}))$. This implies that $\text{int}^*(A_{N^*}) \vee f(\text{int}^*(B_{N^*})) \leq fnS(\text{int}^*(A_{N^*} \vee B_{N^*}))$. Hence (ii). The following example shows that the equality need not hold in proposition 3.13(ii).

3.4 Fuzzy Neutrosophic supra Semi−Closure In Fuzzy Neutrosophic supra Topological Spaces.

In this section, we introduce the concept of fuzzy neutrosophic supra semi-closure and their properties in fuzzy neutrosophic supra topological space.

Definition 3.4:

Let (X_{N^*}, T_{N^*}) be a fuzzy neutrosophic supra topological space. Then for a fuzzy neutrosophic supra set A_{N^*} of (X_{N^*}, T_{N^*}), the fuzzy neutrosophic supra semi-closure of A_{N^*} [$fnSc(A_{N^*})$ for short] is the intersection of all fuzzy neutrosophic supra semi-closed sets of (X_{N^*}, T_{N^*}) contained in A_{N^*}.

That is, $\text{cl}^*(A_{N^*}) = \wedge\{ K_{N^*} : K_{N^*} \textit{ is a } fnSC \textit{ set in } (X_{N^*}, T_{N^*}) \text{ and } K_{N^*} \geq A_{N^*} \}$.

Proposition 3.14:

Let (X_{N^*}, T_{N^*}) be a fuzzy neutrosophic supra topological space. Then for any fuzzy neutrosophic supra sets A_{N^*} and B_{N^*} of a (X_{N^*}, T_{N^*})

(i) $(\text{int}^*(A_{N^*})) \leq fnS(C(\text{cl}^*(A_{N^*})))$,

(ii) $(\text{cl}^*(A_{N^*})) \leq (\text{int}^*(A_{N^*}))$.

Proof:

(i) follows from definition 3.4, $\text{int}^*(A_{N^*}) = \vee\{ G_{N^*} : G_{N^*} \textit{ is a } fnSO \textit{ in } (X_{N^*}, T_{N^*}) \text{ and } G_{N^*} \leq A_{N^*} \}$. Taking complement on bothsides, $(fnS(\text{int}^*(A_{N^*}))) = C(\vee\{G_{N^*} : G_{N^*} \textit{ is a } fnSO \textit{ set in } (X_{N^*}, T_{N^*}) \text{ and } G_{N^*} \leq A_{N^*} \}) = \wedge\{ (G_{N^*}) : C(G_{N^*}) \textit{ is a } fnSC \textit{ set in } (X_{N^*}, T_{N^*}) \textit{ and } C(A_{N^*}) \leq C(G_{N^*}) \}$. Replacing $C(G_{N^*})$ by K_{N^*}, we get $C(fnS(\text{int}^*(A_{N^*}))) = \wedge\{ K_{N^*} : K_{N^*} \textit{ is a } fnSC \textit{ set in } (X_{N^*}, T_{N^*}) \text{ and } K_{N^*} \geq C(A_{N^*})\}$. By definition 3.4, $(\text{int}^*(A_{N^*})) = fnS(C(\text{cl}^*(A_{N^*})))$. This proves (i). By using (i), $C(fnS(C(\text{int}^*(A_{N^*})))) = fnS(C(C(\text{cl}^*(A_{N^*})))) = fnS(\text{cl}^*(A_{N^*}))$. Taking complement on both sides, we get $(C(\text{int}^*(A_{N^*}))) = C(fnS(\text{cl}^*(A_{N^*})))$.

Hence proved (ii).

Proposition 3.15:

Let (X_{N^*}, T_{N^*}) be a fuzzy neutrosophic supra topological space. Then for any fuzzy neutrosophic supra sets A_{N^*} and B_{N^*} of a fuzzyneutrosophic supra topological space (X_{N^*}, T_{N^*}) we have

(i) $A_{N^*} \leq \text{cl}^*(A_{N^*})$

(ii) A_{N^*} is $fnSC$ set in $(X_{N^*},T_{N^*}) \Leftrightarrow cl^*(A_{N^*}) = A_{N^*}$

(iii) $f(cl^*(cl^*(A_{N^*}))) = fnS(cl^*(A_{N^*}))$

(iv) If $A_{N^*} \leq B_{N^*}$ then $cl^*(A_{N^*}) \leq f(cl^*(B_{N^*}))$.

Proof:

(i) follows from definition 3.3

Let A_{N^*} be fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}). By using proposition 3.14, (A_{N^*}) be fuzzy neutrosophic suprasemi-open set in (X_{N^*},T_{N^*}). By proposition 3.12 (ii), $f(C(int^*(A_{N^*}))) = C(A_{N^*}) \Leftrightarrow C(fnS(cl^*(A_{N^*}))) = C(A_{N^*}) \Leftrightarrow fnS(cl^*(A_{N^*})) = A_{N^*}$. Thus (ii) proved.By using (ii), $f(cl^*(cl^*(A_{N^*})))$ $= fnS$ $cl^*(A_{N^*})$). This proved (iii).Since $A_{N^*} \leq B_{N^*}$, $(A_{N^*}) \leq (B_{N^*})$. By proposition 3.14 (iv), $(int^*(B_{N^*})) \leq fnS(C(cl^*(B_{N^*})))$. Taking complement on bothsides, $((int^*(B_{N^*}))) \geq (fn(C(int^*(A_{N^*}))))$. By proposition 3.14 (ii), $cl^*(A_{N^*}) \leq f(cl^*(B_{N^*}))$. This proves (iv).

Proposition 3.16:

Let A_{N^*} be a fuzzy neutrosophic supra set in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}). Then fn $(int^*(A_{N^*})) \leq fnS(int^*(A_{N^*})) \leq A_{N^*} \leq fnS(cl^*(A_{N^*})) \leq fn(cl^*(A_{N^*}))$.

Proof:

It follows from the definitions of corresponding operators.

Proposition 3.17:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space. Then for any fuzzy neutrosophic supra sets A_{N^*} and B_{N^*} of a fuzzyneutrosophic supra topological space (X_{N^*},T_{N^*}) we have

(i) $cl^*(A_{N^*} \vee B_{N^*}) = cl^*(A_{N^*}) \vee f(cl^*(B_{N^*}))$ and

(ii) $cl^*(A_{N^*} \wedge B_{N^*}) \leq cl^*(A_{N^*}) \wedge f(cl^*(B_{N^*}))$.

Proof:

Since $fnS(cl^*(A_{N^*} \vee B_{N^*})) = fnS(cl^*(C(C(A_{N^*} \vee B_{N^*}))))$,by using proposition 3.12(i), $fnS(cl^*(A_{N^*} \vee B_{N^*})) = C(fnS(int^*(C(A_{N^*} \vee B_{N^*})))) = C(fnS(C(A_{N^*})) \wedge C(int^*(B_{N^*})))$.Again using proposition 3.15 (i),$fn(cl^*(A_{N^*} \vee B_{N^*})) = C(fnS(int^*(C(A_{N^*}))) \wedge fnS(int^*(C(B_{N^*}))) = fnS(C(C(A_{N^*})) \vee fnS(cl^*(C(C(B_{N^*})))) = fnS(cl^*(A_{N^*})) \vee fnS(cl^*(B_{N^*}))$. This proved (i)Since $A_{N^*} \wedge B_{N^*} \leq$ and $A_{N^*} \wedge B_{N^*} \leq B_{N^*}$. By using proposition 3.15 (iv), $f(cl^*(A_{N^*} \wedge B_{N^*})) \leq fnS(cl^*(A_{N^*}))$ and $fnS(cl^*(A_{N^*} \wedge B_{N^*}))$ $\leq fnS(cl^*(B_{N^*}))$. This implies that $cl^*(A_{N^*} \wedge B_{N^*}) \leq f(cl^*(A_{N^*})) \wedge fnS(cl^*(B_{N^*}))$. This proves (ii).

Proposition 3.18:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space. Then for any fuzzy neutrosophic supra sets A_{N^*} and B_{N^*} of a fuzzyneutrosophic supra topological space (X_{N^*},T_{N^*}) we have

(i) $cl^*(A_{N^*}) \geq A_{N^*} \vee cl^*(f(int^*(A_{N^*})))$,

(ii) $int^*(A_{N^*}) \leq A_{N^*} \wedge int^*(f(cl^*(A_{N^*})))$,

$(iii)(int^*(fnS(cl^*(A_{N^*})))) \leq fn(int^*(fn(cl^*(A_{N^*}))))$,

$(iv)(int^*(fnS(cl^*(A_{N^*})))) \geq fn(int^*(fnS(cl^*(fnS\ int^*(A_{N^*})))))$

Proof:

By using proposition 3.15 (i), $A_{N^*} \leq f(cl^*(A_{N^*})) \longrightarrow (1)$. Again By using proposition 3.12 (i), $f(int^*(A_{N^*})) \leq A_{N^*}$. Then $f(cl^*(int^*(A_{N^*}))) \leq fnS(cl^*(A_{N^*})) \longrightarrow (2)$. By (1) and (2) we have, $A_{N^*} \vee cl^*(int^*(A_{N^*})) \leq fnS(cl^*(A_{N^*}))$. This proves (i).By proposition 3.12 (i), $f(int^*(A_{N^*})) \leq A_{N^*} \longrightarrow (1)$.

Again by using proposition 3.15 (i), $A_{N^*} \leq f(\text{cl}^*(A_{N^*}))$. Then $f(\text{int}^*(A_{N^*})) \leq \text{int}^*(fnS(\text{cl}^*(A_{N^*})) \longrightarrow (2)$. From (1) and (2) we have $f(\text{int}^*(A_{N^*})) \leq A_{N^*} \wedge fnS(\text{int}^*(fnS(\text{cl}^*(A_{N^*}))))$. This proves (ii). By proposition 3.16, $f(\text{cl}^*(A_{N^*})) \leq fn(\text{cl}^*(A_{N^*}))$. We get $\text{int}^*(f(\text{cl}^*(A_{N^*}))) \leq fn(\text{int}^*(fn(\text{cl}^*(A_{N^*})))$. Hence (iii).By (i), $f(\text{cl}^*(A_{N^*})) \geq A_{N^*} \vee fnS(\text{cl}^*(fnS(\text{int}^*(A_{N^*}))))$. We have $(\text{int}^*(fnS(\text{cl}^*(A_{N^*})))) \geq fn(A_{N^*} \vee \text{int}^*(fnS(\text{cl}^*(fnS(\text{int}^*(A_{N^*}))))))$. Since $\text{int}^*(A_{N^*} \vee B_{N^*}) \geq fn(\text{int}^*(A_{N^*})) \vee fn(\text{int}^*(B_{N^*}))$, $fn(\text{int}^*(fnS(\text{cl}^*(B_{N^*})))) \geq fn(\text{int}^*(A_{N^*})) \vee fn(\text{int}^*(fnS(\text{cl}^*(fnS(\text{int}^*(A_{N^*})))))) \geq fn(\text{int}^*(fnS(\text{cl}^*(fnS(\text{int}^*(A_{N^*}))))))$.Hence proves (iv).

4. FUZZY NEUTROSOPHIC SUPRA SEMI-NOWHERE DENSE SETS

Definition 4.1:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space. A fuzzy neutrosophic supra set A_{N^*} in (X_{N^*},T_{N^*}) is called a fuzzy neutrosophic supra semi - nowhere dense set if there exists no non-zero fuzzy neutrosophic supra semi -open set B_{N^*} in (X_{N^*},T_{N^*}) Such that $B_{N^*} < \text{scl}^*(A_{N^*})$. That is, $\text{sint}^*\text{scl}^*(A_{N^*}) = 0$.

Example 4.1:

Let $X_{N^*} = \{a, b, c\}$. The fuzzy neutrosophic supra sets A_{N^*}, B_{N^*}, A_{N^*}, D_{N^*} and E_{N^*} are defined on X_{N^*} as follows:

A_{N^*}: $X_{N^*} \rightarrow [0,1]$ define as $A_{N^*}(a)=0.7$; $A_{N^*}(b)=0.3$; $A_{N^*}(c)=0.5$,

B_{N^*}: $X_{N^*} \rightarrow [0,1]$ define as $B_{N^*}(a)=0.2$; $B_{N^*}(b)=0.8$; $B_{N^*}(c)=0.6$,

C_{N^*}: $X_{N^*} \rightarrow [0,1]$ define as $C_{N^*}(a)=0.5$; $C_{N^*}(b)=0.7$; $C_{N^*}(c)=0.8$,

D_{N^*}: $X_{N^*} \rightarrow [0,1]$ define as $D_{N^*}(a)=0.9$; $D_{N^*}(b)=0.8$; $D_{N^*}(c)=0.7$,

E_{N^*}: $X_{N^*} \rightarrow [0,1]$ define as $E_{N^*}(a)=0,9$; $E_{N^*}(b)=0.9$; $E_{N^*}(c)=0.6$.

Then $T_{N^*} = \{0, A_{N^*}, B_{N^*}, A_{N^*}, (A_{N^*} \vee B_{N^*}), (A_{N^*} \vee C_{N^*}), (B_{N^*} \vee A_{N^*}), (A_{N^*} \wedge E_{N^*}), (A_{N^*} \wedge D_{N^*}), (B_{N^*} \wedge A_{N^*}), A_{N^*} \vee B_{N^*} \vee A_{N^*}, B_{N^*} \vee (A_{N^*} \wedge A_{N^*}), A_{N^*} \vee (B_{N^*} \vee A_{N^*}), A_{N^*} \wedge (A_{N^*} \vee B_{N^*}), 1\}$ is a fuzzy neutrosophic supra topology. Then non-zero fuzzy neutrosophic supra semi- open sets in (X_{N^*},T_{N^*}) are A_{N^*}, B_{N^*}, C_{N^*}, $(A_{N^*} \vee B_{N^*})$, $(A_{N^*} \vee C_{N^*})$, $(B_{N^*} \vee A_{N^*})$, $(A_{N^*} \wedge B_{N^*})$, $(A_{N^*} \wedge C_{N^*})$, $(B_{N^*} \wedge A_{N^*})$, $A_{N^*} \vee B_{N^*} \vee A_{N^*}$, $B_{N^*} \vee (A_{N^*} \wedge C_{N^*})$, $A_{N^*} \vee (B_{N^*} \vee A_{N^*})$, $C_{N^*} \wedge (A_{N^*} \vee B_{N^*})$, Now the fuzzy neutrosophic sets $1-A_{N^*}$, $1-B_{N^*}$, $1-A_{N^*}$, $1-(A_{N^*} \vee B_{N^*})$, $1-(A_{N^*} \vee C_{N^*})$, $1-{}_{N^*} \wedge D_{N^*})$, $1-(B_{N^*} \vee A_{N^*})$, $1-(A_{N^*} \vee (B_{N^*} \wedge A_{N^*}))$, $1-(B_{N^*} \vee (A_{N^*} \wedge C_{N^*}))$, $1-(A_{N^*} \vee (B_{N^*} \vee A_{N^*}))$,

$1-(C_{N^*} \wedge (A_{N^*} \vee B_{N^*}))$, $1- D_{N^*}$, $1- E_{N^*}$ are fuzzy neutrosophic supra semi-nowhere dense sets in (X_{N^*},T_{N^*}).

Proposition 4.1:

The complement of a fuzzy neutrosophic supra semi- nowhere dense set in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) need not be fuzzy neutrosophic supra semi-nowhere dense set.

Proof:

For example 4.1, $(1-D_{N^*})$ is a fuzzy neutrosophic supra semi- nowhere dense set in (X_{N^*},T_{N^*}) where as $D_{N^*} = 1-(1-D_{N^*})$ is not a fuzzy neutrosophic supra semi-nowhere dense set in (X_{N^*},T_{N^*}).

Proposition 4.2:

If A_{N^*} and B_{N^*} are fuzzy neutrosophic supra semi- nowhere dense set in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}), then $A_{N^*}\vee B_{N^*}$ need not be fuzzy neutrosophic supra semi- nowhere dense set in (X_{N^*},T_{N^*}).

Proof:

For, in Example 4.1, 1 - D_{N^*}, 1 - A_{N^*} are fuzzy neutrosophic supra semi- nowhere dense set in (X_{N^*},T_{N^*}). But (1 - D_{N^*}) $\vee$ (1 - A_{N^*}) = C_{N^*} implies that sint*scl*(C_{N^*}) $\neq$ 0. Therefore union of fuzzy neutrosophic supra semi - nowhere dense sets need not be fuzzy neutrosophic supra semi-nowhere dense set in (X_{N^*},T_{N^*}).

Proposition 4.3:

If the fuzzy neutrosophic supra sets A_{N^*} and B_{N^*} are fuzzy neutrosophic supra semi-nowhere dense sets in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) then $A_{N^*}\wedge B_{N^*}$ is a fuzzy neutrosophic supra semi - nowhere dense set in (X_{N^*},T_{N^*}).

Proof:

Let the fuzzy neutrosophic supra sets A_{N^*} and B_{N^*} be fuzzy neutrosophic supra semi- nowhere dense set in (X_{N^*},T_{N^*}). Now sint*scl*($A_{N^*}\wedge B_{N^*}$) $\leq$ sint*scl*(A_{N^*}) $\wedge$ sint*scl*(B_{N^*}) $\leq 0 \wedge 0$ (since sint* scl*(A_{N^*}) = 0 and sint*scl*(B_{N^*}) = 0). That is, sint*scl*($A_{N^*}\wedge B_{N^*}$) = 0. Hence ($A_{N^*}\wedge B_{N^*}$) is a fuzzy neutrosophic supra semi-nowhere dense set in (X_{N^*},T_{N^*}).

Proposition 4.4:

If A_{N^*} is a fuzzy neutrosophic supra semi-nowhere dense sets in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) then sint*(A_{N^*}) = 0.

Proof:

Let A_{N^*} be a fuzzy neutrosophic supra semi-nowhere dense set in (X_{N^*},T_{N^*}). Then we have sint*scl*(A_{N^*}) = 0.Now $A_{N^*} \leq$ scl*(A_{N^*}) we have sint*(A_{N^*}) $\leq$ sint*scl*(A_{N^*}) = 0. Hence sint*(A_{N^*}) = 0.

Proposition 4.5:

If A_{N^*} is a fuzzy neutrosophic nowhere dense set in a fuzzy neutrosophic topological space (X_{N^*},T_{N^*}) then int*scl*(A_{N^*}) = 0.

Proof:

Let A_{N^*} be a fuzzy neutrosophic supra nowhere dense set in (X_{N^*},T_{N^*}). Then, we have int* cl*(A_{N^*}) = 0 and int*(A_{N^*}) = 0. Now scl*(A_{N^*}) = A_{N^*}. Since A_{N^*} is fuzzy neutrosophic supra semi-closed set in (X_{N^*},T_{N^*}), implies that int* scl*(A_{N^*}) = int*(A_{N^*}) = 0. Hence int* scl*(A_{N^*}) = 0.

Proposition 4.6:

If A_{N^*} is a fuzzy neutrosophic supra semi-nowhere dense set and B_{N^*} is any fuzzy neutrosophic supra set in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}),then ($A_{N^*}\wedge B_{N^*}$) is a fuzzy neutrosophic supra semi-nowhere dense set in (X_{N^*},T_{N^*}).

Proof:

Let A_{N^*} be a fuzzy neutrosophic supra semi-nowhere dense set in (X_{N^*},T_{N^*}). Then sint*scl*(A_{N^*}) = 0. Now sint*scl*($A_{N^*}\wedge B_{N^*}$) $\leq$ sint*scl*(A_{N^*}) $\wedge$ sint*scl*(B_{N^*}) $\leq 0 \wedge$ sint*scl*(B_{N^*}) = 0.That is, sint*scl*($A_{N^*}\wedge B_{N^*}$) = 0. Hence, ($A_{N^*}\wedge B_{N^*}$) is a fuzzy neutrosophic supra semi - nowhere dense set in (X_{N^*},T_{N^*}).

Definition 4.2:

A fuzzy neutrosophic supra set A_{N^*} in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) is called fuzzy neutrosophic supra semi-dense if there exists no fuzzy neutrosophic supra semi-closed set B_{N^*} in (X_{N^*},T_{N^*}) such that $A_{N^*}<B_{N^*}<1$. That is., $scl^*(A_{N^*}) = 1$.

Proposition 4.7:

If A_{N^*} is a fuzzy neutrosophic supra semi-dense and fuzzy neutrosophic supra semi-open set in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) and if $B_{N^*}\leq 1 - A_{N^*}$, then B_{N^*} is a fuzzy neutrosophic supra semi- nowhere dense set in (X_{N^*},T_{N^*}).

Proof:

Let A_{N^*} be a fuzzy neutrosophic supra semi- dense set in (X_{N^*},T_{N^*}). Then we have $scl^*(A_{N^*}) = 1$ and $sint^*(A_{N^*}) = A_{N^*}$. Now $B_{N^*}\leq 1-A_{N^*}$, implies that $scl^*(B_{N^*}) \leq scl^*(1-A_{N^*})$. Then $scl^*(B_{N^*}) \leq 1-sint^*(A_{N^*}) = 1-A_{N^*}$. Hence $scl^*(B_{N^*})\leq scl^*(1-A_{N^*})$, which implies that $sint^*scl^*(B_{N^*}) \leq sint^*(1- A_{N^*}) = 1-scl^*(A_{N^*}) = 1-1 = 0$. That is, $sint^*scl^*(B_{N^*}) = 0$. Hence B_{N^*} is a fuzzy neutrosophic supra semi- nowhere dense set in (X_{N^*},T_{N^*}).

Proposition 4.8:

If A_{N^*} is a fuzzy neutrosophic supra semi-nowhere dense set in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}). Then $1 - A_{N^*}$, is a fuzzy neutrosophic supra semi - dense set in (X_{N^*},T_{N^*}).

Proof:

Let A_{N^*} be a fuzzy neutrosophic supra semi-nowhere dense set in (X_{N^*},T_{N^*}). Then, $sint^*scl^*(A_{N^*}) = 0$. Now $A_{N^*} \leq scl^*(A_{N^*})$ implies that $sint^*(A_{N^*}) \leq sint^*scl^*(A_{N^*}) = 0$. Then $sint^*(A_{N^*}) = 0$ and $scl^*(1-A_{N^*}) = 1- sint^*(A_{N^*}) = 1-0 = 1$ and hence $1- A_{N^*}$ is a fuzzy neutrosophic supra semi-dense set in (X_{N^*},T_{N^*}).

5. FUZZY NEUTROSOPHIC SUPRA SEMI - BAIRE SPACES

Definition 5.1:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space. Then (X_{N^*},T_{N^*}) is called a fuzzy neutrosophic supra semi-Baire space $sint^*[\bigvee_{i=1}^{\infty}(A_{N_i}{}^*)] = 0$, where $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi - nowhere dense sets in (X_{N^*},T_{N^*}).

Proposition 5.1:

If $sint^*(\bigvee_{i=1}^{\infty}(A_{N_i}{}^*)) = 0$ where $sint^*(A_{N_i}{}^*) = 0$ and $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi-closed sets in a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}), then (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi- Baire space.

Proof:

Let $A_{N_i}{}^*$'s be fuzzy neutrosophic supra semi-closed sets in (X_{N^*},T_{N^*}). since $sint^*(A_{N_i}{}^*) = 0$, Then $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi-nowhere dense sets in (X_{N^*},T_{N^*}), therefore we have $sint^*(\bigvee_{i=1}^{\infty}(A_{N_i}{}^*)) = 0$ where $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi-nowhere dense sets in (X_{N^*},T_{N^*}). Hence (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi-Baire space.

Proposition 5.2:

If $scl^*(\bigwedge_{i=1}^{\infty}(A_{N_i}{}^*)) = 1$, where $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi-nowhere dense and fuzzy neutrosophic supra semi-open sets in a fuzzy neutrosophic topological space (X_{N^*},T_{N^*}), then (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi-Baire space.

Proof:

Now scl*($\wedge_{i=1}^{\infty}(A_{N_i}{}^*)$) = 1 implies that 1- scl*($A_{N_i}{}^*$) = 0. Then we have sint*(1-$\wedge_{i=1}^{\infty}(A_{N_i}{}^*)$) = 0, which implies that sint*($\vee_{i=1}^{\infty}(A_{N_i}{}^*)$) = 0. Since $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi-dense sets in (X_{N^*},T_{N^*}), scl*($A_{N_i}{}^*$) = 1and sint*(1 - $A_{N_i}{}^*$)) = 0, where sint*(1 - $A_{N_i}{}^*$) = 0 and (1 - $A_{N_i}{}^*$)'s are fuzzy neutrosophic supra semi-closed sets in (X_{N^*},T_{N^*}). Then, by proposition 5.1, (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi-Baire space.

Proposition 5.3:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra topological space.Then the following are equivalent:

1) (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi-Baire space.
2) sint*(A_{N^*}) = 0 for every fuzzy neutrosophic supra semi-first categoryset A_{N^*} in (X_{N^*},T_{N^*}).
3) scl*(B_{N^*}) = 1 for every fuzzy neutrosophic supra semi-residual set in(X_{N^*},T_{N^*}).

Proof:

(1) $\Longrightarrow$ (2). Let A_{N^*} be a fuzzy neutrosophic supra semi- first category set in (X_{N^*},T_{N^*}). Then A_{N^*} = $\vee_{i=1}^{\infty}(A_{N_i}{}^*)$ where $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi-nowhere dense set in (X_{N^*},T_{N^*}). Now sint*(A_{N^*}) = sint*($\vee_{i=1}^{\infty}(A_{N_i}{}^*)$) = 0 (since ($X_{N^*}$,$T_{N^*}$) is a fuzzy neutrosophic supra semi - Baire space). Therefore, sint*(A_{N^*}) = 0.

(2)$\Longrightarrow$ (3). Let B_{N^*} be a fuzzy neutrosophic supra semi-residual set in (X_{N^*},T_{N^*}). Then, 1 - B_{N^*} is a fuzzy neutrosophic supra semi-first category set in (X_{N^*},T_{N^*}). By hypothesis, sint*(1-B_{N^*}) = 0. Which implies that 1-scl*(B_{N^*}) = 0. Hence we have scl*(B_{N^*}) =1.

(3)$\Longrightarrow$ (1). Let A_{N^*} be a fuzzy neutrosophic supra semi-first category set in (X_{N^*},T_{N^*}), Then A_{N^*} = $\vee_{i=1}^{\infty}(A_{N_i}{}^*)$, where $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi-nowhere dense set in (X_{N^*},T_{N^*}). 1-A_{N^*} is a fuzzy neutrosophic supra semi- residual set in (X_{N^*},T_{N^*}). Since A_{N^*} is a fuzzy neutrosophic supra semi-first category set in (X_{N^*},T_{N^*}). By hypothesis, we have scl*(1-A_{N^*}) = 1.Then 1 - sint*(A_{N^*}) =1, which implies that sint*(A_{N^*}) = 0. Hence sint*($\vee_{i=1}^{\infty}(A_{N_i}{}^*)$) = 0 where $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi - nowhere dense sets in (X_{N^*},T_{N^*}). Hence (X_{N^*},T_{N^*}) is a fuzzy neutrosophic suprasemi - Baire space.

Proposition 5.4:

If a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi-Baire space, then (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi-second category space.

Proof:

Let (X_{N^*},T_{N^*}) be a fuzzy neutrosophic supra semi-Baire space. Then sint*($\vee_{i=1}^{\infty}(A_{N_i}{}^*)$) = 0, where $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi nowhere dense sets in (X_{N^*},T_{N^*}). Then ($\vee_{i=1}^{\infty}(A_{N_i}{}^*)$) $\neq 1_x$. (Suppose ($\vee_{i=1}^{\infty}(A_{N_i}{}^*)$) $\neq 1_x$ implies that sint*($\vee_{i=1}^{\infty}(A_{N_i}{}^*)$) = sint*($1_x$) which implies that 0 = 1, a contradiction). Hence (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi-second category space.

Proposition 5.5:

If a fuzzy neutrosophic supra topological space (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi-Baire space, Then no non-zero fuzzy neutrosophic supra semi-open sets in (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi-first category set in (X_{N^*},T_{N^*}).

Proof:

Suppose that A_{N^*} is a non-zero fuzzy neutrosophic supra semi- open set in (X_{N^*},T_{N^*}), such that $(V_{i=1}^{\infty}(A_{N_i}{}^*))$,where $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi-nowhere dense set in (X_{N^*},T_{N^*}). Then we have $sint^*(A_{N^*}) = sint^*(V_{i=1}^{\infty}(A_{N_i}{}^*))$. Since A_{N^*} is a non- zero fuzzy neutrosophic supra semi-open set in (X_{N^*},T_{N^*}). $sint^*(A_{N^*}) = A_{N^*}$, Then $sint^*(V_{i=1}^{\infty}(A_{N_i}{}^*)) = A_{N^*} \neq 0$. But this is a contradiction to (X_{N^*},T_{N^*}) being a fuzzy neutrosophic supra semi-Baire space in which $sint^*(V_{i=1}^{\infty}(A_{N_i}{}^*)) = 0$.where $A_{N_i}{}^*$'s are fuzzy neutrosophic supra semi- nowhere dense set in (X_{N^*},T_{N^*}). Hence, we must have $A_{N^*} \neq (V_{i=1}^{\infty}(A_{N_i}{}^*))$. Therefore no non-zero fuzzy neutrosophic supra semi - open set in (X_{N^*},T_{N^*}) is a fuzzy neutrosophic supra semi-first category set in (X_{N^*},T_{N^*}).

6. CONCLUSION

In this paper, we introduced and studied a new concept fuzzy neutrosophic supra semi-open set, fuzzy neutrosophic supra semi-closed set, fuzzy neutrosophic supra semi-nowhere dense set, fuzzy neutrosophic supra semi-dense set, fuzzy neutrosophic supra semi-Baire space, in fuzzy neutrosophic supra topological space. Some characteristics of fuzzy neutrosophic supra semi-Baire spaces are established.

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ON FUZZY NEUTROSOPHIC SUPRA REGULAR BAIRE SPACES

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ABSTRACT
In this paper the concept of fuzzy neutrosophic supra Regular open (closed) sets, fuzzy neutrosophic supra regular nowhere dense sets are introduced and studied. By means of fuzzy neutrosophic supra regular nowhere dense set, the concepts of fuzzy neutrosophic supra regular Baire space are introduced and characterization of fuzzy neutrosophic supra regular Baire space are studied.

Keywords: Fuzzy neutrosophic supra Regular open set, Fuzzy neutrosophic supra Regular closed set, Fuzzy neutrosophic supra regular dense set, Fuzzy neutrosophic supra regular nowhere dense set, Fuzzy neutrosophic supra regular first and second category, Fuzzy neutrosophic supra regular Baire spaces.

1. INTRODUCTION

In 1984, Mashhour.et.al [5] introduced the concept of supra topological spaces. In 1987, Abd E-Monsef et.al. [1] introduced Fuzzy supra topological space and defined basic notion of fuzzy supra topological space.

Arockiarani I, Sumathi, and MartinaJency [2] introduced the idea of fuzzy neutrosophic supra sets with the concept of fuzzy neutrosophic topological spaces. In their study, Poongothai and Thangaraj [6] investigated fuzzy settings on supra σ-Baire spaces, while Poongothai and Priya [7] studied fuzzy supra semi Baire spaces.

Building upon Smarandache.F in 2005 [11] Neutrosophic Supra Set, A Generalization of the Intuitionistic Fuzzy Sets. Poongothai and Celine [8] investigated fuzzy supra regular Baire spaces. Salama and Alblowi [10] investigated neutrosophic supra sets and neutrosophic topological spaces.

Through our investigation of neutrosophic supra Regular Baire spaces on fuzzy neutrosophic supra topological spaces, we aim to contribute to the development of the theory of fuzzy neutrosophic supra Regular Baire spaces and provide a foundation for further research in this area.

2. PRELIMINARIES

Definition 2.1. [2] A fuzzy neutrosophic supra set A on the universe of discourse X is defined as, $A = \{x, T_A(x), I_A(x), F_A(x)\}$, $x \in X$ where $T, I, F : X \rightarrow [0, 1]$ and $0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3$. With the condition $0 \leq T_{A^*}(x)+I_{A^*}(x)+F_{A^*}(x) \leq 2$.

Definition 2.2. [2] A fuzzy neutrosophic supra set A is a subset of a fuzzy neutrosophic supra set B. (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$.

Definition 2.3. [2] Let X be a non-empty set, and $A=\{x, T_A(x), I_A(x), F_A(x)\}, B=\{x, T_B(x), I_B(x), F_B(x)\}$ be two fuzzy neutrosophic supra sets. Then

$A \cup B = \{x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\}$

$A \cap B = \{x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\}$

Definition 2.4. [2] The difference between two fuzzy neutrosophic supra sets A and B is defined as $A\backslash B(x)=\{x, \min(T_A(x), F_B(x)), \min(I_A(x), 1-I_B(x)), \max(F_A(x), T_B(x))\}$.

Definition 2.5. [2] A fuzzy neutrosophic supra set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0, I_A(x) = 0, F_A(x) = 10$ for all $x \in X$. It is denoted by 0_N.

Definition 2.6. [2] A fuzzy neutrosophic supra set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by I_N.

Definition 2.7. [2] The complement of a fuzzy neutrosophic set A is denoted by A^c and is defined as $A^c = \{x, T^c_A(x), I^c_A(x), F^c_A(x)\}$, where $T^c_A(x) = F_A(x), I_A{}^c(x) = 1 - I_A(x), F_A{}^c(x) = T_A(x)$. The complement of fuzzy neutrosophic set A can also be defined as $A^c = 1_N - A$.

Definition 2.8. [2] A fuzzy neutrosophic topology on a non-empty set X is a τ_N offuzzy neutrosophic sets in X Satisfying the following axioms.

i) $0_N, 1_N \in \tau$.

ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.

iii) $\cup A_i \in \tau$ for any arbitary family $\{A_i : i \in J\} \in \tau$.

In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic supra set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9. [2] The complement A_c of a fuzzy neutrosophic supra set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set inX.

Definition 2.10. [2] Let (X, T^*) be a fuzzy neutrosophic supra topological space and λ be any fuzzy supra set in (X, T^*). The interior and the closure of λ are defined respectively as follows:

i) $int^*(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T^*\}$;

ii) $cl^*(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T^*\}$.

3. FUZZY NEUTROSOPHIC SUPRA REGULAR OPEN SETS

Definition 3.1. Let (X_{N^*}, T_{N^*})be a fuzzy neutrosophic supra topological space. A fuzzy neutrosophic supra set A_{N^*}, is called a fuzzy neutrosophic supra regular open set if $int^*\left(cl^*(A_{N^*})\right) = A_{N^*}$ and a fuzzy neutrosophic supra set A_{N^*} is called a fuzzy neutrosophic supra regular closed set if $cl^*\left(int^*(A_{N^*})\right) = A_{N^*}$.

Proposition 3.1. Let A_{N^*} be a fuzzy neutrosophic supra set in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then A_{N^*} is a fuzzy neutrosophic supra regular closed set if and only if A_{N^*c} is a fuzzy neutrosophic supra regular closet set.

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular open set

$$int^*\left(cl^*(A_{N^*})\right) = A_{N^*}$$
$$(int^*\left(cl^*(A_{N^*})\right))^c = A_{N^*c}$$
$$cl^*\left(cl^*(A_{N^*})\right)^c = A_{N^*c}$$
$$cl^*\left(int^*(A_{N^*c})\right) = A_{N^*c}$$

A_{N^*c} is a fuzzy neutrosophic supra regular closed set.

Conversely, Let A_{N^*c} be a fuzzy neutrosophic supra regular closed set

$$cl^*\left(int^*(A_{N^*c})\right) = A_{N^*c}$$
$$\left(cl^*\left(int^*(A_{N^*c})\right)\right)^c = (A_{N^*c})^c$$
$$int^*\left(int^*(A_{N^*c})\right)^c = A_{N^*}$$
$$int^*\left(cl^*(A_{N^*c})^c\right) = A_{N^*}$$
$$int^*\left(cl^*(A_{N^*})\right) = A_{N^*}.$$

A_{N^*c} is a fuzzy neutrosophic supra regular open set

Proposition 3.2. Every fuzzy neutrosophic supra regular open set is fuzzy neutrosophic supra open set.

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular open set in (X_{N^*}, T_{N^*}) Then $int^*\left(cl^*(A_{N^*})\right) = A_{N^*}$. By the definition [2.10] of fuzzy neutrosophic supra interior, $int^*(A_{N^*})$ is the largest fuzzy neutrosophic supra open set contained in A_{N^*}. $int^*\left(cl^*(A_{N^*})\right)$ is the largest fuzzy neutrosophic supra open set contained $cl^*(A_{N^*})$. Hence, A_{N^*} is a fuzzy neutrosophic supra open set.

Proposition 3.3. Every fuzzy neutrosophic supra regular closed set is fuzzy neutrosophic supra closed set.

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular closed set in (X_{N^*}, T_{N^*}). Then $cl^*\left(int^*(A_{N^*})\right) = A_{N^*}$. By the definition [2.10] of fuzzy neutrosophic supra closure, $cl^*(A_{N^*})$ is the smallest fuzzy

neutrosophic supra closed set containing A_{N^*}, $cl^*\left(int^*(A_{N^*})\right)$ is the smallest fuzzy neutrosophic supra closed set containing $int^*(A_{N^*})$. Hence, A_{N^*} is a fuzzy neutrosophic supra closed set.

Proposition 3.4.

(1). The fuzzy neutrosophic supra closure of a fuzzy neutrosophic supra open setis a fuzzy neutrosophic supra regular closed set.

(2). The fuzzy neutrosophic supra interior of a fuzzy neutrosophic supra closedset is a fuzzy neutrosophic supra regular open set.

Proof. (1). Let A_{N^*} be a fuzzy neutrosophic supra open set in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}). By the definition [2.10] of fuzzy neutrosophic supra interior,

$int^*\left((A_{N^*})\right) \leq A_{N^*}$

$int^*\left(cl^*(A_{N^*})\right) \leq cl^*(A_{N^*})$

$\Longrightarrow cl^*\left(int^*\left(cl^*(A_{N^*})\right)\right) \leq cl^*\left(cl^*(A_{N^*})\right)$

$\Longrightarrow cl^*\left(int^*\left(cl^*(A_{N^*})\right)\right) \leq cl^*\left((A_{N^*})\right)$ (1)

Since

$A_{N^*} \leq cl^*(A_{N^*})$

$int^*(A_{N^*}) \leq int^*\left(cl^*(A_{N^*})\right)$

$A_{N^*} \leq int^*\left(cl^*(A_{N^*})\right)$

$\Longrightarrow cl^*(A_{N^*}) \leq cl^*\left(int^*\left(cl^*(A_{N^*})\right)\right)$ (2)

From (1) and (2)

$cl^*\left(int^*\left(cl^*(A_{N^*})\right)\right) = cl^*(A_{N^*})$ fuzzy neutrosophic supra closure of a fuzzy neutrosophic supra open set is a fuzzy neutrosophic supra regular closed set.

(2). Let A_{N^*} be a fuzzy neutrosophic supra closed set in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}). By the definition [2.10] of fuzzy neutrosophic supra closure,

$A_{N^*} \leq cl^*(A_{N^*})$

$int^*(A_{N^*}) \leq cl^*\left(int^*(A_{N^*})\right)$

$\Longrightarrow int^*\left(int^*(A_{N^*})\right) \leq int^*\left(cl^*\left(int^*(A_{N^*})\right)\right)$

$\Longrightarrow int^*(A_{N^*}) \leq int^*\left(cl^*\left(int^*(A_{N^*})\right)\right)$ (3)

Since,

$int^*(A_{N^*}) \leq A_{N^*}$

$cl^*\left(int^*(A_{N^*})\right) \leq cl^*(A_{N^*})$

$\Longrightarrow cl^*\left(int^*(A_{N^*})\right) \leq A_{N^*}$

$\Longrightarrow int^*(cl^*(int^*(A_{N^*}))) \leq int^*(A_{N^*})$ (4)

From (3) and (4), $int^*(cl^*(int^*(A_{N^*}))) = int^*(A_{N^*})$ fuzzy neutrosophic supra interior of a fuzzy neutrosophic supra closed set is a fuzzy neutrosophic supra regular open set.

Definition 3.2. Let (X_{N^*}, T_{N^*}) be a fuzzy neutrosophic supra topological space and A_{N^*} be a fuzzy neutrosophic supra set in X_{N^*} , then the fuzzy neutrosophic supra regular closure and fuzzy neutrosophic supra regular interior of A_{N^*} is denoted and defined respectively as

$rcl^*(A_{N^*})$

$= \wedge \{B_{N^*} | B_{N^*} \text{ is a fuzzy neutrosophic supra regular closed set in } X_{N^*} \text{ and } A_{N^*} \leq B_{N^*}\}$,

$rint^*(A_{N^*})$

$= \vee \{B_{N^*} | B_{N^*} \text{ is a fuzzy neutrosophic supra regular open set in } X_{N^*} \text{ and } B_{N^*} \leq A_{N^*}\}$.

Proposition 3.5. For any fuzzy neutrosophic supra set A_N* in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}). Then

(1). $[rint^*(A_{N^*})]^c = rcl^*(A_{N^*{}^c})$

(2). $[rcl^*(A_{N^*})]^c = rint^*(A_{N^*{}^c})$

Proof.

(1). Consider $rint^*(A_{N^*}) = \vee \{B_{N^*} | B_{N^*}$ is a fuzzy neutrosophic supra regular set in $A_{N^*}\}$

$[rint^*(A_{N^*})]^c$

$= 1 - \vee \{B_{N^*} | B_{N^*}$ is a fuzzy neutrosophic supra regular open set in $X_{N^*} \text{ and } B_{N^*} \leq A_{N^*}\}$

$= \wedge \{B_{N^*}{}^c | B_{N^*}{}^c$ is a fuzzy neutrosophic supra regular closed set in $X_{N^*} \text{ and } B_{N^*}{}^c \geq A_{N^*}{}^c\}$

$= rcl^*(A_{N^*}{}^c)$.

(2). $rcl^*(A_{N^*}) = \wedge$

$\{B_{N^*} | B_{N^*}$ is a fuzzy neutrosophic supra regular closed set in $X_{N^*} \text{ and } A_{N^*} \leq B_{N^*}\}$

$[rint^*(A_{N^*})]^c$

$= 1 - \wedge \{B_{N^*} | B_{N^*}$ is a fuzzy neutrosophic supra regular closed set in $X_{N^*} \text{ and } A_{N^*} \leq B_{N^*}\}$

$= \vee \{B_{N^*}{}^c | B_{N^*}{}^c$ is a fuzzy neutrosophic supra regular open set in $X_{N^*} \text{ and } A_{N^*}{}^c \geq B_{N^*}{}^c\}$

$= rint^*(A_{N^*}{}^c)$.

Proposition 3.6. For any two fuzzy neutrosophic supra set A_{N^*} and B_{N^*} in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}) Then if $A_{N^*} \leq B_{N^*}$ implies

(1). $rint^*(A_{N^*}) \leq rint^*(B_{N^*})$

(2). $rcl^*(A_{N^*}) \leq rcl^*(B_{N^*})$

Proof.

(1). $rint^*(A_{N^*}) = \vee$

$\{C_{N^*} | C_{N^*} \text{ is a fuzzy neutrosophic supra regular open set in } X_{N^*} \text{ and } C_{N^*} \leq C_{N^*}\}$

since $rint^*(A_{N^*})$ is the largest fuzy neutrosophic supra regular open set cointained in A_{N^*}

$rint^*(A_{N^*}) \leq A_{N^*}, A_{N^*} \leq B_{N^*}$ implies $rint^*(A_{N^*}) \leq A_{N^*} \leq B_{N^*}$ implies $rint^*(A_{N^*}) \leq B_{N^*}, rint^*(A_{N^*})$ is a fuzzy neutrosophic supra regular open setcontained in B_{N^*}. But $rint^*(B_{N^*})$ is the largest fuzzy neutrosophic supra regular open set contained in $B_{N^*}, rint^*(A_{N^*}) \leq rint^*(B_{N^*})$.

(2). $A_{N^*} \leq B_{N^*}$ if and only if ${A_{N^*}}^c \geq {B_{N^*}}^c$. Consider ${B_{N^*}}^c \leq {A_{N^*}}^c$, $rint^*(B_{N^*})^c \leq rint^*(A_{N^*})^c$, $\left(rcl^*(B_{N^*})\right)^c \leq \left(rcl^*(A_{N^*})\right)^c, rcl^*(A_{N^*}) \leq rcl^*(B_{N^*})$

4. FUZZY NEUTROSOPHIC SUPRA REGULAR NOWHERE DENSE SETS

Definition 4.1. Let (X_{N^*}, T_{N^*}) be a fuzzy neutrosophic supra topological Space. A fuzzy neutrosophic supra set A_{N^*} in (X_{N^*}, T_{N^*}) is called a fuzzy neutrosophic supra regular nowhere dense set if there exists no non-zero fuzzy neutrosophic supra regular open set B_{N^*} in (X_{N^*}, T_{N^*})such that $B_{N^*} < cl^*(A_{N^*})$. That is, $int^*cl^*(A_{N^*}) = 0$, in (X_{N^*}, T_{N^*}).

Example 4.1. Let X_{N^*} = {a, b, c}. The fuzzy neutrosophic supra sets $A_{N^*}, B_{N^*}, C_{N^*}$ and D_{N^*} are defined on X_{N^*} as follows:

$A_{N^*} : X_{N^*} \to [0,1]$ defined as $A_{N^*}(a)$=0.5; $A_{N^*}(b)$=0.7; $A_{N^*}(c)$=0.3

$B_{N^*} : X_{N^*} \to [0,1]$ defined as $B_{N^*}(a)$=0.6; $B_{N^*}(b)$=0.5; $B_{N^*}(c)$=0.8

$C_{N^*} : X_{N^*} \to [0,1]$ defined as $C_{N^*}(a)$=0.4; $C_{N^*}(b)$=0.4; $C_{N^*}(c)$=0.7

$D_{N^*} : X_{N^*} \to [0,1]$ defined as $D_{N^*}(a)$=0.7; $D_{N^*}(b)$=0.3; $D_{N^*}(c)$=0.6

Then $T_{N^*} = \{0, A_{N^*}, B_{N^*}, C_{N^*}, D_{N^*}, (A_{N^*} \vee B_{N^*}), (C_{N^*} \vee D_{N^*}), (A_{N^*} \vee D_{N^*}), (A_{N^*} \vee C_{N^*}), (B_{N^*} \vee D_{N^*}), (A_{N^*} \wedge B_{N^*}), (C_{N^*} \wedge D_{N^*}), (A_{N^*} \wedge D_{N^*}), (A_{N^*} \wedge C_{N^*}), (B_{N^*} \wedge D_{N^*}), 1\}$ is a fuzzy neutrosophic supra topological on X_{N^*}. The non-zero fuzzy neutrosophic supra regular open sets in (X_{N^*}, T_{N^*}) are $A_{N^*}, B_{N^*}, C_{N^*}, D_{N^*}, (A_{N^*} \vee B_{N^*}), (C_{N^*} \vee D_{N^*}), (A_{N^*} \vee D_{N^*}), (A_{N^*} \vee C_{N^*}), (B_{N^*} \vee D_{N^*}), (A_{N^*} \wedge B_{N^*}), (C_{N^*} \wedge D_{N^*}), (A_{N^*} \wedge D_{N^*}), (A_{N^*} \wedge C_{N^*}), (B_{N^*} \wedge D_{N^*})$. Now the fuzzy neutrosophic supra sets $\text{int}^*\text{cl}^*(A_{N^*}) = A_{N^*} \neq 0$ and $int^*cl^*(C_{N^*}) = int^*[1 - (A_{N^*} \vee B_{N^*})] = 0$ in (X_{N^*}, T_{N^*}). Hence A_{N^*} is not a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}) and C_{N^*} in a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*})

Remark 4.1.

1. If A_{N^*} and B_{N^*} are fuzzy neutrosophic supra regular nowhere dense sets in a fuzzy neutrosophic supra topological Space (X_{N^*}, T_{N^*}) then $A_{N^*} \vee B_{N^*}$ need not be a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

2. The complement of a fuzzy neutrosophic supra regular nowhere dense set ina fuzzy neutrosophic supra topological Space need not be a fuzzy neutrosophic supra regular nowhere dense set. For consider the following example.

Example 4.2. Let X_{N^*} = {a, b, c}. The fuzzy neutrosophic supra sets $A_{N^*}, B_{N^*}, and\ C_{N^*}$ are defined on X_{N^*} as follows:

$A_{N^*} : X_{N^*} \to [0,1]$ defined as $A_{N^*}(a)$=0.3; $A_{N^*}(b)$=0.1; $A_{N^*}(c)$=0.2

$B_{N^*} : X_{N^*} \to [0,1]$ defined as $B_{N^*}(a)$=0.7; $B_{N^*}(b)$=0.4; $B_{N^*}(c)$=0.1

$C_{N^*} : X_{N^*} \to [0,1]$ defined as $C_{N^*}(a)$=0.1; $C_{N^*}(b)$=0.2; $C_{N^*}(c)$=0.7

Then $T_{N^*} = \{0, A_{N^*}, B_{N^*}, C_{N^*}, (B_{N^*} \vee C_{N^*}), (B_{N^*} \wedge C_{N^*}), 1\}$ is a fuzzy neutrosophic supra topological on X_{N^*}. On Computation, one can see that fuzzy neutrosophic supra regular nowhere dense sets in (X_{N^*}, T_{N^*}) are $1 - B_{N^*}, 1 - C_{N^*}$ and $1 - (B_{N^*} \vee C_{N^*})$. Now $(1 - B_{N^*}) \vee (1 - C_{N^*}) = 1 - (B_{N^*} \wedge C_{N^*})$ and $int^* cl^*[(1 - B_{N^*}) \vee (1 - C_{N^*})] = int^*(1 - (B_{N^*} \wedge C_{N^*}) = A_{N^*} \neq 0$ and hence $(1 - B_{N^*}) \vee (1 - C_{N^*})$ is not a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}) and $1 - B_{N^*}$ is a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}) where as A_{N^*} is not a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

Definition 4.2. A fuzzy neutrosophic supra set A_{N^*} in a fuzzy neutrosophic supra topological Space $(X_{N^*}, T_{N^*})$is called a fuzzy neutrosophic supra regular dense if there exists no fuzzy neutrosophic supra regular closed set B_{N^*} in (X_{N^*}, T_{N^*}) suchthat $A_{N^*} < B_{N^*} < 1$. That is, $cl^*(A_{N^*}) = 1$, in (X_{N^*}, T_{N^*})

Proposition 4.1. If A_{N^*} is a fuzzy neutrosophic supra regular nowhere dense setin a fuzzy neutrosophic supra topological Space (X_{N^*}, T_{N^*}), then $1 - A_{N^*}$ is a fuzzy neutrosophic supra regular dense set in (X_{N^*}, T_{N^*}).

Proof. Let A_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Then $int^* cl^*(A_{N^*}) = 0$ in (X_{N^*}, T_{N^*}). Now $1 - int^* cl^*(A_{N^*}) = 1 - 0 = 1$ and hence $cl^* int^*(1 - A_{N^*}) = 1$ in (X_{N^*}, T_{N^*}). But $cl^* int^*(1 - A_{N^*}) \leq cl^*(1 - A_{N^*})$ implies that $1 \leq cl^*(1 - A_{N^*})$. That is, $cl^*(1 - A_{N^*}) = 1$ in (X_{N^*}, T_{N^*}). Therefore $1 - A_{N^*}$ is a fuzzy neutrosophic supra dense set in (X_{N^*}, T_{N^*}).

Proposition 4.2. If A_{N^*} is a fuzzy neutrosophic supra regular closed set with $int^*(A_{N^*}) = 0$, in a fuzzy neutrosophic supra topological Space (X_{N^*}, T_{N^*}), then A_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set (X_{N^*}, T_{N^*}).

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular closed set with $int^*(A_{N^*}) = 0$, in (X_{N^*}, T_{N^*}). Then $cl^*(A_{N^*}) = A_{N^*}$ and $int^*(A_{N^*}) = 0$ and hence $int^* cl^*(A_{N^*}) = int^*(A_{N^*}) = 0$ in (X_{N^*}, T_{N^*}),. Therefore A_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set (X_{N^*}, T_{N^*}).

Proposition 4.3. If A_{N^*} is a non-zero fuzzy neutrosophic supra set in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then $A_{N^*} \vee int^* cl^* int^* A_{N^*} \leq cl^* A_{N^*}$, in (X_{N^*}, T_{N^*}).

Proof. Since $cl^*(A_{N^*})$ is a fuzzy neutrosophic supra regular closed set in (X_{N^*}, T_{N^*}), $int^*cl^*int^*(cl^*(A_{N^*})) \leq cl^*(A_{N^*})$. Then $int^*cl^*int^*(A_{N^*}) \leq int^*cl^*int^*\big(cl^*(A_{N^*})\big)$, $A_{N^*} < cl^*(A_{N^*})$ and hence $A_{N^*} \vee int^*cl^*int^*(A_{N^*}) \leq cl^*(A_{N^*})$ in (X_{N^*}, T_{N^*}).

Proposition 4.4. If A_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set ina fuzzy neutrosophic supra regular topological space (X_{N^*}, T_{N^*}), then $int^*(A_{N^*}) = 0$ in (X_{N^*}, T_{N^*}).

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Then $int^*cl^*(A_{N^*})$ in (X_{N^*}, T_{N^*}). Now $int^*(A_{N^*}) \leq int^*cl^*(A_{N^*})$, implies that $int^*(A_{N^*}) \leq 0$, in (X_{N^*}, T_{N^*}). That is $int^*(A_{N^*}) = 0$ in (X_{N^*}, T_{N^*}).

Proposition 4.5. If $A_{N^*} \leq B_{N^*}$ and B_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set in fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then A_{N^*} is also a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

Proof. Now $A_{N^*} \leq B_{N^*}$, implies that $cl^*(A_{N^*}) \leq int^*cl^*(B_{N^*})$ in (X_{N^*}, T_{N^*}). Since $B_{N^*}$is a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). That is $int^*cl^*(A_{N^*}) = 0$. Then $int^*cl^*(A_{N^*}) \leq 0$ in (X_{N^*}, T_{N^*}). That is $int^*cl^*(A_{N^*}) = 0$ in (X_{N^*}, T_{N^*}). Therefore A_{N^*} is a fuzzy neutrosophic supra regular nowhere denseset in (X_{N^*}, T_{N^*}).

Proposition 4.6. If either A_{N^*} or B_{N^*} is a fuzzy neutrosophic supra regular nowhere dense set in fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then $A_{N^*} \wedge B_{N^*}$ is a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

Proof. $int^*cl^*(A_{N^*} \wedge B_{N^*}) \leq int^*[cl^*(A_{N^*}) \wedge cl^*(B_{N^*})] = int^*cl^*(A_{N^*}) \wedge int^*cl^*(B_{N^*})$ in (X_{N^*}, T_{N^*}). If either A_{N^*} or B_{N^*} is a fuzzy neutrosphic supra regular nowhere dense set in (X_{N^*}, T_{N^*}), then either $int^*cl^*(A_{N^*}) = 0$ or $int^*cl^*(B_{N^*}) = 0$. Then either $int^*cl^*(A_{N^*} \wedge B_{N^*}) \leq 0 \wedge int^*cl^*(B_{N^*}) = 0$ (or) $int^*cl^*(A_{N^*} \wedge B_{N^*}) \leq int^*cl^*(A_{N^*}) \wedge 0 = 0$. Therefore, $A_{N^*} \wedge B_{N^*}$ is a fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

Proposition 4.7. If A_{N^*} is a fuzzy neutrosophic supra regular dense set in fuzzy neutrosophic supra topological space(X_{N^*}, T_{N^*}), than A_{N^*} is a fuzzy neutrosophic supra regular open set in(X_{N^*}, T_{N^*}).

Proof. Let A_{N^*} be a fuzzy neutrosophic supra regular dense set in (X_{N^*}, T_{N^*}). Then $cl^*(A_{N^*}) = 1$, in (X_{N^*}, T_{N^*}). Since $cl^*(A_{N^*}) \leq cl^*(A_{N^*}), 1 \leq cl^*(A_{N^*})$. That is $cl^*(A_{N^*}) = 1$, in (X_{N^*}, T_{N^*}). Now $cl^*int^*cl^*(A_{N^*}) = cl^*int^*(A_{N^*}) = cl^*(A_{N^*}) = 1$ and hence $A_{N^*} \leq cl^*int^*cl^*(A_{N^*})$ in (X_{N^*}, T_{N^*}). Therefore A_{N^*} is a fuzzy neutrosophic supra regular open set in (X_{N^*}, T_{N^*}).

Definition 4.3. A fuzzy neutrosophic supra set A_{N^*} in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}). is called a fuzzy neutrosophic supra regular first category set if $A_{N^*} = \bigvee_{i=1}^{\infty}(A_{N_i^*})$ where $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Any other fuzzy neutrosophic supra set in (X_{N^*}, T_{N^*}). Is said to be fuzzy neutrosophic supra regular second category.

Definition 4.4. Let A_{N^*} be a fuzzy neutrosophic supra regular first category set in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}). Then $1 - A_{N^*}$ is called a fuzzy neutrosophic supra regular residual set in (X_{N^*}, T_{N^*}).

5. FUZZY NEUTROSOPHIC SUPRA REGULAR BAIRE SPACES

Definition 5.1. Let (X_{N^*}, T_{N^*}) be a fuzzy neutrosophic supra topological space. Then (X_{N^*}, T_{N^*}) is called a fuzzy neutrosophic supra regular Baire space $int^*\left(\bigvee_{i=1}^{\infty}(A_{N_i^*})\right) = 0$ where $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}).

Example 5.1. Let X_{N^*} = {a, b, c}. The fuzzy neutrosophic supra sets $A_{N^*}, B_{N^*}, C_{N^*}, E_{N^*}, F_{N^*}, D_{N^*}$ *and* G_{N^*} are defined on X_{N^*} as follows:

$A_{N^*} : X_{N^*} \to [0,1]$ defined as $A_{N^*}(a)$=0.5; $A_{N^*}(b)$=0.4; $A_{N^*}(c)$=06

$B_{N^*} : X_{N^*} \to [0,1]$ defined as $B_{N^*}(a)$=0.4; $B_{N^*}(b)$=0.6; $B_{N^*}(c)$=0.3

$C_{N^*} : X_{N^*} \to [0,1]$ defined as $C_{N^*}(a)$=0.6; $C_{N^*}(b)$=0.5; $C_{N^*}(c)$=0.7

$D_{N^*} : X_{N^*} \to [0,1]$ defined as $D_{N^*}(a)$=0.5; $D_{N^*}(b)$=0.6; $D_{N^*}(c)$=0.6

$E_{N^*} : X_{N^*} \to [0,1]$ defined as $E_{N^*}(a)$=0.3; $E_{N^*}(b)$=0.1; $E_{N^*}(c)$=0.7

$F_{N^*} : X_{N^*} \to [0,1]$ defined as $F_{N^*}(a)$=0.6; $F_{N^*}(b)$=0.6; $F_{N^*}(c)$=0.7

$G_{N^*} : X_{N^*} \to [0,1]$ defined as $G_{N^*}(a)$=0.5; $G_{N^*}(b)$=0.5; $G_{N^*}(c)$=0.6

Then $T_{N^*} = \{0, A_{N^*}, B_{N^*}, C_{N^*}, E_{N^*}, F_{N^*}, D_{N^*}, G_{N^*}, 1\}$ is a fuzzy neutrosophic supra topological on X_{N^*}. On computations, one can see that the non-zero fuzzy neutrosophic supra regular open set in (X_{N^*}, T_{N^*}) are $A_{N^*}, C_{N^*}, 1 - B_{N^*}, E_{N^*}, 1 - E_{N^*}, D_{N^*}, G_{N^*}, 1$. Now the fuzzy neutrosophic supra regular nowhere dense set in(X_{N^*}, T_{N^*}) are $1 - D_{N^*}, 1 - G_{N^*}, 1 - F_{N^*}, B_{N^*}, 1 - A_{N^*}$. On computations, $int^*[(1 - D_{N^*}) \vee (1 - G_{N^*}) \vee (1 - F_{N^*}) \vee (B_{N^*}) \vee (1 - A_{N^*})] = int^*(1 - A_{N^*}) = 0$. Hence the fuzzy neutrosophic supra topological Space (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire Space.

Proposition 5.1. If $int^*\left(\bigvee_{i=1}^{\infty}(A_{N_i^*})\right) = 0$, where $(A_{N_i^*}) = 0$ and $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular closed sets in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire space.

Proof. Let $(A_{N_i^*})$'s fuzzy neutrosophic supra regular closed sets in (X_{N^*}, T_{N^*}). Since $int^*(A_{N_i^*}) = 0$, by preposition 4.2, the $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Therefore $int^*\left(\bigvee_{i=1}^{\infty}(A_{N_i^*})\right) = 0$, where$(A_{N_i^*})$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}), implies that (X_{N^*}, T_{N^*}), is a fuzzy neutrosophic supra regular Baire Space.

Proposition 5.2. If $cl^*\left(\wedge_{i=1}^{\infty}(A_{N_i^*})\right) = 1$, where $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular dense and fuzzy neutrosophic supra regular open sets in a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}), then (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire Space.

Proof. Now, $cl^*\left(\wedge_{i=1}^{\infty}(A_{N_i^*})\right) = 1$, implies $1 - cl^*\left(\wedge_{i=1}^{\infty}(A_{N_i^*})\right) = 0$. Then $int^*\left[1 - \wedge_{i=1}^{\infty}(A_{N_i^*})\right] = 0$ in (X_{N^*}, T_{N^*}). This implies that $int^*\left[\vee_{i=1}^{\infty}(1 - A_{N_i^*})\right] = 0$. since $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular dense in (X_{N^*}, T_{N^*}), $cl^*(A_{N_i^*}) = 1$ and $int^*\left((1 - A_{N_i^*}\right) = 1 - cl^*(A_{N_i^*}) = 1 - 1 = 0$. Hence $int^*\left(\vee_{i=1}^{\infty}(1 - A_{N_i^*})\right) = 0$, where $int^*\left((1 - A_{N_i^*}\right) = 0$ and $(1 - A_{N_i^*})$'s are fuzzy neutrosophic supra regular closed sets in (X_{N^*}, T_{N^*}). Then by proposition 5.1, the fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire Space.

Proposition 5.3. Let (X_{N^*}, T_{N^*}) fuzzy neutrosophic supra topological space. Then the following are equivalent.

(1). (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire space.

(2). $int^*(A_{N^*}) = 0$ for every fuzzy neutrosophic supra regular first category set A_{N^*} in (X_{N^*}, T_{N^*}).

(3). $cl^*(B_{N^*}) = 1$ for every fuzzy neutrosophic supra regular residual set B_{N^*} in (X_{N^*}, T_{N^*}).

Proof. (1) ⇒ (2) Let A_{N^*} be a fuzzy neutrosophic supra regular first category set in (X_{N^*}, T_{N^*}). Then $A_{N^*} = \vee_{i=1}^{\infty}(A_{N_i^*})$ where $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Now $int^*(A_{N^*}) = int^*\left(\vee_{i=1}^{\infty}(A_{N_i^*})\right)$. (Since$(X_{N^*}, T_{N^*})$. is a fuzzy neutrosophic supra regular Baire space). Therefore $int^*(A_{N^*}) = 0$, in(X_{N^*}, T_{N^*}).

(2) ⇒ (3) Let B_{N^*} be a fuzzy neutrosophic supra regular residual set in (X_{N^*}, T_{N^*}). Then, $1 - B_{N^*}$ is a fuzzy neutrosophic supra regular first category set in(X_{N^*}, T_{N^*}). By hypothesis, $int^*(1 - B_{N^*}) = 0$, in (X_{N^*}, T_{N^*}). This implies that $1 - cl^*(B_{N^*}) = 0$ and hence $cl^*(B_{N^*}) = 1$, in (X_{N^*}, T_{N^*}).

(3) ⇒ (1) Let A_{N^*} be a fuzzy neutrosophic supra regular first category set in(X_{N^*}, T_{N^*}). Then $A_{N^*} = \vee_{i=1}^{\infty}(A_{N_i^*})$, where $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Since A_{N^*} is a fuzzy neutrosophic supra regular first category set in (X_{N^*}, T_{N^*}), $1 - A_{N^*}$ is a fuzzy neutrosophic supra regular residual set in (X_{N^*}, T_{N^*}). By hypothesis, $cl^*(1 - A_{N^*}) = 1$. Then $1 - int^*(A_{N^*}) = 1$, in (X_{N^*}, T_{N^*}). This implies that $int^*(A_{N^*}) = 0$, in (X_{N^*}, T_{N^*}). Hence $int^*\left(\vee_{i=1}^{\infty}(A_{N_i^*})\right) = 0$, where $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular nowhere dense in (X_{N^*}, T_{N^*}), implies that (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire space.

Definition 5.2. A fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}) is said to be of fuzzy neutrosophic supra regular first category space if $\vee_{i=1}^{\infty}(A_{N_i^*}) = 1_X$, where $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Otherwise (X_{N^*}, T_{N^*})will be called a fuzzy neutrosophic supra regular second category space.

Proposition 5.4. If a fuzzy neutrosophic supra topological space (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire space, then $(X_{N^*}, T_{N^*})$is a fuzzy neutrosophic supra regular second category space.

Proof. Let (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular Baire space. Then, $int^*\left(\bigvee_{i=1}^{\infty}(A_{N_i^*})\right) = 0$ where $(A_{N_i^*})$'s are fuzzy neutrosophic supra regular nowhere dense set in (X_{N^*}, T_{N^*}). Then $\bigvee_{i=1}^{\infty}(A_{N_i^*}) \neq 1_X$ [otherwise $\bigvee_{i=1}^{\infty}(A_{N_i^*}) = 1_X$, implies that where, $int^*\left(\bigvee_{i=1}^{\infty}(A_{N_i^*})\right) =, int^*(1_X) = 1_X \neq 0$, a contradiction]. Hence (X_{N^*}, T_{N^*}) is a fuzzy neutrosophic supra regular second category space.

6. CONCLUSION

In this paper, we introduced and studied a new concept fuzzy neutrosophic supra Regular open set, fuzzy neutrosophic supra Regular closed set, fuzzy neutrosophic supra Regular nowhere dense set, fuzzy neutrosophic supra Regular dense set, fuzzy neutrosophic supra Regular Baire space, in fuzzy neutrosophic supra topological space. Some characteristics of fuzzy neutrosophic supra Regular Baire spaces are established

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SUPRA RESIDUAL SETS AND SUPRA BAIRE SPACE FUNCTIONS ON FUZZY NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT

In this paper several characterization of fuzzy neutrosophic supra residual sets are established and the criteria of the fuzzy neutrosophic supra residual of fuzzy neutrosophic supra sets on fuzzy neutrosophic supra topological space.

Keywords: Fuzzy neutrosophic supra residual set, Fuzzy supra Semi F_σ −set, Fuzzy supra Semi G_δ − set, Fuzzy neutrosophic supra nowhere dense set, Fuzzy neutrosophic supra first and second category, Fuzzy neutrosophic supra Baire spaces.

1. INTRODUCTION

The notion of a fuzzy set introduced by L.A. Zadeh(14)in 1965, both pure and applied Mathematicians. Since then, the notion of fuzziness has been applied for the study in all branches of Mathematics. The concept of fuzzy topological spaces , introduced by Chang in 1968(5) has been extensively studied and applied in many field , including artificial intelligence, decision making, and images processing .

Supra Baire spaces, introduced by Haworth and McCoy in 1977 [6], are an important concept in topology. In 1983, Mashhour et al. [7] extended the notion of supra topological spaces. Abd El-Monsef and Ramadan in 1987 [1] investigated the concept of fuzzy supra topological spaces, which has since been further studied by researchers such as Ahmed, Chandra Chetia, and others [2].

The concept of supra Baire spaces in fuzzy setting was introduced and studied by E. Poongothai and G. Thangaraj in (8). Also introduced and studied fuzzy supra semi Baire spaces by (9).

Building upon previous research, Arockiarani l and Martina Jency [4] further studied fuzzy neutrosophic sets and fuzzy neutrosophic topological spaces.

In this paper, we explain the concept of supra residual sets on fuzzy neutrosophic topological space. We investigate several fundamental properties and characterization of these sets.

2. PRELIMINARIES

Definition 2.1 [3]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle\ x \in X$ where $T, I, F: X \rightarrow [0,1]$ and $0 \leq \{T_A(x) + I_A(x) + F_A(x)\} \leq 3$.

Definition 2.2 [3]:

A fuzzy neutrosophic set A is a subset of a fuzzy neutrosophic set B (i.e.,) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$

Definition 2.3 [3]:

Let X be a non-empty set, and $A = T_A(x), I_A(x), F_A(x),\ B = T_B(x), I_B(x), F_B(x)$ be two fuzzy neutrosophic sets. Then

$$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x))\rangle$$

$$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x))\rangle$$

Definition 2.4 [3]:

The difference between two fuzzy neutrosophic sets A and B is defined as $A\backslash B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1\text{-}I_B(x)), \max(F_A(x), T_B(x))\rangle$

Definition 2.5 [3]:

A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T(x) = 0, I(x) = 0, F_A(x) = 1$ for all $x \in X$ It is denoted by 0_N .

Definition 2.6 [3]:

A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T(x) = 1, I(x) = 1, F_A(x) = 0$ for all $x \in X$ It is denoted by 0_N.

Definition 2.7 [3]:

The complement of a fuzzy neutrosophic set A is denoted by A^C and is defined as

$A^C = \langle T_{A^C}(x), I_{A^C}(x), F_{A^C}(x)\rangle$

where $T_{A^C}(x) = F_A(x), I_{A^C}(x) = 1 - I_A(x), F_{A^C}(x) = T_A(x)$

The complement of fuzzy neutrosophic set A can also be defined as $A^C = 1_N\text{-}A$

Definition 2.8 [12]:

Let $(X_N{}^*, T_N{}^*)$ be a fuzzy neutrosophic supra topological space. Then $(X_N{}^*, T_N{}^*)$ is called fuzzy neutrosophic Baire Space if $fn\left(int^*\left(\bigvee_{i=1}^{\infty}(X_{N_i}{}^*)\right)\right) = 0$ where $(X_{N_i}{}^*)$'s are fuzzy neutrosophic supra nowhere dense set in $(X_N{}^*, T_N{}^*)$.

Definition 2.9 [4]:

The complement A^C of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Theorem 2.10 [12]:

Let (X_N, T^*) be a fuzzy neutrosophic supra topological space. Then the following are equivalent:

(1) (X_N, T^*) is a fuzzy neutrosophic supra Baire Space

(2) $fn\ int^*(A_N) = 0$ for every fuzzy neutrosophic supra first category set (A_N) in (X_N, T^*)

(3) $fn\ cl^*(B) = 1$ for every fuzzy neutrosophic supra residual set (B_N) in (X_N, T^*).

3. FUZZY NEUTROSOPHIC SUPRA RESIDUAL SET

Proposition 3.1

Let (X_N, T^*) be a fuzzy neutrosophic supra topological space. A fuzzy set A_N in (X_N, T^*) is called a fuzzy neutrosophic supra residual set if $A_N = \wedge_{i=1}^{\infty} A_{N_i}$, where the fuzzy sets (A_{N_i}),s are such that $fn\ cl^*\left[fn\ int^*\left(A_{N_i}\right)\right] = 1_N$ in (X_N, T^*)

Proposition 3.2

If A_N is a fuzzy neutrosophic supra G_δ-set in a fuzzy neutrosophic supra topological space (X_N, T^*) such that $fn\ cl^*(A_N) = 1$, then A_N is a fuzzy neutrosophic supra residual set in (X_N, T^*).

Proof.

Let A_N be a fuzzy neutrosophic supra G_δ-set in (X_N, T^*). Then $A_N = \wedge_{i=1}^{\infty}(A_{N_i})$, where $A_{N_i} \in T^*$. Now $fn\ cl^*(A_N) = fn\ cl^*\left[\wedge_{i=1}^{\infty}(A_{N_i})\right]$ in (X_N, T^*). But $fn\ cl^*\left[\wedge_{i=1}^{\infty}(A_{N_i})\right] \leq \wedge_{i=1}^{\infty} fn\ cl^*(A_{N_i})$ in $(X_N T^*)$. Then $fn\ cl^*(A_N) \leq \wedge_{i=1}^{\infty} fn\ cl^*(A_{N_i})$ in (X_N, T^*). Since A_N is a fuzzy neutrosophic supra dense set in (X_N, T^*), $fn\ cl^*(A_N) = 1$ and hence $1 \leq fn\ cl^*\left[\wedge_{i=1}^{\infty}(A_{N_i})\right]$. That is, $fn\ cl^*\left[\wedge_{i=1}^{\infty}(A_{N_i})\right] = 1$ in (X_N, T^*). This implies that $fn\ cl^*\left(A_{N_i}\right) = 1$ in (X_N, T^*) and hence $fn\ cl^*\left[fn\ int^*\left(A_{N_i}\right)\right] = fn\ cl^*\left(A_{N_i}\right) = 1$ in (X_N, T^*). Thus $A = \wedge_{i=1}^{\infty}(A_{N_i})$, where $fn\ cl^*\left[fn\ int^*\left(A_{N_i}\right)\right] = 1$, implies that A_N is a fuzzy neutrosophic supra residual set in (X_N, T^*).

Definition 3.3

Let (X_N, T^*) fuzzy neutrosophic supra topological space. A fuzzy set A_N is called a fuzzy neutrosophic supra semi-open set if $A_N \leq fn\ cl^*[fn\ int^*(A_N)]$. The complement of a fuzzy neutrosophic supra semi-open set is called a fuzzy neutrosophic supra semi-closed set in (X_N, T^*).

Definition 3.4

A fuzzy set A in a fuzzy neutrosophic supra topological space (X_N, T^*), is called a fuzzy neutrosophic supra semi G_δ-set in (X_N, T^*) if $A = \wedge_{i=1}^{\infty}(A_{N_i})$, where (A_{N_i})'s are fuzzy neutrosophic supra semi-closed sets in (X_N, T^*).

Definition 3.5

A fuzzy set A in a fuzzy neutrosophic supra topological space (X_N, T^*) is called a fuzzy neutrosophic supra semi-F_σ-set in (X_N, T^*) if $A = \vee_{i=1}^{\infty}(A_i)$ where (A_i)'s are fuzzy neutrosophic supra semi-closed sets in (X_N, T^*).

Proposition 3.6

If A_N is a fuzzy neutrosophic supra residual set in a fuzzy neutrosophic supra topological space (X_N, T^*), then A_N is a fuzzy neutrosophic supra semi-.G_δ set.

Proof.

Let A_N be a fuzzy neutrosophic supra residual set in a fuzzy neutrosophic supra topological space(X_N, T^*). Then, $A_N = \wedge_{i=1}^{\infty}(A_{N_i})$, where fuzzy sets $(A_{N_i})'$s are such that $fn\ cl^*\left[fn\ int^*\left(A_{N_i}\right) = 1_N\right]$ in (X_N, T^*). Now $fn\ cl^*\left[fn\ int^*\left(A_{N_i}\right)\right] = 1_N$ in (X_N, T^*), implies that $A_{N_i} \leq fn\ cl^*\left[fn\ int^*\left(A_{N_i}\right)\right]$. Then $(A_{N_i})'$s are fuzzy neutrosophic supra semi-open sets in $(X_N T^*)$ and thus $\wedge_{i=1}^{\infty}(A_{N_i})$ is a fuzzy neutrosophic supra semi G_δ-set in (X_N, T^*).

Proposition 3.7

If A_N is a fuzzy neutrosophic supra first category set in a fuzzy neutrosophic supra topological space (X_N, T^*), then A_N is a fuzzy neutrosophic supra semi-F_σ set in (X_N, T^*).

Proof.

Let If A_N is a fuzzy neutrosophic supra first category set in (X_N, T^*). Then $1 - A_N$ is a fuzzy neutrosophic supra residual set in (X_N, T^*) and hence, by property (3.6), $1 - A_N$ is a fuzzy neutrosophic supra semi-G_δ set in (X_N, T^*). Therefore A_N is a fuzzy neutrosophic supra semi-F_σ set in (X_N, T^*).

Proposition 3.8

If $fn\ cl^*\ (B_N)$ is a fuzzy neutrosophic supra first category set in a fuzzy neutrosophic supra Baire Space (X_N, T^*), then A is a fuzzy neutrosophic supra nowhere dense set in (X_N, T^*).

Proof.

If $fn\ cl^*\ (B_N)$ is a fuzzy neutrosophic supra first category set in (X_N, T^*). Since (X_N, T^*) is a fuzzy neutrosophic supra Baire Space, by theorem (2.10) $fn\ int^*[fncl^*(A)] = 0$ in (X_N, T^*). Therefore A is an fuzzy neutrosophic supra nowhere dense set in (X_N, T^*).

Proposition 3.9

If $fn\ int^*(B_N)$ is A fuzzy neutrosophic supra residual set in a fuzzy neutrosophic supra Baire Space (X_N, T^*), then $1 - B$ is a fuzzy neutrosophic supra nowhere dense set in (X_N, T^*).

Proof.

Let $fn\ int^*(B_N)$ be a fuzzy neutrosophic supra residual set in (X_N, T^*). Since (X_N, T^*) is a fuzzy neutrosophic supra Baire Space, by theorem (2.10), $fn\ cl^*[fn\ int^*(B_N)] = 1$ in (X_N, T^*). This implies that $1 - fn\ cl^*[fn\ int^*(B_N)] = 0$ and hence $fn\ int^*[fn\ cl^*(1 - B_N)] = 0$ in (X_N, T^*). This implies that $1 - B_N$ is a fuzzy neutrosophic supra nowhere dense set in (X_N, T^*).

Proposition 3.10

If $fn\left[int^*(B_{N_i})\right]$'s $(i = 1\ to\ \infty)$ are fuzzy neutrosophic supra residual sets in a fuzzy neutrosophic supra Baire Space (X_N, T^*), then $fn\ cl^*\left[\wedge_{i=1}^{\infty}(B_{N_i})\right] = 1$ in (X_N, T^*).

Proof.

Let $fn[int^*(B_N)]$'s $(i = 1\ to\ \infty)$ be fuzzy neutrosophic supra residual sets in (X_N, T^*). Since (X_N, T^*) is fuzzy neutrosophic supra Baire Space by proposition (3.9), $1 - B_{N_i}$'s are fuzzy neutrosophic supra nowhere dense sets in (X_N, T^*) and hence $fn\ int^*\left[\bigvee_{i=1}^{\infty}(1 - B_{N_i})\right] = 0$ in (X_N, T^*). This implies that $fn\ int^*\left[1 - \bigvee_{i=1}^{\infty}(B_{N_i})\right] = 0$. Then $1 - fn\ cl^*\left[\bigvee_{i=1}^{\infty}(B_{N_i})\right] = 0$ in (X_N, T^*) and thus $fn\ cl^*\left[\bigvee_{i=1}^{\infty}(B_{N_i})\right] = 1$ in (X_N, T^*).

4. FUZZY NEUTROSOPHIC SUPRA BAIRE SPACE FUNCTIONS

Definition 4.1

Let (X_N, T^*) be a fuzzy neutrosophic supra topological space. Then (X_N, T^*) is called fuzzy neutrosophic supra Baire Space if $fn\ int^*\left[\bigvee_{i=1}^{\infty}(A_{N_i})\right] = 0$, where A_{N_i}'s are fuzzy neutrosophic supra nowhere dense sets is (X_N, T^*)

Definition 4.2

A function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is said to be fuzzy neutrosophic supra open if the image of every fuzzy neutrosophic supra open set in $(X_N, T^*)$is fuzzy neutrosophic supra open in (Y_N, S^*).

Definition 4.3

A function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is called a fuzzy continuous if $f^{-1}(A_N)$ fuzzy neutrosophic supra open in (X_N, T^*) for each fuzzy neutrosophic supra open set is A_N in (Y_N, S^*).

Definition 4.4

A function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is called a fuzzy neutrosophic supra semi-continuous if ${f^{-1}}^*(A_N)$ is fuzzy neutrosophic supra semi-open in (X_N, T^*) for each fuzzy neutrosophic supra open set is A_N in (Y_N, S^*).

Definition 4.5

A function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is called a fuzzy neutrosophic supra somewhat continuous if $A_N \in S^*$ and $f^{-1}A_N \neq 0$ implies that there exist a fuzzy neutrosophic open set B_N in (X_N, T^*). Such that $B_N \neq 0$ and $B_N \leq {f^{-1}}^*(A_N)$.

Definition 4.6

A function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is called a fuzzy neutrosophic supra somewhat open

if $A_N \in T^*$ and $B_N \neq 0$ implies than there exist a fuzzy neutrosophic supra open set B_N in (Y_N, S^*). Such that $B_N \neq 0$ and $B_N \leq f(A_N)$.

Theorem 4.7

Let (X_N, T^*) be a fuzzy neutrosophic supra topological space. Then the following are equivalent.

1) (X_N, T^*) is a fuzzy neutrosophic supra Baire Space,
2) $fn\big(int^*(A_N)\big) = 0$ for every fuzzy neutrosophic supra first category set in A_N in (X_N, T^*).
3) $fn\big(cl^*(B_N)\big) = 1$, for every fuzzy neutrosophic supra residual set B_N in (X_N, T^*).

Proof.

Let f^* be a function from the fuzzy neutrosophic supra topological space (X_N, T^*) to the (Y_N, S^*), Underwhat conditions on "f^*" may we assert that if (X_N, T^*) is a fuzzy neutrosophic supra Baire Space, then (Y_N, S^*) is a fuzzy neutrosophic supra Baire Space. If may be notised that the fuzzy neutrosophic supra continuous image of a fuzzy neutrosophic supra Baire Space may fail to be a fuzzy neutrosophic supra Baire Space. For, consider the following example.

Example 4.8

Let $X = \{a, b\}$. The fuzzy neutrosophic sets A_N, B_N and C_N are defined on X as follows

$$A_N = \left(X, \left(\frac{a}{0.7}, \frac{b}{0.7}\right), \left(\frac{a}{0.6}, \frac{b}{0.7}\right), \left(\frac{a}{0.4}, \frac{b}{0.3}\right)\right)$$

$$B_N = \left(X, \left(\frac{a}{0.6}, \frac{b}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right)\right)$$

$$C_N = \left(X, \left(\frac{a}{0.6}, \frac{b}{0.7}\right), \left(\frac{a}{0.7}, \frac{b}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}\right)\right)$$

Then,

$T^* = \{0, A_N, B_N, C_N, A_N \vee B_N, B_N \vee C_N, A_N \wedge B_N, B_N \wedge C_N, 1\}$ and $S^* = \{0, A_N, B_N, A_N \vee B_N, A_N \wedge B_N, 1\}$ are fuzzy neutrosophic topologies on X. Now the fuzzy neutrosophic supra nowhere dense sets in (X_N, T^*) are $1 - A_N$ are $1 - (A_N \vee B_N)$ and $\big[(1 - A_N) \vee \big(1 - (A_N \vee B_N)\big)\big] = 1 - A_N$ and $fn\, int^*(1 - A_N) = 0$. Hence (X_N, T^*) is a fuzzy neutrosophic supra Baire Space. Then fuzzy neutrosophic supra nowhere dense sets is (X_N, S^*) are $(1 - A_N), (1 - B_N)$ and $1 - (A_N \vee B_N)$ this implies that $\big[(1 - A_N) \vee (1 - B_N) \vee \big(1 - (A_N \vee B_N)\big)\big] = 1 - (A_N \wedge B_N)$ and $fn\, int^*[1 - (A_N \vee B_N)] = A_N \wedge B_N \neq 0$. Hence (X_N, S^*) is not a fuzzy neutrosophic supra Baire Space.

Define a function $f^*: (X_N, T^*) \longrightarrow (X_N, S^*)$ by $f^*(a) = a$ and $f^*(b) = b$. Clearly f^* is a fuzzy neutrosophic supra continuous function from the fuzzy neutrosophic supra Baire Space. (X_N, T^*) to the fuzzy neutrosophic supra topological space. (X_N, S^*) which is not a fuzzy neutrosophic supra Baire Space.

Example 4.8

Let $X = \{a, b, c\}$. Then fuzzy sets A_N, B_N and C_N are defined on X as follows:

$$A_N = \left(X, \left(\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.4}\right)\right)$$

$$B_N = \left(X, \left(\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.5}\right), \left(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.6}\right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right)$$

$$C_N = \left(X, \left(\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.7}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.6}\right)\right)$$

Then,

$T^* = \{0, A_N, 1\}$ and $S^* = \{0, A_N, B_N, A_N \vee B_N, B_N \vee C_N, A_N \wedge B_N, B_N \wedge C_N, 1\}$

Are fuzzy neutrosophic topological on X. Now the fuzzy neutrosophic supra nowhere dense sets in (X_N, T^*) are $1 - A_N, 1 - B_N$ and $1 - (A_N \vee B_N) = 1 - A_N$ and $fn\ int^*(1 - A_N) = 0$. Hence (X_N, T^*) is a fuzzy neutrosophic supra Baire Space.

The fuzzy neutrosophic supra nowhere dense sets in (X_N, S^*) are $1 - A_N, 1 - B_N, 1 - C_N$, $1 - (A_N \vee B_N)$ and $1 - (B_N \vee C_N)$ and

$(1 - A_N) \vee (1 - B_N) \vee (1 - C_N) \vee \big(1 - (A_N \vee B_N)\big) \vee \big(1 - (B_N \vee C_N)\big) = 1 - A_N \wedge B_N$.

Now, $fn\ int^*\big(1 - (A_N \wedge B_N)\big) = B_N \wedge C_N \neq 0$. Hence (X_N, T^*) is not a fuzzy neutrosophic supra Baire Space.

Define a function $f^*: (X_N, T^*) \longrightarrow (X_N, S^*)$ by $f^*(a) = a$ and $f^*(b) = b$ and $f^*(c) = c$ clearly f^* is a fuzzy neutrosophic supra open function from the fuzzy neutrosophic supra Baire space (X_N, T^*) to the fuzzy neutrosophic supra topological space (X_N, S^*) which is not a fuzzy neutrosophic supra Baire Space.

Proposition 4.1

If a function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is fuzzy neutrosophic supra continuous, 1-1, onto and fuzzy neutrosophic supra open function, then for any fuzzy set A_N in (X_N, T^*). $fn\ int^* cl^*\ [f^*(A_N)] \leq f^*\big[int^*\big(cl\ (A_N)\big)\big]$.

Proof.

Let A_N be a fuzzy set in (X_N, T^*). Then $A_N \leq fn\ cl^*(A_N)$ in (X_N, T^*) implies that $1 - fn\ cl^*(A_N) \leq (1 - A_N)$. Then $f^*(1 - fn\ cl^*(A_N)) \leq f^*(1 - A_N)$.

Since f is a fuzzy neutrosophic supra open function and $1 - fn\ cl^*(A_N)$ is a fuzzy neutrosophic supra open set is (Y_N, S^*). Such that $f^*(1 - fn\ cl^*(A_N)) \leq f^*(1 - A_N)$. But $fn\ int\ f^*(1 - A_N)$ in the largest fuzzy neutrosophic supra open in (Y_N, S^*). Such that $fn\ int^*[f^*(1 - A_N)] \leq f^*(1 - A_N)$.

Hence we have that $f^*\big(1 - fn\ cl^*(A_N)\big) \leq fn\ int^* f^*(1 - A_N)$.

Since f^*in 1-1, and onto, $f^*(1 - A_N) = 1 - f^*(A_N)$. Hence $f^*\big(fn\ cl^*(A_N)\big) \leq fn\ int^*(1 - f^*(A_N))$ which implies that $1 - f^*\big(fn\ cl^*(A_N)\big) \leq \big(1 - fnint^*\big(f^*(A_N)\big)\big)$. Then $fn\ cl^*[f^*(A_N)] \leq f^*[fncl^*(A_N)]$, which implies that $\big(fn\ int^* cl^*\big(f^*(A_N)\big)\big) \leq \big(fn\ int^*(f^*[fn\ cl^*(A_N)])\big)$.

Therefore we have $f^{-1^*}[fn\ int^* cl^*\big(f^*(A_N)\big)] \leq f^{-1^*}[fn\ int^* f^*(fn\ cl^*(A_N))]$. (1)

Now $fn\ int^*(f^*[fn\ cl^*(A_N)])$ is an fuzzy neutrosophic supra open set in (Y_N, S^*). Since f^* is fuzzy neutrosophic supra continuous. $f^{-1^*}\left[fn\ int\left(f^*\big(fn\ cl^*(A_N)\big)\right)\right]$ is an fuzzy neutrosophic supra open set in (X_N, T^*) . Hence $f^{-1^*}\left[fn\ int\left(f^*\big(fncl^*(A_N)\big)\right)\right] = int\ f^{-1^*}\left[fn\ int\left(f^*\big(fncl^*(A_N)\big)\right)\right] \leq fn\ int^*\big(cl^*(A_N)\big)$ [Since f^* is 1-1].

That is $f^{-1^*}\left[fn\ int^*\left(f\big(fn\ cl^*(A_N)\big)\right)\right] \leq fn\ int^*\big(cl(A_N)\big)$ (2)

From (1) & (2), we have $f^{-1^*}\left[fn\ int^* cl^*[f^*(A_N)]\right] \leq fn\ int^*\left(cl^*(A_N)\right)$ which implies that $f^*\ f^{-1^*}(fn\ int^*\ cl^*[f^*(A_N)]) \leq f^*\left(fn\ int^*\left(cl^*(A_N)\right)\right)$.

Since f^* is onto, we have $(fn\ int^*\ cl^*[f^*(A_N)]) \leq f^*\left(fn\ int^*\left(cl^*(A_N)\right)\right)$.

Proposition 4.2

If a function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is fuzzy neutrosophic supra continuous and fuzzy neutrosophic supra open function, then for any fuzzy set A_N in (X_N, T^*) $f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) \leq fn\ int^*(cl^*[\ f^*(A_N)])$.

Proof.

Let A_N be any fuzzy set in (X_N, T^*). Then $fn\ int^*\left(cl^*\ (A_N)\right)$ is a fuzzy neutrosophic supra open set is (X_N, T^*). Since f^* is a fuzzy neutrosophic supra open function, $f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right)$ is a fuzzy neutrosophic supra open set is (Y_N, S^*). Now $f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) \leq f^*\left(fn\ cl^*(A_N)\right)$. Since f^* is a continuous, $f^*\left(fn\ cl^*\ (A_N)\right) \leq fn\ cl^*\left(f^*(A_N)\right)$,

Hence we have $f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) \leq fn\ cl^*\left(f^*(A_N)\right)$. But $fn\ int^*\left(cl^*\ f^*(A_N)\right) \leq fn\ cl^*\left(f(A_N)\right)$.

Therefore, we have $f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) \leq fn\ int^*(cl^*[\ f^*(A_N)])$.

Proposition 4.3

If a function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is fuzzy neutrosophic supra continuous and fuzzy neutrosophic supra open 1-1 and onto function, then for any fuzzy set A_N in (X_N, T^*),

$f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) = fn\ int^*(cl^*[\ f(A_N)])$.

Proof.

Let A_N be any fuzzy set in (X_N, T^*). Since f^* is fuzzy neutrosophic supra continuous, 1-1, onto and fuzzy neutrosophic supra open function, by Proposition (4.1), we have

$(fn\ int^*\ cl^*[f^*(A)]) \leq f^*\left(fn\ int^*\left(cl^*(A)\right)\right)$ (1)

Since f is a fuzzy neutrosophic supra continuous and a fuzzy neutrosophic supra open function, by Proposition 4.2, we have $f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) \leq fn\ int^*(cl^*[\ f^*(A_N)])$ (2)

From (1)&(2), we have $f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) = fn\ int^*(cl^*[\ f(A_N)])$

Proposition 4.4

Let the function $f^*: (X_N, T^*) \to (Y_N, S^*)$ from a fuzzy neutrosophic supra topological space (X_N, T^*) into another fuzzy neutrosophic supra topological space (Y_N, S^*) is fuzzy neutrosophic supra continuous fuzzy neutrosophic supra open 1-1 and onto function, then for any fuzzy set A_N in (X_N, T^*), A_N is a fuzzy neutrosophic supra nowhere dense set in (X_N, T^*) if and only if $f^*(A_N)$ is a fuzzy neutrosophic supra nowhere dense set in (Y_N, S^*).

Proof.

Let A_N be a fuzzy neutrosophic supra nowhere dense set is (X_N, T^*). Then, $fn\ int^*\left(cl^*\ (A_N)\right) = 0$. Since f is $fuzzy\ neutrosophic\ supra$ continuous, fuzzy neutrosophic supra open, 1-1 and onto function, by Proposition (4.3), we have, $f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) = fn\ int^*(cl^*[\ f^*(A_N)])$. Then, $fn\ int^*\left(cl^*\ (A_N)\right) = f^*(0) = 0$. Hence $f^*(A_N)$ is a fuzzy neutrosophic supra nowhere dense set in (Y_N, S^*).

Conversely, Let $f^*(A_N)$ be a fuzzy neutrosophic supra nowhere dense set in (Y_N, S^*).

Then, $fn\ int^*\left(cl^*\ (A_N)\right) = 0$. Hence $f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) = fn\ int^*(cl^*[\ f^*(A_N)])$ implies that $f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) = 0$, Therefore $f^{-1^*}f^*\left(fn\ int^*\left(cl^*\ (A_N)\right)\right) = f^{-1^*}(0) = 0$. Since f^* is 1-1, $fn\ int^*\left(cl^*\ (A_N)\right) = 0$, Hence A_N is a fuzzy neutrosophic supra nowhere dense set in (X_N, T^*).

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This collection of research explores the evolving landscape of topology by integrating **fuzzy set theory** with **neutrosophic sets**, which account for membership, non-membership, and indeterminacy degrees. The primary focus of these works is the systematic development and characterization of **Baire spaces** within various fuzzy neutrosophic environments.

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ISBN 978-1-59973-848-2
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